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String Fluid Cosmological Model with Magnetic Field in Bimetric Theory of Gravitation

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Abstract

In this paper, LRS Bianchi type I string fluid cosmological model with magnetic field in bimetric theory of gravitation is investigated by assuming barotropic equation of state for pressure and density and assuming the bulk viscosity to be inversely proportional to the scalar expansion. The source of energy momentum tensor is a bulk viscous fluid containing one dimensional string with electromagnetic field. The physical and geometrical properties of the model are discussed. The bulk viscosity affected the whole properties of the model.

Keywords: Gravitation; Electromagnetic field; Cosmology

MSC 2010 No.: 83-02, 83C50, 83F05

1. Introduction

Recently, a lot of interest is developed in finding the cosmological models related to Rosen's bimetric theory of gravitation. The Rosen's bimetric theory is the theory of gravitation based on two metrics. One is the fundamental metric tensor g_{ij} that describes the gravitational potential and the second metric γ_{ij} refers to the flat space–time and describes the inertial forces associated with the acceleration of the frame of reference. The theory agrees with the present observational facts pertaining to theories of gravitation given by Rosen (1973, 1977), Isrelit (1980) and Karade (1981). Accordingly at every point of space-time, there are two metrics:

$$ds^2 = g_{ij}dx^i dx^j, \tag{1}$$

$$d\eta^2 = \gamma_{ij}dx^i dx^j. \tag{2}$$

The field equations of Rosen's (1973) bimetric theory of gravitation are

$$N_i^j - \frac{1}{2}N\delta_i^j = -8\pi k T_i^j, \tag{3}$$

where $N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj}g_{si}\,|_p\right)\big|_r$, $N = N_i^i$, $k = \sqrt{\frac{g}{\gamma}}$ together with $g = det\ (g_{ij})$ and $\gamma = det(\gamma_{ij})$. Here, the vertical bar (|) stands for γ-covariant differentiation and T_i^j is the energy–momentum tensor of matter fields.

The bimetric theory of gravitation is free from the singularities which occur in general relativity. Several aspects of the bimetric theory of gravitation have been studied by many authors such as Reddy et al. (1998), Katore et al. (2006), Khadekar et al. (2007), Borkar et al. (2009, 2010), Sahoo et al. (2010) and Gaikwad et al. (2011).

As the magnetic field is present in galactic and intergalactic spaces, the various string cosmological models under different physical conditions with and without a magnetic field in general relativity have been investigated by many researchers like Bali et al. (2004, 2007). String fluid cosmological models in general relativity have been studied by Tripathy et al. (2008).

It is realized that the work of Tripathy et al. (2008) can be extended to the bimetric theory of gravitation and therefore, in this paper, we have taken up the study of String fluid cosmological model with magnetic field by solving Rosen's field equations. Here the effect of viscosity and the magnetic field upon the Locally Rotationally Symmetric (LRS) Bianchi type I universe filled with massive string is closely watched.

2. The Metric and Field Equations

We Consider the LRS Bianchi Type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2), \tag{4}$$

where A, B and C are functions of cosmic time t only. The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{5}$$

Assume that the energy momentum tensor containing a bulk viscous fluid with one dimensional strings and electromagnetic field.

$$T_i^j = (\rho + p)u_i u^j + p g_i^j - \lambda x_i x^j + E_i^j - \xi\theta\left(u_i u^j + g_i^j\right), \tag{6}$$

where ρ is the rest energy density of the system, λ is the string tension density and ξ(t) is the time dependent coefficient bulk viscosity, p the pressure of the cosmic fluid and θ is the expansion scalar. $u^i = (\,0, 0, 0, 1\,)$ is the four velocity and choosing $x^i = (\,\frac{1}{A}, 0, 0, 0\,)$ as the direction of the string, such that

$$g_{ij}u^i u^j = -g_{ij}x^i x^j = 1 \text{ and } u^i x_i = 1. \tag{7}$$

Assume that particles are attached to the strings and the energy density for the particles is given by

$$\rho_p = \rho - \lambda. \tag{8}$$

E_{ij} is the part of the energy momentum tensor corresponding to the electromagnetic field and is defined as

$$E_{ij} = \frac{1}{4\pi}\left(g^{sp}F_{is}F_{jp} - \frac{1}{4}g_{ij}F_{sp}F^{sp}\right), \tag{9}$$

where F^{sp} is the electromagnetic field tensor. The magnetic field is taken along the x-direction, so that the only non-vanishing component of F^{sp} is F_{23} . Due to the assumption of finite electrical conductivity, we have $F_{14} = F_{24} = F_{34} = 0$.

