1.4 Quadratic Equations

Solving a Quadratic Equation

Completing the Square

The Quadratic Formula

Quadratic Equation in One Variable

An equation that can be written in the form

$$ax^2 + bx + c = 0$$

where *a*, *b*, and *c* are real numbers with $a \neq 0$, is a **quadratic equation**. The given form is called **standard form**.

Second-degree Equation

A quadratic equation is a **second-degree equation**.

This is an equation with a squared variable term and no terms of greater degree.

$$x^2 = 25, \quad 4x^2 + 4x - 5 = 0, \quad 3x^2 = 4x - 8$$

Zero-Factor Property

If a and b are complex numbers with $ab = 0$, then $a = 0$ or $b = 0$ or both.

Example 1

USING THE ZERO-FACTOR PROPERTY

Solve $6x^2 + 7x = 3$

Solution:

$$6x^2 + 7x = 3$$

$$6X^2 + 7X - 3 = 0$$ Standard form

$$(3x - 1)(2x + 3) = 0$$ Factor.

$$3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$ Zero-factor property.

Example 1

USING THE ZERO-FACTOR PROPERTY

Solve $6x^2 + 7x = 3$

Solution:

$$3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$

Zero-factor property.

$$3x = 1 \quad \text{or} \quad 2x = -3$$

Solve each equation.

$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

Square Root Property

If $x^2 = k$, then

$$x = \sqrt{k} \quad or \quad x = -\sqrt{k}$$

Example 2

USING THE SQUARE ROOT PROPERTY

Solve each quadratic equation.

a. $x^2 = 17$

Solution:

By the square root property, the solution set is

$$\left\{\pm\sqrt{17}\right\}$$

Example 2

USING THE SQUARE ROOT PROPERTY

Solve each quadratic equation.

b. $x^2 = -25$

Solution:

Since $\sqrt{-1} = i,$

the solution set of $x^2 = -25$

is $\{\pm 5i\}.$

Example 2

USING THE SQUARE ROOT PROPERTY

Solve each quadratic equation.

c. $(x-4)^2 = 12$

Solution:

Use a generalization of the square root property.

$$(x-4)^2 = 12$$

$$x - 4 = \pm\sqrt{12}$$ Generalized square root property.

$$x = 4 \pm \sqrt{12}$$ Add 4.

$$x = 4 \pm 2\sqrt{3}$$ $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

Solving A Quadratic Equation By Completing The Square

To solve $ax^2 + bx + c = 0$, by completing the square:

Step 1 If $a \neq 1$, divide both sides of the equation by a.

Step 2 Rewrite the equation so that the constant term is alone on one side of the equality symbol.

Step 3 Square half the coefficient of x, and add this square to both sides of the equation.

Step 4 Factor the resulting trinomial as a perfect square and combine like terms on the other side.

Step 5 Use the square root property to complete the solution.

Example 3

USING THE METHOD OF COMPLETING THE SQUARE $a = 1$

Solve $x^2 - 4x - 14 = 0$ by completing the square.

Solution

Step 1 This step is not necessary since $a = 1$.

Step 2 $x^2 - 4x = 14$

Add 14 to both sides.

Step 3 $x^2 - 4x + 4 = 14 + 4$

$\left[\frac{1}{2}(-4)\right]^2 = 4$; add 4 to both sides.

Step 4 $(x - 2)^2 = 18$

Factor; combine terms.

Example 3

USING THE METHOD OF COMPLETING THE SQUARE $a = 1$

Solve $x^2 - 4x - 14 = 0$ by completing the square.

Solution

Step 4 $(x-2)^2 = 18$ Factor; combine terms.

Step 5 $x - 2 = \pm\sqrt{18}$ Square root property.

Take *both* roots.

$x = 2 \pm \sqrt{18}$ Add 2.

$x = 2 \pm 3\sqrt{2}$ Simplify the radical.

The solution set is $\left\{2 \pm 3\sqrt{2}\right\}$.

Example 4

USING THE METHOD OF COMPLETING THE SQUARE $a \neq 1$

Solve $9x^2 - 12x + 9 = 0$ by completing the square.

