



Effects of Chemical Reaction and Radiation Absorption on MHD Flow of Dusty Viscoelastic Fluid

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Abstract

This investigation is undertaken to study the effects of heat source and radiation absorption on unsteady hydro-magnetic heat and mass transfer flow of a dusty viscous incompressible, electrically conducting fluid between two vertical heated, porous, parallel plates in the presence of chemical reaction under the influence of a transverse applied magnetic field. Initially, the channel walls as well as the dusty fluid are assumed to be at the same temperature and the mass is assumed to be present at low level concentration so that it is constant everywhere. It is also assumed that the dusty particles are non-conducting, solid, spherical and equal in sizes, these are uniformly and symmetrically distributed in the flow field. The governing equations are solved analytically using the perturbation technique. Non-dimensional velocity, temperature, concentration and skin-friction are discussed through graphs for various physical parameters entering into the problem. It is found that velocity of the dusty particles is less than that of the dusty fluid and the skin-friction of the dusty particles is greater than that of the dusty fluid. It is observed that the temperature is minimal at the centre of the channel and decreases towards the plates whereas the concentration is minimal at the center of the channel but increases towards the plates.

Keywords: MHD; heat and mass transfer; radiation absorption; heat source; chemical reaction

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Nomenclature

- B_0 - Magnetic field
 σ - Electric conductivity
 M - Magnetic field parameter
 C - Species concentration in the fluid
 P_r - Prandtl number
 C_w - Concentration of the plate
 S_c - Schmidt number
 C_0 - Initial uniform concentration at T_0
 T_w - Plate temperature
 c_p - Specific heat at constant pressure
 T_0 - Initial temperature
 g - Acceleration due to gravity
 t - Time
 G_r - Thermal Grashof number
 A - Decay parameter
 N_0 - Number density of the dusty particles
 K - Resistance coefficient of dusty particles
 D' - Coefficient of proportionality for the absorption of radiation
 u, v - Velocities of dusty fluid and dusty particles respectively
 S - Dimensionless heat source or sink parameter
 Q - Dimensional heat source/sink parameter
 ϕ - Radiation absorption parameter
 G_m - Modified Grashof number
 y : - Co-ordinate axes normal to the plates
 k : - Dimensionless permeability parameter
 K_1 : - Dimensional permeability of the porous medium
 K_2 : - Dimensional chemical reaction parameter
 K_r : - Dimensionless chemical reaction parameter
 κ : - Thermal diffusivity
 ρ : - Density

1. Introduction

In nature, the availability of pure air or water is scarce. Air and water are contaminated with impurities like CO₂, NH₃, O₂ etc. or salts in water. The influence of dusty particles on free-forced convective flow of dusty viscous fluids has attained some importance in many industrial applications such as wastewater treatment, power plant piping, combustion and petroleum transport. Flows with heat and mass transfer of electrically conducting fluids in channels under

the influence of a transverse magnetic field occur in MHD power generators, the cooling of nuclear reactor, boundary layer control in aerodynamics and crystal growth. Until recently, this study has been largely concerned with the flow of heat and mass transfer characteristics in various physical situations.

On the other hand, mass diffusion rates can be altered tremendously with chemical reactions. The chemical reaction effects depend on whether the reaction is homogeneous or heterogeneous. This, in-turn depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in the solution. In the majority of cases, a chemical reaction depends on the concentration of the species itself. A reaction is said to be of the first order, if the rate of reaction is directly proportional to the concentration itself, Cussler (1988). A few representative areas of interest are those in which heat and mass transfer, combined with chemical reaction play an important role like in food processing and polymer production. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Alexander (1980) studied the analytical solution for mass transfer with a chemical reaction of the first order. Das et al. (1994) have studied the effects of the homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer.

The nature of dusty viscous and viscoelastic fluids under the influence of different physical conditions has been studied by several researchers notably by Saffman (1962), Micheal and Norey (1968), Raptis and Perdikis (1985). Singh (2002) proposed the study of free convection and mass diffusion of a dusty viscoelastic (Walter's Liquid Model-B) fluid flowing between two heated porous plates in porous media in the presence of a magnetic field. Kumar and Srivastava (2005) studied the effects of chemical reaction on MHD flow of dusty viscoelastic (Walter's Liquid Model-B) liquid with heat source/sink. Mbeledogu and Ogulu (2007), Patil and Kulkarni (2008) proposed the study of convective heat and mass transfer in an incompressible viscous Boussinesq fluid in the presence of a chemical reaction of the first order. Osalusi et al. (2008), Afify (2009) and Beg et al. (2009) have studied the effects of thermo diffusion (Soret effects) on MHD mixed convection heat and mass transfer of an electrically conducting fluid. Nandeppanavar et al. (2010) have proposed a study on heat transfer in a Walter's Liquid B Fluid over an impermeable stretching sheet with heat source/sink.