From Maxwell's equation, $F_{[ij,k]} = 0$, we write

$$F_{23} = -F_{32} = H = Constant. \tag{10}$$

For the LRS Bianchi Type-I metric considered in equation (4), the components of the energy momentum tensor of electromagnetic field are

$$E_{11} = -\frac{H^2A^2}{8\pi B^4} = -\delta A^2, \tag{11}$$

$$E_{22} = E_{33} = \frac{H^2}{8\pi B^2} = \delta B^2, \tag{12}$$

$$E_{44} = \frac{H^2}{8\pi B^4} = \delta. \tag{13}$$

Equations (6) now yields

$$T_1^1 = (p - \lambda - \delta - \xi\theta), \qquad T_2^2 = T_3^3 = (p + \delta - \xi\theta), \qquad T_4^4 = (\rho + \delta). \tag{14}$$

So that Rosen's field equation (3) leads to

$$-\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{2\ddot{B}}{B} - \frac{2\dot{B}^2}{B^2} = 16\pi AB^2(-p + \lambda + \delta + \xi\theta), \tag{15}$$

$$\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} = 16\pi AB^2(-p - \delta + \xi\theta), \tag{16}$$

$$\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} + \frac{2\ddot{B}}{B} - \frac{2\dot{B}^2}{B^2} = 16\pi AB^2(\rho + \delta), \tag{17}$$

where $\dot{A} = \frac{\partial A}{\partial t}$, $\dot{B} = \frac{\partial B}{\partial t}$, $\ddot{A} = \frac{\partial^2 A}{\partial t^2}$, $\ddot{B} = \frac{\partial^2 B}{\partial t^2}$. Denoting $\alpha = \frac{\dot{A}}{A}$ and $\beta = \frac{\dot{B}}{B}$ the field equations (15) to (17) can be reduced to

$$\dot{\alpha} + 2\alpha^2 + 2\dot{\beta} = 16\pi e^{\int(\alpha+2\beta)dt}(-p + \lambda + \delta + \xi\theta), \tag{18}$$

$$\dot{\alpha} = 16\pi e^{\int(\alpha+2\beta)dt}(-p - \delta + \xi\theta), \tag{19}$$

$$\dot{\alpha} + 2\dot{\beta} = 16\pi e^{\int(\alpha+2\beta)dt}(\rho + \delta). \tag{20}$$

3. Solution of Field Equations with Physical Significance

These equations (18-20) are three differential equations in seven unknowns $\alpha, \beta, \rho, p, \lambda, \xi$ and θ.Therefore, in order to have solution, we assume four conditions: first we assume a linear relationship between α and β as

$$\alpha = k\beta, \quad k > 0. \tag{21}$$

Second, the coefficient of viscosity varies with the expansion scalar such that [Saha (2005) and Bali et al. (2007)]

$$\xi\theta = \xi_0 = \text{constant}. \tag{22}$$

The third assuming condition is the proper rest energy density ρ and the string tension density λ satisfies the equation $\rho = \gamma\lambda$, where γ is an arbitrary constant. The fourth assuming condition, for a baratropic cosmological fluid is $p = \epsilon\rho$, $0 \leq \epsilon \leq 1$. Using the four conditions stated above for a set of three differential equations (18)-(20), by straight forward calculations, we have

$$\frac{1}{16\pi e^{\int \theta dt}}\left[M\dot{\beta} + N\beta^2\right] = Q,$$

or

$$\frac{1}{L^2}\left[M\dot{\beta}+N\beta^2\right]=Q, \tag{23}$$

where,

$$16\pi e^{\int \theta dt}=L^2$$

and

$$M=2k\epsilon\gamma+(2k+2)\gamma-(2k+1), \tag{24}$$

$$N=-2k^2\epsilon\gamma+2k^2\gamma, \tag{25}$$

$$Q=(2\gamma-1)\xi_0. \tag{26}$$

The expansion scalar θ and the shear scalar σ for the metric given by

$$\theta=u^l{}_{|l}=\alpha+2\beta, \tag{27}$$

$$\sigma^2=\frac{1}{2}\left(\alpha^2+2\beta^2-\frac{\theta^2}{3}\right)=\left[\frac{1}{3}\frac{(k-1)^2}{(k+2)^2}\right]\theta^2. \tag{28}$$

