10TH EDITION

COLLEGE ALGEBRA

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Education

1.1 Linear Equations

Basic Terminology of Equations
Solving Linear Equations
Identities

Equations

An equation is a statement that two expressions are equal.

$x + 2 = 9$ $11x = 5x + 6x$ $x^2 - 2x - 1 = 0$

To solve an equation means to find all numbers that make the equation true. The numbers are called **solutions** or **roots** of the equation. A number that is a solution of an equation is said to *satisfy* the equation, and the solutions of an equation make up its **solution set**. Equations with the same solution set are **equivalent equations**.

Addition and Multiplication Properties of Equality

For real numbers *a*, *b*, and *c*:

If $a = b$, then $a + c = b + c$.

That is, the same number may be added to both sides of an equation without changing the solution set.

Addition and Multiplication Properties of Equality

For real numbers *a*, *b*, and *c*:

If $a = b$ and $c \neq 0$, then $ac = bc$.

That is, both sides of an equation may be multiplied by the same nonzero number without changing the solution set.

Linear Equation in One Variable

A linear equation in one variable is an equation that can be written in the form

$$\boldsymbol{ax + b = 0,}$$

where a and b are real numbers with $a \neq 0$.

Example 1 **SOLVING A LINEAR EQUATION**

Solve $3(2x-4)=7-(x+5)$

Solution: $3(2x-4)=7-(x+5)$

Be careful with signs.

$$6x-12=7-x-5$$ Distributive Property

$$6x-12=2-x$$ Combine terms.

$$6x-12+x=2-x+x$$ Add *x* to each side.

$$7x-12=2$$ Combine terms.

$$7x-12+12=2+12$$ Add 12 to each side.

$$7x=14$$ Combine terms.

Divide each side by 7.

$$\frac{7x}{7}=\frac{14}{7}$$

$$x=2$$

Example 1

SOLVING A LINEAR EQUATION

Check:

A check of the solution is recommended.

$$3(2x-4)=7-(x+5)$$ Original equation

$$3(2\cdot 2-4)=7-(2+5)$$? Let $x = 2$.

$$3(4-4)=7-(7)$$?

$$0=0$$ True

Example 2

CLEARING FRACTIONS BEFORE SOLVING A LINEAR EQUATION

Solve $\frac{2t+4}{3}+\frac{1}{2}t=\frac{1}{4}t-\frac{7}{3}$

Solution:

$$12\left(\frac{2t+4}{3}+\frac{1}{2}t\right)=12\left(\frac{1}{4}t-\frac{7}{3}\right)$$

Multiply by 12, the LCD of the fractions.

Distribute the 12 to *all* terms within parentheses!

$$4(2t+4)+6t=3t-28$$ Distributive property

$$8t+16+6t=3t-28$$ Distributive property

$$14t+16=3t-28$$ Combine terms.

Example 2

CLEARING FRACTIONS BEFORE SOLVING A LINEAR EQUATION

Solve $$\frac{2t+4}{3}+\frac{1}{2}t=\frac{1}{4}t-\frac{7}{3}$$

Solution:

$$14t+16=3t-28$$ Combine terms.

$$11t=-44$$ Subtract 3*t*; subtract 16.

$$t=-4$$ Divide by 11.

Identities, Conditional Equations, and Contradictions

An equation satisfied by every number that is a meaningful replacement for the variable is called an **identity**.

$$3(x+1) = 3x + 3$$

Identities, Conditional Equations, and Contradictions

An equation satisfied by some numbers but not others, such as $2x = 4$, is called a **conditional equation**.

$$2x = 4$$

Identities, Conditional Equations, and Contradictions

An equation that has no solution, such as $x = x + 1$, is called a **contradiction**.

$$x = x + 1$$

Example 3

IDENTIFYING TYPES OF EQUATIONS

Decide whether this equation is an identity, a conditional equation, or a contradiction.

a. $-2(x+4)+3x=x-8$

Solution: $-2(x+4)+3x=x-8$

$-2x-8+3x=x-8$ Distributive property

$x-8=x-8$ Combine terms

$0=0$ Subtract *x* and add 8.

Example 3

IDENTIFYING TYPES OF EQUATIONS

Decide whether this equation is an identity, a conditional equation, or a contradiction.

a. $-2(x+4)+3x = x-8$

Solution:

$0 = 0$ Subtract *x* and add 8.

When a true statement such as 0 = 0 results, the equation is an identity, and the solution set is **{all real numbers}**.

Example 3

IDENTIFYING TYPES OF EQUATIONS

Decide whether this equation is an identity, a conditional equation, or a contradiction.

b. $5x - 4 = 11$

Solution: $5x - 4 = 11$

$5x = 15$ Add 4.

$x = 3$ Divide by 5.

This is a conditional equation, and its solution set is {3}.

Example 3

IDENTIFYING TYPES OF EQUATIONS

Decide whether this equation is an identity, a conditional equation, or a contradiction.

c. $3(3x-1)=9x+7$

Solution: $3(3x-1)=9x+7$

$9x-3=9x+7$ Distributive property

$-3=7$ Subtract 9*x*.

When a false statement such as −3 = 7 results, the equation is a contradiction, and the solution set is the **empty set** or **null set**, symbolized by ϕ .

Identifying Linear Equations as Identities, Conditional Equations, or Contradictions

1. If solving a linear equation leads to a true statement such as $0 = 0$, the equation is an **identity**. Its solution set is **{all real numbers}.**
2. If solving a linear equation leads to a single solution such as $x = 3$, the equation is **conditional**. Its solution set consists of a single element.
3. If solving a linear equation leads to a false statement such as $-3 = 7$, then the equation is a **contradiction**. Its solution set is ϕ.