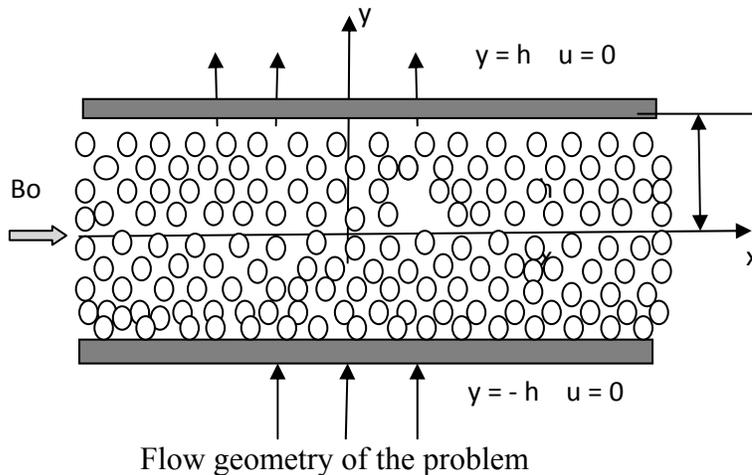
Makinde and Chinyoka (2010) analyzed MHD transient flows and heat transfer of a dusty fluid. In a channel with Navier slip condition. Sharma et al. (2010) have discussed the unsteady MHD heat and mass transfer free convective flow of a viscous fluid through a Darcian porous regime adjacent to a moving semi-infinite vertical plate. The governing equations were solved with Element free Galerkin method. Prakash et al. (2010) have investigated the effects of thermal diffusion and chemical reaction on MHD flow of a dusty viscous incompressible and electrically conducting fluid between two vertical heated porous plates with heat source/sink. Sivaraj and Kumar (2012) analyzed the problem of steady, mixed convective and laminar flow of viscoelastic and viscous fluids in a vertical porous channel filled with viscoelastic fluid in one region and viscous fluid in another region. Mastanrao et al. (2013) have studied chemical reaction and combined buoyancy effects of thermal and mass diffusion on MHD convective flow

along an infinite vertical porous plate in the presence of Hall current with variable suction and heat generation.

2. Mathematical Analysis

In this problem, we considered the effects of heat source (or sink) and radiation absorption on the unsteady dusty flow of a viscous, incompressible, slightly conducting, viscoelastic fluid between two heated infinite vertical porous parallel plates ($2h$ distance apart) located at the $y = h$ and $y = -h$ planes and extending from $x = -\infty$ to ∞ and from $z = -\infty$ to ∞ under the influence of a uniform magnetic field in the presence of chemical reaction of the first order. The x -axis is taken along the flow midway of the plates ($y = 0$) and y -axis is taken normal to it. Let u and v be the velocities of dusty fluid and dusty particles respectively in x -direction. Initially, the channel walls as well as the dusty fluid are assumed to be at the same temperature T_0 with concentration levels C_0 (assumed to be present at low level and uniformly distributed everywhere). At time $t > 0$, the temperature of the walls is instantaneously raised to T_w and the species concentration is raised to C_w . It is also assumed that,

1. The flow in the x -direction is driven by a constant pressure gradient $\frac{\partial p}{\partial x}$ with negligible body forces.
2. A uniform magnetic field with magnetic flux density vector B_0 is applied in the positive y -direction. This is also assumed to be also the total magnetic field.
3. There is no externally applied electric field and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number.
4. The dusty particles are non-conducting, solid, spherical and equal in size. They are distributed uniformly and symmetrically everywhere in the flow field with number density N_0 in the entire flow.
5. There exists a chemical reaction in the mixture.
6. The plates are assumed to be infinite in the x and z -directions so that the physical quantities do not change in these directions.