The energy density corresponding to the particles loaded on the strings is

$$\rho_p=\rho-\lambda=(\gamma-1)\lambda\,.$$

There are three possible cases in equation (23) depending upon the values of M, N and Q

Case I: For $N\neq 0$, and $M\neq 0$, we have (a) $\frac{Q}{N}>0$ or (b) $\frac{Q}{N}<0$.

Case I (a) Assuming $\frac{Q}{N}>0$ and taking $\frac{Q}{N}=\mu^2$, then equation (23) gives

$$\frac{\dot{\beta}}{\beta^2-L^2\frac{Q}{N}}=-\frac{N}{M}, \tag{29}$$

from which, we obtain

$$\beta=\mu+\frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}. \tag{30}$$

Using $\alpha=\frac{\dot{A}}{A}$ and $\alpha=k\beta$, $k>0$, from above equation (30), we have

$$A=a_1 e^{\mu kt}\left(1-b_1 e^{-2L\mu Nt/M}\right)^{Mk}/_{LN}\,, \tag{31}$$

$$B = b_1 e^{\mu t}(1 - be^{-2L\mu Nt/M})^{M/_{LN}}, \tag{32}$$

where $a_1, b_1, b > 0$ and $b \neq 1$ are constants. Choosing $a_1 = b_1 = 1$, the metric (4) can be written a

$$ds^2 = -dt^2 + \left(1 - b_1 e^{-\frac{2L\mu Nt}{M}}\right)^{\frac{2MK}{LN}} e^{2\mu kt} dx^2 + \left(1 - b_1 e^{-\frac{2L\mu Nt}{M}}\right)^{\frac{2M}{LN}} e^{2\mu t}(dy^2 + dz^2). \tag{33}$$

The electromagnetic field, string tension density λ, the energy density ρ, the isotropic pressure p, the particle density ρ_p, the scalar of expansion θ, shear tensor σ and the bulk viscosity ξ, the spatial volume V for the model (33) are

$$\delta = \frac{H^2}{8\pi(1-b_1 e^{-2L\mu Nt/M})^{4M/_{LN}} \; e^{4\mu t}}, \tag{34}$$

$$\lambda = (\rho/\gamma) = \left(\rho_p/\gamma - 1\right) = (p/\epsilon\gamma),$$

$$\lambda = \left(\frac{1}{\gamma-1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2+2)\right]\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right)^2\right\}, \tag{35}$$

$$\rho = \left(\frac{\gamma}{\gamma-1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2+2)\right]\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right)^2\right\}, \tag{36}$$

$$p = \left(\frac{\epsilon\gamma}{\gamma-1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2+2)\right]\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right)^2\right\}, \tag{37}$$

$$\rho_p = \left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2+2)\right]\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right)^2\right\}, \tag{38}$$

$$\theta = (k+2)\beta = (k+2)\left(\mu + \frac{2\mu b_1 e^{-\frac{2L\mu Nt}{M}}}{1-b_1 e^{-\frac{2L\mu Nt}{M}}}\right), \tag{39}$$

$$\sigma = \frac{1}{\sqrt{3}}(k-1)\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right), \tag{40}$$

$$\xi = \frac{\xi_0}{(k+2)\left(\mu + \frac{2\mu b_1 e^{-2L\mu Nt/M}}{1-b_1 e^{-2L\mu Nt/M}}\right)}, \tag{41}$$

and

$$V = \sqrt{-g} = AB^2 = \quad e^{(k+2)\mu t}\left(1 - b_1 e^{-2L\mu Nt/M}\right)^{(k+2)\frac{M}{LN}}, \tag{42}$$

respectively.

