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On Calculation of Failure Probability for Structures Designed Based on Magnitudes of Historical Event

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Abstract

During their operational life, structures may be subject to various types of live load caused by events such as earthquakes, high speed winds, etc. Given the design life of a structure, the probability for a specific live load to cause a failure depends on the magnitude of the load structure it is designed to withstand (designed load). In this article, methods are developed for calculation of the failure probability for structures designed to withstand loads comparable to historical loads at the site of interest.

Keywords: Failure Probability, Live Load, Designed Load, Extreme Values, Records, Tail Modeling

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1. Introduction

In the design of important structures, consideration of the live loads caused by events such as earthquakes, winds, etc., is of prime importance. The probability that a specific live load causes failure during the life of the structure depends on its designed lifetime and the designed load; that is, the magnitude of the load structure it is designed to withstand. It also depends on material properties as well as the geometry of the structure. See Chowdhury and Rao (2009), Khaleel and Simonen (2009), Madsen et al. (2006), Kolen et al. (2013), Gerrard, and Tsanakas (2011), Koulouriotis, and Botsaris (2015), Bracegirdle and Marshall (2012), Nicolas and Bromwich (2014), Klemenc (2015) and references therein for details. If the designed load, M, is larger than M_1, the magnitude of the largest load in the history of the corresponding site, then calculation of failure probability is not straight forward. For this, the usual approach has been to fit one of the three limiting extreme value distributions to, for example, the yearly maxima of the load considered. See DeHann and Ferreira (2006), Cole (2001), Ahsanullah and Kirmani (2008), and Beirlant et al. (2004) for more recent developments of extreme values and their analysis. Of three extreme value distributions, the type I (Gumbel distribution) has often been preferred despite the fact that it has no upper bound. This is because type III distribution with an upper bound usually leads to unreliable results and, in some cases, to an estimate for the upper bound that is smaller than those that have already occurred. For this case, a method based on modeling the tail of a distribution is presented in this article.

Other problems related to the use of extreme value distribution are:

1. The estimating procedures are complicated and require numerical calculations.

2. Their application requires a moderate or large sample.

3. Since only the largest loads of each period (e.g. a year) are used, information contained, for example, in the second largest events of that period is not utilized. This point is particularly important when the time span of the available data is short.

4. Missing observations corresponding to the years with no events (e.g. earthquake). To overcome this certain arbitrary assumptions are usually made for the periods (years) without recorded data. For example, one popular approach has been to extend the intervals during which extremes were extracted until the amount of missing data became a desired percentage of data. In the case of earthquakes the time intervals were found to vary between 1 to 15 years leading to few maxima and hence inaccurate results, see e.g. Burton (1979).

Now if M is taken to be smaller than M_1 then, in general, estimation of the failure probability could cause similar difficulties. However, for $M = M_1$ that is, designs based on the largest event of the past, the probability calculations are particularly easy. Here, we will present two different methods for this case. We will also present a method for $M = M_2$ or, more generally, $M = M_m$ where M_m is the m^{th} largest event of the past. The latter two cases are useful when there is enough past data so that designs for the second or third largest event of the past satisfy the desired safety requirements. Also, as pointed out earlier, we will also present a method for the case $M > M_1$ utilizing the relevant results of a theory known as threshold theory.

2. *M* = *M₁*, Method 1

This method uses a relatively new development known as the theory of records. The theory has many interesting results. See Ahsanullah (1995), Arnold et al (1998), Glick (1978), and Gulati and Padgett (2003) for details.

Briefly, if we register a set of observations in chronological sequence, the observation X_i will be called a record high or an upper record if it exceeds all previous values in the sequence. If we assume that ties have zero probability, then in a random sequence, X_i is a record high if and only if $X_i = \max(X_1, X_2, \dots X_i)$. Noting that all *i* ranks are equally likely for X_i, the probability for X_i to be an upper record is then $\frac{1}{i}$. Using this, the theoretical expected number of record highs in a chronological sequence of *n* independent and identically distributed observations is the sum of corresponding probabilities; that is,

$$E\{R_n\} = 1 + \frac{1}{2} + \cdots + \frac{1}{n} = \sum_{i=1}^{n} \frac{1}{i}\,.$$

Here, R_n denotes the number of the record highs among the *n* observations. Note that the above also presents the expected number of record lows which can be defined similarly.