Under these assumptions, with Boussinesq's approximation, the fluid motion, temperature and concentration are governed by the following set of equations.

$$\frac{\partial u}{\partial t} = g\beta(T - T_0) + g\beta'(C - C_0) + \nu \left(1 - K_0 \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(v - u) - \frac{\sigma}{\rho} B_0^2 u - \frac{\nu}{K_1} u, \quad (1)$$

$$m \frac{\partial v}{\partial t} = K(u - v), \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{Q}{\rho c_p}(T - T_0) + \frac{D'}{\rho c_p}(C - C_0), \quad (3)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_l(C - C_0). \quad (4)$$

At $t = 0$, the temperature and concentration level changes according to the following laws:

$$T = T_0 + (T_w - T_0)(1 - e^{-at}),$$

$$C = C_0 + (C_w - C_0)(1 - e^{-at}),$$

with the following initial and boundary conditions

$$t = 0 : u = 0 = v, T = T_0, y \in (-d, d),$$

$$t > 0 : u = 0 = v, T = T_0 + (T_w - T_0)(1 - e^{-at}),$$

$$C = C_0 + (C_w - C_0)(1 - e^{-at}) \text{ for } y = -d, \quad (5)$$

$$u = 0 = v, T = T_0 + (T_w - T_0)(1 - e^{-at}),$$

$$C = C_0 + (C_w - C_0)(1 - e^{-at}) \text{ for } y = d.$$

On introducing the following non-dimensional quantities

$$y^* = \frac{y}{d}, u^* = \frac{ud}{\nu}, v^* = \frac{vd}{\nu}, t^* = \frac{\nu t}{d^2}, \lambda = \frac{mN_0}{\rho},$$

$$T^* = \frac{T - T_0}{T_w - T_0}, C^* = \frac{C - C_0}{C_w - C_0}, a^* = \frac{d^2 a}{\nu}, k = \frac{d^3 K_1}{\nu^2}.$$

We have the following governing equations which are dimensionless

$$\frac{\partial u}{\partial t} = G_r T + G_m C + \left(1 - E \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{w}(v - u) - \left(M^2 + \frac{1}{k}\right)u, \quad (6)$$

$$w \frac{\partial v}{\partial t} = u - v, \quad (7)$$

$$\frac{\partial^2 T}{\partial y^2} - \text{Pr} \frac{\partial T}{\partial t} - ST + \varphi C = 0, \quad (8)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - Sc Kr C. \quad (9)$$

Now, initial and boundary conditions (5) according to new system reduce to

$$\begin{aligned} t = 0 : u = 0 = v, T = 0 \text{ in } y \in (-1, 1), \\ t > 0 : u = 0 = v, T = 1 - \exp(-at), C = 1 - \exp(-at) \text{ for } y = -1, \\ t > 0 : u = 0 = v, T = 1 - \exp(-at), C = 1 - \exp(-at) \text{ for } y = 1, \end{aligned} \quad (10)$$

where

$$M^2 = \frac{\sigma B_0^2 d^2}{\rho \nu} \text{ (Hartmann number),}$$

$$w = \frac{m \nu}{K d^2} \text{ (Relaxation time parameter for particles),}$$

$$G_r = \frac{g \beta (T_w - T_0) d^3}{\nu^2} \text{ (Grashof number),}$$

$$G_m = \frac{g \beta' (C_w - C_0) d^3}{\nu^2} \text{ (Modified Grashof number),}$$

$$Sc = \frac{\nu}{D} \text{ (Schmidt number),}$$

$$S = \frac{d^2 Q}{\kappa} \text{ (Heat source/sink parameter),}$$

$$Kr = \frac{K_l d^2}{\nu} \text{ (Chemical reaction parameter),}$$

$$E = \frac{K_0 \nu}{d^2} \text{ (Viscoelastic parameter),}$$

$$\text{Pr} = \frac{\mu c_p}{\kappa} \text{ (Prandtl number),}$$

$$\phi = \frac{\rho c_p d^2 (C_w - C_0) D'}{\kappa (T_w - T_0)} \text{ (Radiation absorption parameter).}$$

3. Method of Solution

To solve the equations (6) to (9) subject to the boundary conditions (10) according to Pop (1968), we assume

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y,t) e^{-at} + \dots, \\ v(y,t) &= v_0(y) + \varepsilon v_1(y,t) e^{-at} + \dots, \\ T(y,t) &= T_0(y) + \varepsilon T_1(y,t) e^{-at} + \dots, \\ C(y,t) &= C_0(y) + \varepsilon C_1(y,t) e^{-at} + \dots. \end{aligned} \quad (11)$$