In this case for $\frac{Q}{N} > 0$, the value of δ is finite, as $t \to 0$ and as $t \to \infty, \delta \to 0$. So that the magnetic field in the model disappear for very large value of cosmic time t. ρ, p, ρ_p, λ all are finite when $t \to 0$ and decrease when t increases. The coefficient of bulk viscosity $\xi = \frac{\xi_0}{\theta}$ exists for a whole range of cosmic time t. The model has shear for $k \neq 1$ and for $k = 1$, shear vanishes. μ, θ are positive, we infer that universe is expanding without affecting the magnetic field and the expansion rate decreasing with increasing time. All properties of the model are affected by the factor Q only. The value $\frac{\sigma}{\theta} \neq 0$ for $k \neq 1$ and therefore the model does not isotropize and for $k = 1$, it isotropizes for whole t.

Case I (b) Assuming $\frac{Q}{N} < 0$ and taking $\frac{Q}{N} = -\nu^2$. Here,

$$\beta = L\, \nu\, tan\left(\frac{LQ}{M\nu} t + k_1\right), \tag{43}$$

which gives

$$A = a_2 \left[cos\left(\frac{LQ}{M\nu} t + k_1\right)\right]^{kM/N}, \tag{44}$$

$$B = b_2 \left[cos\left(\frac{LQ}{M\nu} t + k_1\right)\right]^{M/N}, \tag{45}$$

where $a_2,\ b_2, k_1$ are constants.

Choosing the coordinates $x = X, y = Y, z = Z$ and $\left(\frac{Q}{M\nu} t + k_1\right) = \frac{Q}{M\nu} T$, then the model (4) takes the form

$$ds^2 = -dt^2 + \left[cos\left(\frac{LQ}{M\nu} T\right)\right]^{2kM/N} dX^2 + \left[cos\left(\frac{LQ}{M\nu} T\right)\right]^{2M/N} (dY^2 + dZ^2). \tag{46}$$

The physical quantities of the model are

$$\delta = \frac{H^2}{8\pi\left[cos\left(\frac{LQ}{M\nu} T\right)\right]^{4M/N}}, \tag{47}$$

$$\lambda = \left(\frac{1}{\gamma - 1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2 + 2)\right]\left[L\, \nu\, tan\left(\frac{LQ}{M\nu} T\right)\right]^2\right\}, \tag{48}$$

$$\rho = \left(\frac{\gamma}{\gamma - 1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2 + 2)\right]\left[L\, \nu\, tan\left(\frac{LQ}{M\nu} T\right)\right]^2\right\}, \tag{49}$$

$$p = \left(\frac{\epsilon\gamma}{\gamma - 1}\right)\left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2 + 2)\right]\left[L\, \nu\, tan\left(\frac{LQ}{M\nu} T\right)\right]^2\right\}, \tag{50}$$

$$\rho_p = \left\{(k+1)\frac{Q}{M} - \frac{1}{L^2}\left[(k+1)\frac{N}{M} + (2k^2+2)\right]\left[L\,\nu\,tan\left(\frac{LQ}{M\nu}T\right)\right]^2\right\}, \tag{51}$$

$$\theta = (k+2)\beta = (k+2)\left[L\,\nu\,tan\left(\frac{LQ}{M\nu}T\right)\right], \tag{52}$$

$$\sigma = \frac{1}{\sqrt{3}}(k-1)\left[L\,\nu\,tan\left(\frac{LQ}{M\nu}T\right)\right], \tag{53}$$

$$\xi = \frac{\xi_0}{(k+2)\left[L\,\nu\,tan\left(\frac{LQ}{M\nu}T\right)\right]}, \tag{54}$$

$$V = \sqrt{-g} = AB^2 = \left[cos\left(\frac{LQ}{M\nu}T\right)\right]^{(k+2)M/N}. \tag{55}$$

In this case $\frac{Q}{N} < 0$, it is seen that the physical quantities $\delta, \lambda, \rho, p, \rho_p$ all are having finite values and changing harmonically with the growth of cosmic time, since the cosine function is present in it. Therefore our model is representing the oscillatory universe. The bulk viscosity affecting the physical quantities of the model and the model is shearing for $k \neq 1$. The shear in the model disappears when $k = 1$. The value $\frac{\sigma}{\theta} \neq 0$ for $k \neq 1$ suggests the model does not isotropize, and it isotropizes for $k = 1$.