The theory of records also has some simple and, in some cases, counter-intuitive results. A simple result relevant to the problem considered here is the following. Consider an initial sequence of n_1 observations, $X_1, X_2, \dots X_{n_1}$ and a further batch of n_2 observations, $X_{n_1+1}, \dots X_{n_1+n_2}$. The probability that this additional batch contains no new record is

$$P\{R_{n_1} = R_{n_1+n_2}\} = P\{\max(X_1, X_2, \dots X_{n_1}) = \max(X_1, \dots X_{n_1+n_2})\} = \frac{n_1}{n_1+n_2},$$

that is, the ratio of the initial sample size divided by the total number of observations. Note that the results mentioned above are distribution free. Another useful result is the following. As (sample size) $n \to \infty$, the frequency of the upper records among observations indexed by $an < i \le bn$ tends to a Poisson count with mean $\ln(b/a)$. For example, for $a = 1$ and $b = 2$ and large *n*

$$P(\text{exactly } k \text{ records among } (n+1)\text{th and } (2n)\text{th observations}) = \frac{(\ln 2)^k}{2k!}\,.$$

To apply these results suppose that we have data on a particular type of load (e.g. wind) for n_1 years and the designed load is M_1, the largest event of the past. Then the probability (reliability) of having no future load greater than M_1 during the next n_2 years (the designed life of the structure) is simply $\frac{n_1}{n_1+n_2}$ provided that the rate of occurrences of the event of interest remains unchanged. Thus, under this method both the failure probability and reliability depend on the amount of past data. For this case the application of the result involving Poisson count provides the same answer. The Poisson count is particularly useful for cases where failure occur as a result of accumulation of several occurrences of the load.

3. $M = M_1$, Method 2

Consider the probability

$$P(X > \mathrm{M}_1) = p,$$

where M_1 is the magnitude of the largest load of the past n_1 loads. Since M_1 is a random variable so is p. Using the properties of the order statistics, it is easy to show that p has a Beta distribution

$$f(p) = n_1(1-p)^{n_1-1}, \qquad 0 \le p \le 1,$$

with mean $\frac{1}{n_1+1}$ and variance $\frac{n_1}{(n_1+1)^2(n_1+2)}$. Since for a relatively large n_1 the variance of p is small (e.g. for n_1= 100, it is less than 10^{-4}), when applying these results we could replace p either by its mean $\frac{1}{n_1+1}$ or its median $1-2^{-\frac{1}{n_1}}$. Proceeding in this way the failure probability corresponding to a designed life of n_2 (e.g years) are respectively $1-\left(\frac{n_1}{n_1+1}\right)^{n_2} \approx 1-e^{-\frac{n_2}{n_1+1}}$ and $1-2^{-\frac{n_2}{n_1}}$. Note that since $\frac{1}{n+1} \ge 1-2^{-\frac{1}{n}}$ the approximation based on the mean will always result in a larger probability. For example, for $n_1 = 100$ and $n_2 = 30$, these probabilities are respectively 0.258 and 0.188. For this example the method based on theory of records gives $\frac{30}{130} = 0.231$. The same is true for other values of n_1 provided that they are greater than n_2. This may serve as a reason to recommend the first method. In fact, the main advantage of the first method is exactness of the result it is based on.

4. Waiting Time Analysis

In the above analysis we only considered the size of the future events. Considering the designed life of a structure we could analyze the failure probability using the waiting time to the next record. This can be done by utilizing an interesting and somehow surprising result regarding the waiting times between records for independent and identically distributed events. The expected value of W_r, the waiting time between the $(r\text{-}1)^{th}$ and r^{th} records is infinite but its median is finite (see e.g. Glick (1978)). Table 1 presents medians together with their ratios for the first few records.

Table1. Medium Waiting Times and Their Ratios for the First Few Records

Record Number r	2	3	4	5	6	7	8
Median (W_r)	4	10	26	69	183	490	1316
$\mathrm{Med}(W_r)/\mathrm{Med}(W_{r-1})$		2.50	2.60	2.65	2.65	2.68	2.69

Additionally,

$$\frac{\text{Median}(W_{r+1})}{\text{Median}(W_r)} \approx e$$

and

$$\ln(W_r)/r \to 1.$$

Also, ln (W_r) is approximately equivalent to the arrival time sequence of a Poisson process.

To apply this result we could estimate the waiting time as the average of times between the observed records.

5. Methods For $M = M_m$

5.1. Theory of Exceedances

The theory of exceedances deals with the number of times a specified threshold such as designed load is exceeded. Assuming independent and identically distributed events (loads) we may wish to determine the probability of *r* exceedances in the next *n* trials. This is clearly a Bernoulli experiment with two possible outcomes: "exceedance" or "not exceedance." Thus, the number of exceedances has a binomial distribution with parameters *n*, $p(x)$, where $p(x)$ is the probability of exceedance of the level *x*.