Using (11) in equations (6), (7), (8) and (9) and comparing the coefficients of the harmonic and non-harmonic terms, we obtain the following set of equations:

$$u_0'' - \left(M^2 + \frac{1}{k} \right) u_0 + \frac{\lambda}{w} (v_0 - u_0) = -G_r T_0 - G_m C_0, \quad (12)$$

$$(1 + aE) u_1'' - \left(M^2 + \frac{1}{k} - a \right) u_1 + \frac{\lambda}{w} (v_1 - u_1) = -G_r T_1 - G_m C_1, \quad (13)$$

$$u_0 = v_0, \quad (14)$$

$$u_1 = (1 - a w) v_1, \quad (15)$$

$$T_0'' - S T_0 + \phi C_0 = 0, \quad (16)$$

$$T_1'' - (S - a \text{Pr}) T_1 = -\phi C_1, \quad (17)$$

$$C_0'' - Sc Kr C_0 = 0, \quad (18)$$

$$C_1'' - Sc (Kr - a) C_1 = 0, \quad (19)$$

where primes represent differentiation with respect to y and further the boundary conditions are reduced to:

$$u_0 = v_0 = 0 = u_1 = v_1, T_0 = C_0 = 1,$$

$$T_1 = C_1 = -\frac{1}{\varepsilon}, \text{ at } y = -1.$$

and

$$u_0 = v_0 = 0 = u_1 = v_1, T_0 = C_0 = 1,$$

$$T_1 = C_1 = -\frac{1}{\varepsilon}, \text{ at } y = 1. \quad (20)$$

By solving the equations (12) to (19) under the boundary conditions (20), we get the solutions for the velocity (dusty fluid and dusty particles), temperature and concentration fields as follows:

$$u(y,t) = \frac{A_6 \cosh A_0 y}{\cosh A_0} - \frac{A_5 \cosh \sqrt{S} y}{\cosh \sqrt{S}} + \frac{(A_5 - A_6) \cosh M_1 y}{\cosh M_1} + \left[\frac{A_7 \cosh A_3 y}{\cosh A_3} + \frac{A_8 \cosh A_1 y}{\cosh A_1} - \frac{A_9 \cosh M_2 y}{\cosh M_2} \right] e^{-at}, \quad (21)$$

$$v(y,t) = \frac{A_6 \cosh A_0 y}{\cosh A_0} - \frac{A_5 \cosh \sqrt{S} y}{\cosh \sqrt{S}} + \frac{(A_5 - A_6) \cosh M_1 y}{\cosh M_1} + \frac{1}{1-aw} \left[\frac{A_7 \cosh A_3 y}{\cosh A_3} + \frac{A_8 \cosh A_1 y}{\cosh A_1} - \frac{A_9 \cosh M_2 y}{\cosh M_2} \right] e^{-at}, \quad (22)$$

$$T(y,t) = \frac{(1 + A_2) \cosh \sqrt{S} y}{\cosh \sqrt{S}} - \frac{A_2 \cosh A_0 y}{\cosh A_0} + \left[\frac{A_4 \cosh A_1 y}{\cosh A_1} - \frac{(1 + A_4) \cosh A_3 y}{\cosh A_3} \right] e^{-at}, \quad (23)$$

$$C(y,t) = \frac{\cosh A_0 y}{\cosh A_0} - \frac{\cosh A_1 y}{\cosh A_1} e^{-at}, \quad (24)$$

where

$$A_0 = \sqrt{S}, \quad A_1 = \sqrt{Sc(Kr - a)}, \quad A_2 = \frac{\phi}{A_0^2 - S},$$

$$A_3 = \sqrt{S - aPr}, \quad A_4 = \frac{\phi}{A_1^2 - A_3^2}, \quad M_1^2 = M^2 + \frac{1}{k},$$

$$M_2^2 = \frac{1}{1+aE} \left(M_1^2 - a - \frac{a\lambda}{1-aw} \right), G_{r1} = \frac{G_r}{1+aE}, G_{m1} = \frac{G_m}{1+aE},$$

$$A_5 = \frac{-G_r(1+A_2)}{S-M_1^2}, A_6 = \frac{G_r A_2 - G_m}{A_0^2 - M_1^2}, A_7 = \frac{G_{r1}(1+A_4)}{A_3^2 - M_2^2},$$

$$A_8 = \frac{G_{m1} - G_{r1}A_4}{A_1^2 - M_2^2}, A_9 = A_7 + A_8.$$

4. Skin-Friction

Let τ_f and τ_p be the skin friction for dusty fluid and dusty particles respectively. Then

$$\tau_f = \left[\frac{\partial u}{\partial y} \right]_{y=1} = A_0 A_6 \tanh A_0 - A_5 \sqrt{S} \tanh \sqrt{S} + M_1 (A_5 - A_6) \tanh M_1$$

$$+ [A_3 A_7 \tanh M_3 + A_1 A_3 \tanh A_1 - M_2 A_9 \tanh M_2] e^{-at}, \quad (25)$$

$$\tau_p = \left[\frac{\partial v}{\partial y} \right]_{y=1} = A_0 A_6 \tanh A_0 - A_5 \sqrt{S} \tanh \sqrt{S} + M_1 (A_5 - A_6) \tanh M_1$$

$$+ \frac{1}{1-aw} [A_3 A_7 \tanh M_3 + A_1 A_3 \tanh A_1 - M_2 A_9 \tanh M_2] e^{-at}. \quad (26)$$

