4.1 Synthetic Division

Synthetic Division
Evaluating Polynomial Functions Using the Remainder Theorem
Testing Potential Zeros

Division Algorithm

Let $f(x)$ and $g(x)$ be polynomials with $g(x)$ of lower degree than $f(x)$ and $g(x)$ of degree one or more. There exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = g(x)q(x) + r(x),$$

where either $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $g(x)$.

Synthetic Division

Synthetic division is a shortcut method of performing long division with polynomials.

It is used only when a polynomial is divided by a first-degree binomial of the form $x - k$, where the coefficient of x is 1.

Synthetic Division

Additive inverse → $4\overline{)\,3 \quad -2 \quad 0 \quad -150}$

$$\begin{array}{r|rrrr} 4 & 3 & -2 & 0 & -150 \\ & & 12 & 40 & 160 \\ \hline & 3 & 10 & 40 & 10 \end{array}$$

Signs changed

Quotient → $3x^2 + 10x + 40 + \dfrac{10}{x-4}$ ← Remainder (10)

With synthetic division it is helpful to change the sign of the divisor, so the –4 at the left is changed to 4, which also changes the sign of the numbers in the second row. To compensate for this change, subtraction is changed to addition.

Caution ***To avoid errors, use 0 as the coefficient for any missing terms, including a missing constant, when setting up the division.***

Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Solution Express $x + 2$ in the form $x - k$ by writing it as $x - (-2)$. Use this and the coefficients of the polynomial to obtain

$$-2\overline{)\;5 \quad -6 \quad -28 \quad -2}.$$ ← Coefficients

***x* + 2 leads to –2**

Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Solution Bring down the 5, and multiply:

$-2(5) = -10$

$$\begin{array}{r|rrrr} -2 & 5 & -6 & -28 & -2 \\ & \downarrow & -10 & & \\ \hline & 5 & & & \end{array}$$

Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Solution Add –6 and –10 to obtain –16. Multiply –2(–16) = 32.

$$\begin{array}{r|rrrr} -2 & 5 & -6 & -28 & -2 \\ & & -10 & 32 & \\ \hline & 5 & -16 & & \end{array}$$

Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Solution Add –28 and 32 to obtain 4. Finally, –2(4) = – 8.

$$\begin{array}{r|rrrr} -2 & 5 & -6 & -28 & -2 \\ & & -10 & 32 & -8 \\ \hline & 5 & -16 & 4 & \end{array}$$

Add columns. Watch your signs.

▶ Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Solution Add –2 and –8 to obtain –10.

-2	5	–6	–28	–2
		–10	32	–8
	5	–16	4	–10 ← Remainder

Quotient: 5 –16 4

Example 1

USING SYNTHETIC DIVISION

Use synthetic division to divide

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}.$$

Since the divisor $x - k$ has degree 1, the degree of the quotient will always be written one less than the degree of the polynomial to be divided. Thus,

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2} = 5x^2 - 16x + 4 + \frac{-10}{x + 2}.$$

Remember to add $\frac{\text{remainder}}{\text{divisor}}$.

Special Case of the Division Algorithm

For any polynomial $f(x)$ and any complex number k, there exists a unique polynomial $q(x)$ and number r such that

$$f(x) = (x - k)q(x) + r.$$

For Example

In the synthetic division in Example 1,

$$\underbrace{5x^3 - 6x^2 - 28x - 2}_{f(x)} = \underbrace{(x+2)}_{=(x-k)}\underbrace{(5x^2 - 16x + 4)}_{q(x)} + \underbrace{(-10)}_{+\ r}.$$

Here $g(x)$ is the first-degree polynomial $x - k$.

Remainder Theorem

If the polynomial $f(x)$ is divided by $x - k$, the remainder is equal to $f(k)$.

Remainder Theorem

A simpler way to find the value of a polynomial is often by using synthetic division. By the remainder theorem, instead of replacing x by –2 to find $f(-2)$, divide $f(x)$ by $x + 2$ using synthetic division as in Example 1. Then $f(-2)$ is the remainder, –10.

$$\begin{array}{r|rrrr} -2 & 5 & -6 & -28 & -2 \\ & & -10 & 32 & -8 \\ \hline & 5 & -16 & 4 & -10 \end{array} \longleftarrow f(-2)$$

Example 2 **APPLYING THE REMAINDER THEOREM**

Let $f(x) = -x^4 + 3x^2 - 4x - 5$. Use the remainder theorem to find $f(-3)$.

Solution Use synthetic division with $k = -3$.

$$\begin{array}{r|rrrrr} -3 & -1 & 0 & 3 & -4 & -5 \\ & & 3 & -9 & 18 & -42 \\ \hline & -1 & 3 & -6 & 14 & -47 \end{array} \leftarrow \text{Remainder}$$

By this result, $f(-3) = -47$.

Testing Potential Zeros

A **zero** of a polynomial function f is a number k such that $f(k) = 0$. **The real number zeros are the *x*-intercepts of the graph of the function.**

The remainder theorem gives a quick way to decide if a number k is a zero of the polynomial function defined by $f(x)$. Use synthetic division to find $f(k)$; if the remainder is 0, then $f(k) = 0$ and k is a zero of $f(x)$. A zero of $f(x)$ is called a **root** or **solution** of the equation $f(x) = 0$.

Example 3 DECIDING WHETHER A NUMBER IS A ZERO

Decide whether the given number k is a zero of $f(x)$.

a. $f(x) = x^3 - 4x^2 + 9x - 6; \quad k = 1$

Solution

Proposed zero →

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 9 & -6 \\ & & 1 & -3 & 6 \\ \hline & 1 & -3 & 6 & 0 \end{array}$$

← $f(x) = x^3 - 4x^2 + 9x - 6$

← Remainder

Since the remainder is 0, $f(1) = 0$, and 1 is a zero of the polynomial function defined by $f(x) = x^3 - 4x^2 + 9x - 6$. An x-intercept of the graph $f(x)$ is 1, so the graph includes the point (1, 0).

Example 3 DECIDING WHETHER A NUMBER IS A ZERO

Decide whether the given number k is a zero of $f(x)$.

b. $f(x) = x^4 + x^2 - 3x + 1; \quad k = -4$

Solution Remember to use 0 as coefficient for the missing x^3-term in the synthetic division.

Proposed zero →

$$
\begin{array}{r|rrrrr}
-4 & 1 & 0 & 1 & -3 & 1 \\
 & & -4 & 16 & -68 & 284 \\
\hline
 & 1 & -4 & 17 & -71 & 285
\end{array}
$$

← Remainder

The remainder is not 0, so –4 is not a zero of $f(x) = x^4 + x^2 - 3x + 1$. In fact, $f(-4) = 285$, indicating that (–4, 285) is on the graph of $f(x)$.