Note that here $p(x) = P(X > x) = 1 - F(x)$, where $F(x)$ is the distribution function of *X*. Using this, the probability of *r* exceedances of level *x* in the next *n* trials is

$$\binom{n}{r}[1-F(x)]^r F^{n-r}(x) \quad ,0 \le r \le n.$$

Moreover, if rather than a fixed level we make the level *x* dependent on *n*, x_n say, and increase that with *n* in such a way that the following condition is satisfied:

$$\lim_{n\to\infty} n[1-F(x_n)] = \tau; \quad 0 \le \tau \le \infty,$$

then the probabilities of *r* exceedances of level x_n can be approximated by a Poisson distribution with parameter τ.

We can also determine the probability distribution of the number of exceedances in the next *N* trials of the m^{th} largest observation in the past *n* trials. This is useful when a design load smaller than the magnitude of some past events is chosen. Suppose that p_m is the probability of exceedance of the m^{th} largest observation in the past *n* trials, then using the properties of the order statistics, it can be shown that p_m has Beta distribution with density function

$$f(p_m) = \frac{p_m^{m-1}(1-p_m)^{n-m}}{B(m,n-m+1)}; \quad 0 \le p_m \le 1.$$

Using this, and the fact that the probability of r exceedances in the next N trials is binomial with parameters N and p_m, it can be shown that the mean and the variance of the number of exceedances of the m^{th} largest observation in future N trials are respectively:

$$\frac{Nm}{n+1} \quad \text{and} \quad \frac{Nm(n-m+1)(N+n+1)}{(n+1)^2(n+2)}.$$

To demonstrate**,** suppose that in the site of interest the yearly maximum wind speed in miles per hour during the last 40 years has been 70. Then, the mean and variance of the number of exceedances of 70 during the next 30 years would respectively be:

$$30/41 = 0.732 \quad \text{and} \quad (30)\,(40)\,(71)/\,(41)^2(42) = 1.207.$$

If the second largest wind speed was 67, then the mean and variance of the number of exceedances of the 67 in the next 30 years would respectively be:

$$(30)(2)/41 = 1.464 \quad \text{and} \quad (30)\,(2)\,(39)\,(71)/\,(41)^2(42) = 2.353.$$

As a different example consider the yearly maximum wind speed during the last 60 years. Suppose that as a design load we want to choose a value in order to have an average of 4 exceedances of that value in the next 20 years. This is useful when the structure of interest fails with accumulation of stress due to several loads.

Using the formula for the mean, we get: $20m/61 \approx 4 \Rightarrow m \approx 12$. This means that the value to be chosen is the 12th largest wind speed in past data.

Finally, suppose that K is the number of occurrences up to the first exceedance. The possible values for K are 0, 1, 2, …, and we have

$$P(K=k) = \frac{1}{n+1}\left(1-\frac{1}{n+1}\right)^k, \quad k = 0,\ 1,\ 2,\ \ldots.$$

This is a geometric distribution with expected value and variance equal to, respectively:

$$n+1, \qquad n(n+1)\ .$$

This means that in order to have an exceedance we need an average of $n+1$ occurrences.

5.2. Return Periods

Another useful concept regarding the failure probability is return period. Consider an event whose probability of occurrence in a unit period of time (normally one year) is p. Assume that occurrences of such events in different periods are independent. Then we have a sequence of Bernoulli experiments (occurrence or non-occurrence). Also, the time (measured in unit periods) to the first occurrence is a geometric random variable with parameter p and mean of $1/p$. This motivates the following definition.

Definition.

Let A be an event, and T the random time between consecutive occurrences of A. The mean value τ, of the random variable T is called the return period of the event A.

Note that if $F(x)$ is the distribution function of the yearly maximum of a random variable, the return period of that random variable to exceed the value x is $1/[1-F(x)]$ years. Similarly, if $F(x)$ is the distribution function of the yearly maximum of a random variable, the return period of the variable to go below the value x is $1/F(x)$ years.

To demonstrate, suppose that the distribution function of the yearly maximum discharge in cubic meters per second of a river at a given location has the following extreme value distribution;

$$F(x) = exp\left[-exp\left(-\frac{x-38.5}{7.8}\right)\right].$$

The return periods of yearly maximum discharges of 60 and 70 are then:

$$\tau_{60} = 1/\,(1 - F(60)) = 16.25 \text{ years} .$$
$$\tau_{70} = 1/\,(1 - F(70)) = 57.24 \text{ years} .$$

Also to have a return period of 50 years, the design load, s, is 68.94 as it must satisfy the equation $1/\,(1- F(s)) =50$.