5. Results and Discussion

To get a physical insight into the problem, the numerical evaluation of the analytical results for velocity, skin-friction for a dusty fluid, dusty particles, also temperature and concentration profiles for adusty fluid have been calculated. The effects of various physical parameters like magnetic parameter M , heat source parameter S , radiation absorption parameter ϕ , chemical reaction parameter K_r , permeability parameter k , the Schmidt number Sc and the Prandtl number Pr on velocity, temperature, concentration and skin-friction are displayed through graphs.

The concentration profiles for different values of the Schmidt number is considered 0.6 which corresponds to water-vapor, 0.22 (Helium), 0.78 (Carbon monoxide), 0.96 (Methonal) and chemical reaction parameter K_r (0.1, 0.3, 0.5 & 0.7) with $a = 0.9$, $S = 0.2$, $t = 1$, $Pr = 0.71$ (usually for air) of the dusty fluid are displayed through Figures 1 and 2 respectively. From these Figures it is seen that increasing values of the Schmidt number Sc and the chemical reaction parameter K_r decreases the concentration. It is also observed that concentration is minimum at the centre of the channel ($y = 0$) and increases towards the plates. The fluid temperature profiles for various

values of Heat source/sink parameter S (0.2, 0.4, 0.6 & 0.8), the Prandtl number P_r (0.50 & 0.71) and the radiation absorption parameter ϕ (1.0, 2.0, 3.0 & 4.0) with $a = 0.2$ and $t = 1.0$ are illustrated through the Figures 3 and 4. It is found that with the increase of the heat source/sink parameter and the Prandtl number, the temperature decreases while it increases with the increase of the radiation absorption parameter. It is also observed that the temperature is minimum at the centre of channel ($y = 0$) and decreases moving towards the plates.

The behavior of the velocity of the liquid and the dusty particles under the influence of the magnetic field parameter M (1.0, 2.0, 3.0 & 4.0), the radiation absorption parameter ϕ (1.0, 2.0, 3.0 & 4.0) and the heat source parameter S (0.0, 0.1, 0.2 & 0.3) with $S_c = 2.0$, $K_r = 0.2$, $a = 0.9$, $P_r = 0.71$, $w = 0.5$, $E = 1$, $d = 1$, $k = 10$, $G_r = 10$, $G_m = 5$ and $t = 1.0$ are shown in Figures 5 - 7 respectively. From these Figures it is observed that by increasing the values of the magnetic field parameter we decrease the velocity of the dusty fluid and dusty particles absolutely while increasing the radiation absorption parameter and heat source/sink parameter. It is also noticed that the velocity is maximum at the centre of channel ($y = 0$) and decreasing towards the plates.

The velocity profiles for different values of the Schmidt parameter S_c (0.22, 0.60 & 0.78) and the permeability parameter k (0.0, 0.01, 0.02 & 0.03) with $M = 4.0$, $K_r = 0.2$, $a = 0.9$, $P_r = 0.71$, $w = 0.5$, $E = 1$, $\phi = 5.0$, $d = 1$, $G_r = 10$, $G_m = 5$ and $t = 1.0$ are displayed in Figures 8 and 9 respectively. It is found that increasing the values of the Schmidt parameter and permeability parameter increases the velocity for both liquid and dusty particles. It is also observed that the velocity is maximum at the centre of channel ($y = 0$) and decreases towards the plates. In the above investigation, it is very interesting to note that the velocity of the dusty particles is less than the velocity of the dusty fluid. Skin-friction is presented in Figures 10 and 11 against the magnetic field parameter (M) and the heat source parameter (S) respectively. From these Figures it is seen that the skin friction increases with increasing the magnetic field parameter M while it decreases with increasing heat source parameter S . It is also observed that the skin friction of the dusty particles is greater than that of the dusty fluid.