Case II: For $N = 0$, and $M \neq 0$, the equation (23) reduces to $\frac{1}{L^2}M\dot{\beta} = Q$, and then equation (25) gives $\gamma = 0$ or $\epsilon = 1$. So that the equation $\frac{1}{L^2}M\dot{\beta} = Q$, yields

$$\frac{\dot{B}}{B} = \beta = \left(\frac{L^2Q}{M}t + k_2\right), \tag{56}$$

On integrating equation (56), we get

$$B = b_3\,exp.\left(\frac{L^2Q}{2M}t^2 + k_2t\right), \tag{57}$$

$$A = a_3\,exp.\left(\frac{kL^2Q}{2M}t^2 + kk_2t\right), \tag{58}$$

where

a_3, b_3, k_2

are constants. With this choice of $a_3 = b_3 = k_2 = 1$, the metric having the form

$$ds^2 = -dt^2 + \left[exp.\left(\frac{kL^2Q}{M}t^2 + 2kt\right)\right]dx^2 + \left[exp.\left(\frac{L^2Q}{M}t^2 + 2t\right)\right](dy^2 + dz^2). \tag{59}$$

In case II, for $N = 0$, and $M \neq 0$, we have $\gamma = 0$ or $\epsilon = 1$.

For $\gamma = 0$, we write

$$\delta = \frac{H^2}{8\pi \, exp.\left[4\left(\frac{L^2 Q}{2M} t^2 + t\right)\right]}, \tag{60}$$

$$\rho_p = -\lambda = \left\{(k+1)\frac{Q}{M} - \frac{2}{L^2}(k^2+1)\left(\frac{L^2 Q}{M} t + 1\right)^2\right\}, \tag{61}$$

$$\rho = p = 0. \tag{62}$$

For

$$\epsilon = 1,$$

$$\delta = \frac{H^2}{8\pi \, exp.\left[4\left(\frac{L^2 Q}{2M} t^2 + t\right)\right]}, \tag{63}$$

$$p = \rho = \gamma\lambda = \left(\frac{\gamma}{\gamma - 1}\right)\left\{(k+1)\frac{Q}{M} - \frac{2}{L^2}(k^2+1)\left(\frac{L^2 Q}{M} t + 1\right)^2\right\}, \tag{64}$$

$$\rho_p = \left\{(k+1)\frac{Q}{M} - \frac{2}{L^2}(k^2+1)\left(\frac{L^2 Q}{M} t + 1\right)^2\right\}, \tag{65}$$

and

$$\theta = (k+2)\beta = (k+2)\left(\frac{L^2 Q}{M} t + 1\right), \tag{66}$$

$$\sigma = \frac{1}{\sqrt{3}}(k-1)\left(\frac{L^2 Q}{M} t + 1\right), \tag{67}$$

$$\xi = \frac{\xi_0}{(k+2)\left(\frac{L^2 Q}{M} t + 1\right)}, \tag{68}$$

$$V = \sqrt{-g} = AB^2 = exp.\left[\left(\frac{k+2}{2}\right)\frac{L^2 Q}{M} t^2 + (k+2)t\right]. \tag{69}$$

In this case, for $N = 0$, and $M \neq 0$, we have $\gamma = 0$ or $\epsilon = 1$. For $\gamma = 0$, our model becomes a vacuum universe with magnetic field and the particle density is the negative of the string tension density in it. For $\epsilon = 1$, the model having magnetic field with pressure equal to the energy density. In both cases, the magnetic field is present in the beginning and then its effect gradually decreases with an increase in the cosmic time and it disappears completely when $t \longrightarrow \infty$. The universe expanding in nature and the bulk viscosity in the model decreases with an increase in time and finally vanishes for a very large value of t. The model has shearing and the shear continuously increases with time for $k \neq 1$ and for $k = 1$, the shear vanishes. Here it is noted that the bulk viscosity plays the major role. If it is zero then $Q = 0$ and in this case our model (59) takes the form

$$ds^2 = -dt^2 + [exp.(2kt)]dx^2 + [exp.(2t)](dy^2 + dz^2). \tag{70}$$

Its physical behavior is observed by the following quantities.