6. Methods for $M > M_1$

6.1. Tail Modeling

In this approach the probabilities of future large (small) values are calculated by developing models for the upper tail (lower) of the distribution for possible values (loads). Here, one usually assumes that the tail of the distribution belongs to a given parametric family and proceeds to do inference using excesses; that is, the amount by which large values exceed some predetermined value y_o. In what follows we will focus on large values.

It has been shown that the natural parametric family of distributions to consider for excesses is the generalized Pareto distribution (GPD);

$$P(Y \le y) = 1 - \left(1 - \frac{ky}{\sigma}\right)^{1/k},$$

where Y represents the magnitude of the loads. See for example Pickands (1975) for theoretical foundation of this approach. Here $\sigma > 0$ and $-\infty < k < \infty$ are unknown parameters. The range of Y is $0 < y < \infty$ for $k \leq 0$, and $0 < y < \sigma/k$ for $k > 0$. The limit $k \rightarrow 0$ of the GPD is the exponential distribution. The use of this model was motivated by the following considerations.

- The GPD arises as a class of limit distributions for the excess over a threshold, as the threshold is increased.
- If Y has the distribution $H(y; \sigma, k)$ and $y' > 0$, $\sigma - ky' > 0$, then the conditional distribution of $Y - y'$ given $Y > y'$ is $H(y; \sigma - ky', k)$. This is a 'threshold stability' property; if the threshold is increased by an arbitrary amount y´, then the GPD form of the distribution remains unchanged.
- If N is a Poisson random variable with mean λ and $Y_1, Y_2, \ldots, Y_N$ are independent excesses with distribution $H(y; \sigma, k)$, then the maximum of Y_i's has a generalized extreme value distribution given below

$$P(\max(Y_1, Y_2, \ldots, Y_N) \leq y) = \exp\{-\lambda(1 - ky/\sigma)^{1/k}\}.$$

Thus, if N denotes the number of excesses in, say, a year and $Y_1, Y_2, \ldots, Y_N$ denote the excesses, then the annual maximum has one of the classical extreme value distributions. This is in line with frequent use of extreme value distributions for modeling large loads.

The GPD includes three specific forms:

1. Long tail Pareto distribution.
2. Medium tail exponential distribution.
3. Short tail distribution with an endpoint.

Most classical distributions have tails that behave like one of these three forms.

Turning to application we first note that, like most asymptotic results, application of this approach is not free of difficulties. Here, obvious problems are the choice of a parametric family, determination of the threshold, and more importantly, difficulties of dealing with intractable likelihood equations. For the latter a major problem is the following: The maximum likelihood works well if $k < 1/2$, but goes haywire otherwise, see e.g. Smith (1987). Additionally, like the type III extreme value distribution the use of short tail distribution with an endpoint usually leads to unreliable results and in some cases to an estimate of the endpoint smaller than some values already occurred.

To remedy these difficulties, some suggestions have been made. For example, it is shown that it is possible to obtain a good estimate for the tail using methods that do not appeal to the likelihood principle. Hill (1975) and Davis and Resnick (1984) have proposed one such method for doing this. Their approach is easy to use and is applicable to a wide class of distribution functions possessing medium or long tails. It is also in line with the use of type I extreme value distribution mentioned earlier despite the fact that it has no upper bound. These authors assume a tail model of the form $F(y) = cy^{-a}$, $y > y_0$ and use a random sample $Y_1, Y_2, \ldots, Y_n$ to estimate the parameters based on the upper $m = m(n)$ order statistics (m largest values). Here m is a sequence of integers chosen such that $m \to \infty$ and $m/n \to 0$. In this approach c is estimated using the empirical $1 - m/n$ quantile, $Y_{(m+1)}$ and $1/a$ is estimated by;

$$\hat{a} * (n/m) = m^{-1} \sum_{i=1}^{m} ln\, Y_{(i)} - ln\, Y_{(m+1)} .$$

Statistical theory regarding these estimators is well established. The Pareto-tail estimate $\overline{F}(y)$, representing the upper tail is then:

$$\overline{F}(y) = \frac{m}{n} \left(\frac{y}{Y_{(m+1)}} \right)^{-1/\hat{a}*(n/m)} , \quad y > Y_{(m+1)} .$$

Here, the only problem that remains is the selection of $m(n)$ as there are infinitely many choices. For example, one obvious choice is integer part of n^r where $0 < r < 1$.