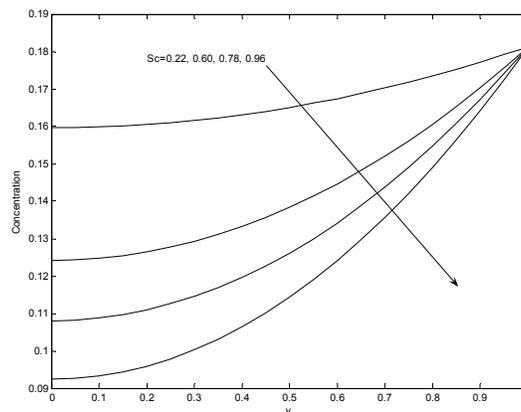


Figure 1. Concentration profiles for different S_c

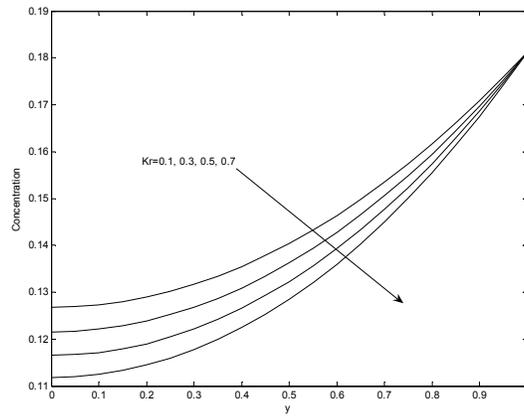


Figure 2. Concentration profiles for different K_r

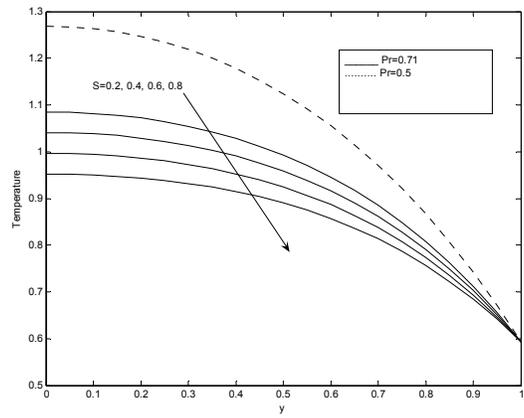


Figure 3. Temperature profiles for different S and P_r

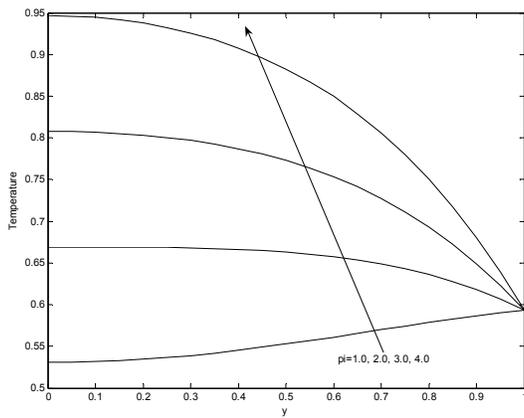


Figure 4. Temperature profiles for different ϕ

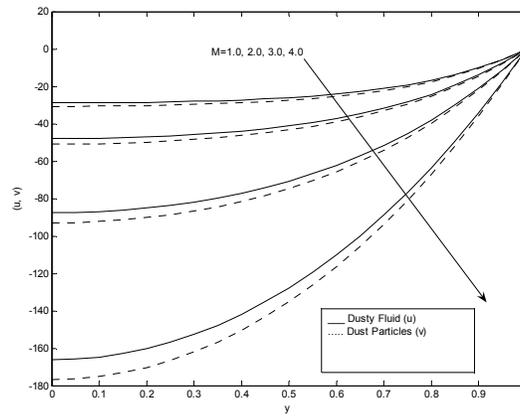


Figure 5. Velocity profiles for different M

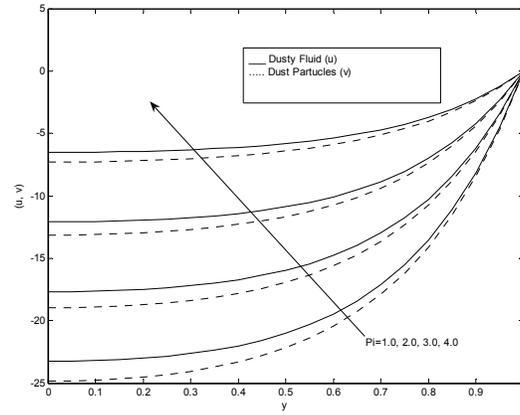


Figure 6. Velocity profiles for different ϕ

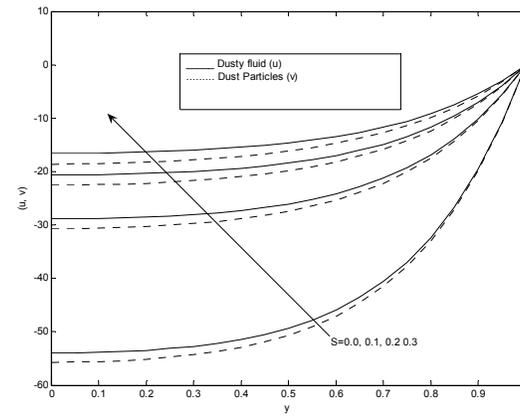


Figure 7. Velocity profiles for different S

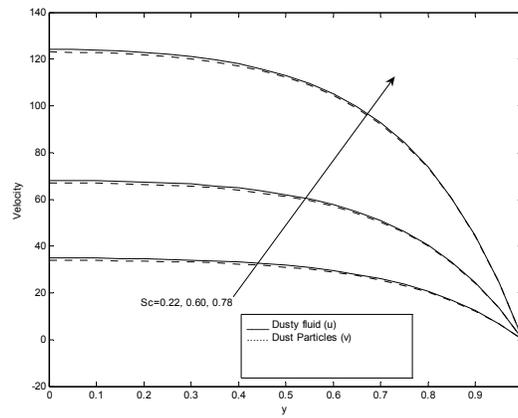


Figure 8. Velocity profiles for different S_c

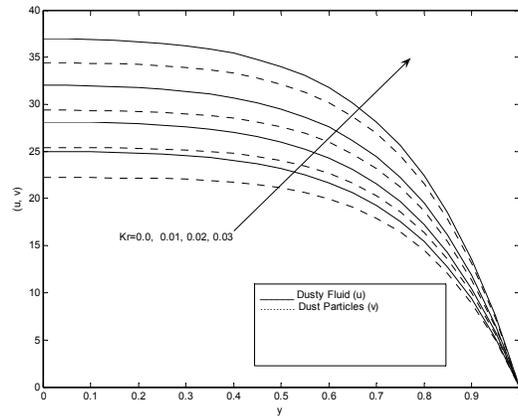


Figure 9. Velocity profiles for different k

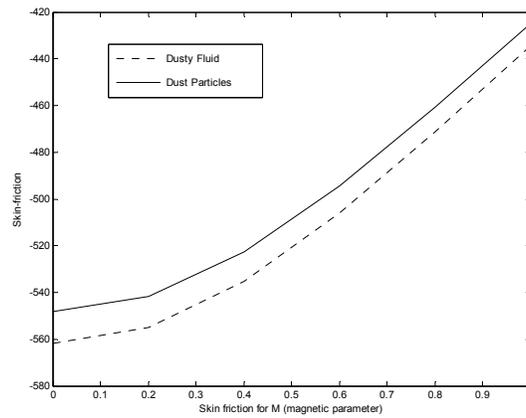


Figure 10. Skin-friction for different M (Magnetic parameter)