Solution

$$9x^2 - 12x + 9 = 0$$

$$x^2 - \frac{4}{3}x + 1 = 0 \qquad \text{Divide by 9. (Step 1)}$$

$$x^2 - \frac{4}{3}x = -1 \qquad \text{Add } -1\text{. (Step 2)}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = -1 + \frac{4}{9} \qquad \left[\frac{1}{2}\left(-\frac{4}{3}\right)\right]^2 = \frac{4}{9}; \text{ add } \frac{4}{9}$$

Example 4

USING THE METHOD OF COMPLETING THE SQUARE $a = 1$

Solve $9x^2 - 12x + 9 = 0$ by completing the square.

Solution

$$x^2 - \frac{4}{3}x + \frac{4}{9} = -1 + \frac{4}{9}$$

$\left[\frac{1}{2}\left(-\frac{4}{3}\right)\right]^2 = \frac{4}{9}$; add $\frac{4}{9}$

$$\left(x - \frac{2}{3}\right)^2 = -\frac{5}{9}$$

Factor, combine terms. (Step 4)

$$x - \frac{2}{3} = \pm\sqrt{-\frac{5}{9}}$$

Square root property

Example 4

USING THE METHOD OF COMPLETING THE SQUARE *a* = 1

Solve $9x^2 - 12x + 9 = 0$ by completing the square.

Solution

$$x - \frac{2}{3} = \pm\sqrt{-\frac{5}{9}}$$

Square root property

$$x - \frac{2}{3} = \pm\frac{\sqrt{5}}{3}i$$

$\sqrt{-a} = i\sqrt{a}$

Quotient rule for radicals

$$x = \frac{2}{3} \pm \frac{\sqrt{5}}{3}i$$

Add ⅔.

Example 4 **USING THE METHOD OF COMPLETING THE SQUARE a = 1**

Solve $9x^2 - 12x + 9 = 0$ by completing the square.

Solution

$$x = \frac{2}{3} \pm \frac{\sqrt{5}}{3}i$$

Add ⅔.

The solution set is $\left\{\frac{2}{3} \pm \frac{\sqrt{5}}{3}i\right\}$.

Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 5

USING THE QUADRATIC FORMULA (REAL SOLUTIONS)

Solve $x^2 - 4x = -2$

Solution:

$$x^2 - 4x + 2 = 0$$

Write in standard form.

Here $a = 1$, $b = -4$, $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula.

Example 5

USING THE QUADRATIC FORMULA (REAL SOLUTIONS)

Solve $x^2 - 4x = -2$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula.

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

The fraction bar extends under *–b.*

Example 5

USING THE QUADRATIC FORMULA (REAL SOLUTIONS)

Solve $x^2 - 4x = -2$

Solution:

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

The fraction bar extends under –*b*.

$$= \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$\sqrt{16 - 8} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

Example 5

USING THE QUADRATIC FORMULA (REAL SOLUTIONS)

Solve $x^2 - 4x = -2$

Solution:

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$\sqrt{16-8} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

$$= \frac{2\left(2 \pm \sqrt{2}\right)}{2}$$

Factor out 2 in the numerator.

Factor first, then divide.

$$= 2 \pm \sqrt{2}$$

Lowest terms.

The solution set is $\left\{2 \pm \sqrt{2}\right\}$.

Example 6 USING THE QUADRATIC FORMULA (NONREAL COMPLEX SOLUTIONS)

Solve $2x^2 = x - 4$.

Solution:

$$2x^2 - x + 4 = 0$$ Write in standard form.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(4)}}{2(2)}$$ Quadratic formula; $a = 2$, $b = -1$, $c = 4$

Use parentheses and substitute carefully to avoid errors.

$$= \frac{1 \pm \sqrt{1 - 32}}{4}$$

Example 6 **USING THE QUADRATIC FORMULA (NONREAL COMPLEX SOLUTIONS)**

Solve $2x^2 = x - 4$.

Solution:

$$= \frac{1 \pm \sqrt{1 - 32}}{4}$$

$$x = \frac{1 \pm \sqrt{-31}}{4} \qquad \sqrt{-1} = i$$

The solution set is $\left\{ \frac{1}{4} \pm \frac{\sqrt{31}}{4} i \right\}$.