In the absence of bulk viscosity, from equation (26), $\xi_0 = 0$ gives $Q = 0$.

For $\gamma = 0$, we write

$$\delta = \frac{H^2}{8\pi\,[exp.(4t)]}, \tag{71}$$

$$\rho_p = -\lambda = -\frac{2}{L^2}(k^2 + 1), \tag{72}$$

$$\rho = p = 0, \tag{73}$$

and for $\epsilon = 1$, we get

$$\delta = \frac{H^2}{8\pi\,[exp.(4t)]}, \tag{74}$$

$$p = \rho = \gamma\lambda = \left(\frac{\gamma}{\gamma-1}\right)\rho_p = \left(\frac{\gamma}{\gamma-1}\right)\left\{-\frac{2}{L^2}(k^2 + 1)\right\}. \tag{75}$$

The values of θ, σ, ξ and V will be

$$\theta = (k + 2), \tag{76}$$

$$\sigma = \frac{1}{\sqrt{3}}(k - 1), \tag{77}$$

$$V = \sqrt{-g} = AB^2 = exp.[(k + 2)t]. \tag{78}$$

In the absence of bulk viscosity, from equations (71-78), it is observed that the physical behavior of the model is the same as that of the model in the presence of bulk viscosity except the fact that the expansion in the model is stripped off, if $\xi = 0$.

Case III: For $N \neq 0$, and $M = 0$, the equation (23) reduces to gives that, $\beta^2 = L^2\frac{Q}{N}$.

For $\frac{Q}{N} > 0$, we have

$$\beta = \pm\, L\sqrt{\frac{Q}{N}}, \tag{79}$$

On integrating equation (79), we get

$$B = b_4\, exp.\left(\pm\, Lt\sqrt{\frac{Q}{N}}\right), \tag{80}$$

$$A = a_4\, exp.\left(\pm\, kLt\sqrt{\frac{Q}{N}}\right), \tag{81}$$

where a_4, b_4 are constants.

With the choice of $a_4 = b_4 = 1$, the metric (4) has the form

$$ds^2 = -dt^2 + \left[exp.\left(\pm 2\, kL\sqrt{\frac{Q}{N}}\ t\right)\right] dx^2 + \left[exp.\left(\pm 2\, L\sqrt{\frac{Q}{N}}\ t\right)\right](dy^2 + dz^2). \tag{82}$$

The physical properties of the model are

$$\delta = \frac{H^2}{8\pi\left[exp.\left(\pm 4L\sqrt{\frac{Q}{N}}\, t\right)\right]}, \tag{83}$$

$$\lambda = -\left(\frac{2k^2+2}{\gamma-1}\right) L^2\, \frac{Q}{N}, \tag{84}$$

$$\rho = -\left[\frac{\gamma(2k^2+2)}{\gamma-1}\right] L^2\, \frac{Q}{N}, \tag{85}$$

$$p = -\left[\frac{\epsilon\gamma(2k^2+2)}{\gamma-1}\right] L^2\, \frac{Q}{N}, \tag{86}$$

$$\rho_p = -2(k^2+1)L^2\, \frac{Q}{N}, \tag{87}$$

$$\theta = (k+2)\beta = \pm(k+2)\ \ L\sqrt{\frac{Q}{N}}, \tag{88}$$

$$\sigma = \pm\frac{1}{\sqrt{3}}(k-1)\left(\ L\sqrt{\frac{Q}{N}}\right), \tag{89}$$

$$\xi = \frac{\xi_0}{\pm(k+2)\ \ L}\sqrt{\frac{N}{Q}}, \tag{90}$$

$$V = \sqrt{-g} = AB^2 = (k+2)\ exp.\left[\pm\ (k+2)L\sqrt{\frac{Q}{N}}\ t\right]. \tag{91}$$

Here, the physical quantities λ, ρ, p and ρ_p are all independent of cosmic time and having constant values. As the model contains positive and negative signs, it has two different natures.