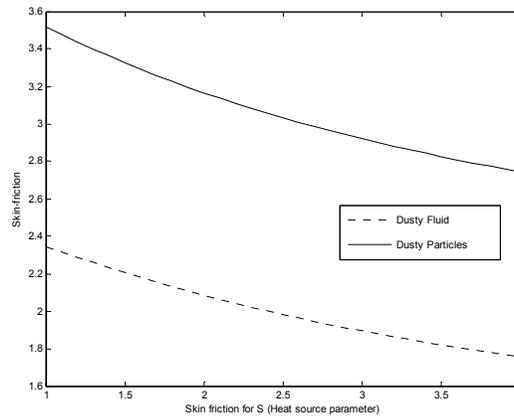


Figure 11. Skin-friction for different S (Heat source parameter)

6. Conclusions

It is found that with the increase of the heat source/sink parameter and the Prandtl number the temperature decreases while it increases with the increase of the radiation absorption parameter. It is also noticed that the temperature is minimum at the centre of the channel and decreases towards the plates whereas the concentration is minimum at the centre of the channel but increases towards the plates. It is observed that with increasing values of the magnetic parameter, the velocity of dusty fluid as well as velocity of dusty particles decreases, while it increases with the increase of the radiation absorption parameter and heat source parameter. It is also found that the velocity of the dusty particles is less than that of the dusty fluid and the kin-friction of the dusty particles is greater than that of the dusty fluid.