Although this is a problem, the situation provides us with an opportunity to utilize further relevant information contained in data, and improve the estimates. Since in most applications data are naturally ordered in time we could, in addition to the original observed values, consider information contained in the records values, their times of occurrences, and the inter-record times (the time between successive records). The last two sequences are particularly relevant and their influence on prediction of future records is clear. In what follows we will present a method that utilizes information contained in the most recent records.

Assume that the data contains r records. Let T_r denote the time between the last and penultimate record values and t_r denote the time the last record has held to date. It can be shown that the following choice proposed by Tata (personal communication) involving T_r and t_r satisfies the conditions $m \to \infty$ and $m/n \to 0$

$$m(n) = \sqrt{eT_r} + \sqrt{t_r} = \sqrt{2.718282T_r} + \sqrt{t_r} .$$

Comparison using simulated data on Beta distribution (unfavorable cases) shows that this is almost always a better choice compared to choices such as, for example, $m(n) = [n^{1/2}]$. Further, the results very much depend on the last value of T_r as expected. In fact, the time between the two latest records being not inordinately large (or small) is fairly essential for the reasonably accurate estimate. This applies to choices such as $m(n) = [n^{1/2}]$ as well, although it does not depend on the T_r. In what follows we describe application of this method using the flood data for Bloomsburg, Pennsylvania.

6.2. Application to Bloomsburg Flood Data

Consider the data representing the historical crest for Susquehanna River at Bloomsburg, Pennsylvania. Susquehanna River in Bloomsburg begins to flood when water level exceeds 19 feet. Since 1850, there have been thirty eight floods exceeding 19 feet. They are listed in table 2 below.

Table 2. Historical crest for Susquehanna River at Bloomsburg, Pennsylvania

32.75 ft on 09/09/2011	25.09 ft on 04/04/2005	22.20 ft on 03/24/1948
32.70 ft on 03/09/1904	24.40 ft on 04/06/1984	22.20 ft on 04/12/1993
31.20 ft on 06/25/1972	24.20 ft on 12/15/1983	22.00 ft on 12/16/1901
28.64 ft on 06/28/2006	24.00 ft on 03/11/1964	21.60 ft on 03/09/1956
28.20 ft on 03/18/1865	23.50 ft on 02/14/1984	21.40 ft on 03/30/2005
27.80 ft on 03/19/2004	23.40 ft on 03/16/1986	21.20 ft on 10/17/1955
27.50 ft on 09/27/1975	23.40 ft on 01/01/1943	21.10 ft on 03/03/1950
27.12 ft on 09/19/2004	23.20 ft on 04/03/1993	20.80 ft on 04/08/1958
26.90 ft on 03/03/1902	22.80 ft on 04/02/1960	20.60 ft on 04/16/1983
26.86 ft on 01/21/1996	22.67 ft on 03/07/1964	20.51 ft on 01/15/2005
25.70 ft on 04/02/1940	22.57 ft on 03/12/2011	20.10 ft on 02/27/1961
25.20 ft on 05/29/1946	22.50 ft on 03/13/1936	19.80 ft on 03/26/1994
25.20 ft on 03/09/1979	22.30 ft on 03/28/1913	

To apply the above method, we need to choose an integer $m(n)$ depending on n such that $m(n) \to \infty$ and $m(n)/n \to 0$ as $n \to \infty$. Using

$$m(n) = \sqrt{eT_r} + \sqrt{t_r} = \sqrt{2.718282T_r} + \sqrt{t_r}$$

and T_r = 107 and t_r = 1 we obtain $m(n)$ = 18 since $y_1 = 32.75, y_2 = 32.70, y_3 = 31.2, \ldots, y_{18} = 23.5$. Using these for $y > y_{18} = 23.5$ we get the following tail model:

$$\bar{F}(y) = 18/38(y/24.2)^{-7.142857}.$$

From this model we can calculate the values of $P(Y > 32.75)$, $P(Y > 33)$ and $P(Y > 34)$ as 0.055, 0.052 and 0.042 respectively. Also, using this model the probability of a record flood in the next 100 years is 67.7%.

7. Conclusion

In the design of important structures, consideration of the live loads such as earthquakes, winds, etc., is of prime importance. The probability that a specific live load will exceed the designed load some time during the life of the structure depends on its designed lifetime and the designed load. If the designed load is

1. larger or smaller than the magnitude of the largest load in the history of a site, then calculation of failure probability is not straight forward.

2. equal to the magnitude of the largest or more generally the m^{th} largest load in the history of a site, then calculation of failure probability is particularly easy.

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