The positive sign represents an expanding universe and negative sign represents a contracting universe. The magnetic field depends upon cosmic time. For expanding universe, the magnetic field is finite in the beginning and decreases gradually and then vanish at $t \to \infty$. For the contracting universe, the magnetic field goes on decreasing the cosmic time. The model does not isotropize for $k \neq 1$ and it isotropizes for $k = 1$.

It is observed that our work is the generalization of that of Tripathy et al. (2008) in bimetric theory of gravitation. If $L = 1$, then our observations match the results of Tripathy et al. (2008).

4. Conclusion

We have investigated four different LRS bianchi type I string fluid cosmological models with magnetic fluid in bimetric theory of gravitation, depending upon the values of $\frac{Q}{N}, N$ and M. In case I (a), by assuming $\frac{Q}{N} > 0$, we deduced the model (33). In the beginning, the magnetic field was present in the model but later on it disappeared for very very large value of cosmic time. The physical quantities p, ρ, ρ_p, λ all are finite in the begaining and then decrease when t increases. The model is bulk viscous and has shear for $k \neq 1$ and for $k = 1$, the shear vanishes. The universe isexpanding without affecting the magnetic field and the expansion rate decreases with increasing time. The value $\frac{\sigma}{\theta} \neq 0, k \neq 1$ and therefore the model does not isotropize for $k \neq 1$ and it isotropizes for $k = 1$. In case I (b), we have assumed $\frac{Q}{N} < 0$ and we deduced the model (46). In this model, it is observed that the physical quantities $\delta, \lambda, \rho, p, \rho_p$ given in equations (47-51) all are finite and changing harmonically with the growth of cosmic time, since the cosine function is present in it. Therefore in this case, our model represents the oscillatory universe. The bulk viscosity affects all the physical parameters of the model as is seen from the equations (48 - 55). The model has shear for $k \neq 1$ and the shear in the model disappears for $k = 1$. Also the model does not isotropize for $k \neq 1$ but isotropizes for $k = 1$. In case II, we have assummed $N = 0, M = 0$ and deduced the model given by equation (59). In this model, we have $\gamma = 0$ or $\epsilon = 1$.

For $\gamma = 0$, our model (59) becomes a vacuum universe with magnetic field and with the particle density which is the negative of the tension density. For $\epsilon = 1$, the model has a magnetic field and pressure equal to the energy density. For $\gamma = 0$ as well as $\epsilon = 1$, the magnetic field is present in the beginning and its effect gradually decrease with an increase in the cosmic time and it completely disappears for a very large value of cosmic time. The universe is expanding in nature and the bulk viscosity in the model decreases with increasing time and finally vanishes at a later time. The model has shear which continuously increases with time for $k \neq 1$ and the shear vanishes for $k = 1$.

Here it is noted that the bulk viscosity is playing the major role in the model. If the bulk viscosity is zero then $Q = 0$ and in this case our model (59) takes over the model (70). In the absence of bulk viscosity ξ, the physical behavior of the model (70) is observed from the results (71-78). It is pointed out that, for $\gamma = 0$, we get the vacuum universe with the particle density equal to the negative of the string density and for $\epsilon = 1$, the model (70) does not exist, when $\gamma = 1$ as its pressure and density becomes infinite. In the absence of bulk viscosity, our model (70) has shear for $k \neq 1$ and the expansion in the model is switched off in both $\gamma = 0$ as well as $\epsilon = 1$ for $\xi = 0$. Lastly, we have deduced the model given by equation (82) contains positive as well as negative signs, it has two different natures. The positive sign represents an expanding universe and the negative sign represents a contracting universe.

In this model, the magnetic field depends upon cosmic time. For an expanding universe, the magnetic fields are finite in the beginning and decrease gradually and then vanish at late time. For contracting universe, the magnetic field goes on decreasing with cosmic time. The model does not isotropize for $k \neq 1$ and it isotropizes for $k = 1$.

Our work is the generalization of that of Tripathy et al. (2008) in bimetric theory of gravitation. In particular, $L = 1$, our observation matched with the result of Tripathy et al. (2008) .

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