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REFERENCES

- Afify, A. A. (2009). Similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective Heat and Mass transfer over a stretching surface considering suction or injection, *Comm, in Nonlinear Science and Numerical Simulation*, Vol. 14, No. 5, pp. 2202-2214.
- Alexander, Apelblat. (1980). Mass transfer with a chemical reaction of the first order, *The Chemical Engineering Journal*, Vol. 19, pp. 19-37

- Beg, O. A., Baker, A. Y. and Prasad, V. R. (2009). Numerical study of free convection magneto hydrodynamic Heat and Mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects, *Computational Materials Science*, Vol. 46, No. 1, pp. 57-65.
- Chambre, P. L. and Young, J. D. (1958). On the diffusion of chemically reactive species in a laminar boundary layer flow, *The Physics of Fluids*, Vol. 1, 1958, pp. 48-54.
- Cussler, E. L. (1988). Diffusion Mass Transfer in Fluid Systems, *Cambridge University Press*.
- Das, U. N., Deka, R. K. and Soundalgekar, V. M. (1994). Effects on mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, *Foushung im ingenieurwesen*, Vol. 60, pp. 284-287.
- Kumar, D. and Srivastava, R. K. (2005). Effects of chemical reaction on MHD flow of Dusty viscoelastic (Walter's Liquid Model-B) Liquid with heat source/sink, *Proc. of National Seminar on Mathematics and Computer Science*, Meerut, pp.105-112.
- Makinde, O. D. and Chinyoka, T. (2010). MHD Transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition, *Computers and Mathematics with Applications*, Vol. 60, pp. 660 - 669.
- Mastanrao, S., Balamurugan, K. S., Varma, S. V. K. and Raju, V. C. C. (2013). Chemical reaction and Hall effects on MHD convective flow along an infinite vertical porous plate with variable suction and heat generation, *Applications and Applied Mathematics*, Vol. 8, Issue.1, pp.268-288.
- Mbeledogu, I. U. and Ogulu, A. (2007). Heat and Mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative Heat transfer, *Int. J. of Heat and Mass Transfer*, Vol. 50, No. 9-10, pp.1902-1908.
- Michael, D. H. and Norey, P. W. (1968). The laminar flow of a dusty gas between rotating cylinders, *The Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 21, pp. 375-388.
- Nadeppanavar, M. M, Abel, M. S. and Tawade, J. (2010). Heat transfer in a Walter's Liquid B-Fluid over an incompressible stretching sheet with non-uniform Heat source/sink and elastic deformation, *Communications in Nonlinear Science and Numerical simulation*, Vol. 15, No. 7, pp. 1791-1802.
- Osalusi, E, Side, J. and Harris, R. (2008). Thermal diffusion and Thermo effect on combined Heat and Mass transfer of a study MHD convective and slip flow due to a rotating disk and viscous dissipation and ohmic heating, *Int. Comm. in Heat and Mass Transfer*, Vol. 35, No. 8, pp. 908-915.
- Patil, P. M. and Kulkarni, P. S. (2008). Effects of chemical reaction on free convection flow of a polar fluid through a porous medium in the presence of thermal sciences, Vol. 47, No. 8, pp.1043-1054.
- Prakash, O., Devendra Kumar, Y.K. and Dwivedi, Y. K. (2010). Effects of thermal diffusion and chemical reaction on MHD flow of dusty viscoelastic (Walter's Liquid Model-B) fluid, *J. Electromagnetic Analysis and applications*, Vol.2, pp. 581-587.
- Pop, I. (1968). *Rev. Roum. Physics*, Vol. 13, pp. 41.
- Raptis, A. and Perdikis, C. P. (1985). Oscillatory flow through a porous medium by the presence of free convective flow, *Int. J. of Engineering Science*, Vol. 23, pp51-55.
- Saffman, P. G. (1962). On the stability of laminar flow of a dusty gas, *Journal of Fluid Mechanics*, Vol. 13, No. 1, pp.120-129.

- Sharma, R., Bhargava, R. and Bhargava, P. (2010). A numerical solution of steady MHD convection Heat and Mass transfer on a semi-infinite vertical porous moving plate using Element Free Galerkin Method, *Computational Materials Science*, Vol. 48, No. 3, pp. 537-543.
- Singh, N. P, Singh, A. K, Yadav, M. K. and Singh, A. K. (2002). *Acta Ciencia Indica*, Vol. XXVIII M, No.1, pp. 089.
- Sivaraj. R. and Rushi Kumar. B. (2012). MHD mixed convective flow of viscoelastic and viscous fluids in a vertical porous channel, *Applications and Applied Mathematics*, Vol.7, Issue.1, pp. 99 -116.