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Analytic Solution for the Drainage of Sisko Fluid Film Down a Vertical Belt

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Abstract

This paper deals with the drainage of Sisko fluid film down a vertical belt. It provides an approximate solution of the resulting non-linear and inhomogeneous ordinary differential equation using perturbation method (PM) and Adomian decomposition method (ADM). Comparison of the results obtained by both methods demonstrate that these series solutions are strictly identical but ADM is easy to compute and can be extended to any higher order. The important physical quantities like velocity profile, volume flow rate, average film velocity, shear stress, force exerted by the fluid film and vorticity vector are derived. The effects of fluid behaviour index, Stokes number and Sisko fluid parameter on some of these physical quantities are observed. Furthermore, we also made a comparison between the Sisko fluid film and Newtonian fluid film.

Keywords: Thin film flow**,** Sisko fluid model, drainage, perturbation method**,** Adomian decomposition method

AMS-MSC 2010 No.: 76A20, 76D08

1. Introduction

Thin film flows have extensively been investigated theoretically and experimentally by many scientists Van Rossum (1958), Raghuraman (1971), O' Brien and Schwartz (2002), Mayers (2005), Siddiqiui et al. (2007). Such flows have found many applications in our nature and industry. The motion of rain drop down a window pane, in eye the tear films, flow of water on stalactites hanging from the roof of a limestone cave, the acclivity of buoyant magma below the solid rocks and the spreading of lava on volcanoes are the naturally occurring examples. In industry, tertiary oil recovery, fabrication of microchip, and in many coating processes its application are found. In all these examples, repetitive feature is that fluid in contact with rigid boundary is drain down due to gravity. Typically, it consists of an extent of liquid bounded by a solid boundary (vertical belt) and with a free surface (usually air). The thickness is much shorter than the length of the contacting object so that the flow takes place mainly in the longer dimensions under the action of gravity. The velocity component perpendicular to the object is much smaller than the velocity component along the object.

We will consider the particular example of the motion of thin liquid film down an infinity long vertical belt which can be extended to any frame of coatings and lubrication process, O' Brien and Schwartz (2002).

Over the past two decades, much attention has been paid to the study of non-Newtonian fluids because of their profuse industrial and technological applications. It is a broad class of fluids so, there is not a single model that can describe all the properties of non-Newtonian fluids. Therefore several constitutive equations are proposed to predict the physical structure and behavior of such fluids. Among these, comparatively simple model, named Sisko fluids, is capable of describing shear thinning and thickening phenomena, which commonly exists in nature. Such fluids are well known and have many industrial applications. It is the most appropriate model for the flow of greases. Waterborne coatings and metallic automotive basecoat where polymeric suspensions are used, cement slurries, lubricating greases, most psueodoplastic fluids and drilling fluids are some of its industrial applications, Sisko (1958), Siddiqiui et al. (2009), Mekheimer and El Kot (2012).

Most of the natural and industrial occurring problems when are modeled show non-linearity and few of them show linearity. Non-linearity increases the mathematical complexity of the problems which reduces the chance of getting exact solutions. For this reason, various techniques such as perturbation method (PM), Adomian decomposition method (ADM), homotopy analysis method (HAM), optimal homotopy analysis method (OHAM),homotopy perturbation method (HPM), optimal homotopy perturbation method (OHPM) and some others have been developed to find approximate solutions of these type of problems.

These techniques have been used successfully to get solutions of many problems of industrial and technological importance. PM Bush (2000), Ji-Huan He (2006) and ADM Adomian (1987), Wazwaz (2009), Siddiqiui et al. (2010), Dita and Grama (1997) will be used in this paper to analyze the non-linear behavior of drainage from a vertical belt. PM is well known and widely used approximate method which relies on the existence of a relatively small parameter. ADM recently get attention of the researchers because it can be used to get any higher order solution easily and it does not require linearization, perturbation or any other restrictive assumption.

The intent of the present theoretical study is to investigate the thin film flow due to the drainage of Sisko fluid down a vertical belt. We extend the work of Siddiqiui et al. (2007) for the drainage problem and present its analytic solution using PM and ADM. Comparison of the solutions obtained by these methods will be provided. We shall find the physical quantities such as velocity profile, volume flow rate, average film velocity, shear stress exerted by the belt on fluid film, force exerted by the fluid film on belt surface and the vorticity vector. Physical insights to these physical quantities will also be given.

The rest of paper is organized in 6 sections: the governing equations of motion and the constitutive equation for incompressible Sisko fluid model are presented in section 2, problem is formulated in section 3, section 4 contains the solution of the problem using PM and ADM and includes physical quantities such as volume flow rate, average film velocity, shear stress exerted by the belt on fluid film, force exerted by the fluid film on belt surface and the vorticity vector, influences of fluid behaviour index n, Sisko fluid parameter β and Stokes number S_t are discussed through tables and graphs in section 5 and finally concluding remarks are given in section 6.

2. Basic Equations

The basic equations governing the motion of an incompressible fluid neglecting the thermal effects are

$$div\mathbf{V} = 0, \tag{1}$$

$$\rho \frac{D\mathbf{V}}{Dt} = \rho\mathbf{f} - \nabla p + div\mathbf{S}, \tag{2}$$

where $\mathbf{V}$ is the velocity vector, ρ is the constant density, $\mathbf{f}$ is the body force per unit mass, p is the dynamic pressure, $\mathbf{S}$ is the extra stress tensor and $\frac{D}{Dt}$ is the material time derivative. The constitutive equation for incompressible Sisko fluid Sisko (1958), Siddiqiui et al. (2009), Mekheimer and El Kot (2012) is given by

$$\mathbf{S} = \left[a + b\left(\sqrt{\frac{1}{2} tr\mathbf{A}_1^2} \right)^{n-1} \right]\mathbf{A}_1 \ , \tag{3}$$

where A_1 is the Rivlin-Ericksen tensor:

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \qquad \mathbf{L} = grad\,\mathbf{V}, \tag{4}$$

a, b are material constants and n is the fluid behaviour index. If $a = 0$ the equations for the power law fluid model and if $b = 0$ for Newtonian fluid are obtained.

3. Problem Formulation

Consider steady, parallel, laminar flow of an incompressible Sisko fluid slowly flowing down an infinite vertical belt. As a result, a thin uniform fluid film of thickness δ is formed in contact with stationary air. The geometry of the problem is shown in the Figure 1. We choose an xz-coordinate system such that x-axis is normal to the belt and z-axis along the belt in downward direction. We assume that the fluid completely wets the belt, there is no applied (force) pressure driving the flow and fluid fall under the action of gravity. Therefore, the only velocity component is in z-direction. Accordingly we assume that

$$\mathbf{V} = [0, 0, w(x)], \qquad \mathbf{S} = \mathbf{S}(x). \tag{5}$$

Equation of continuity (1) is identically satisfied by profile (5). Equation (3) upon using equation (4) and profile (5)

$$\mathbf{S} = \left[a + b\left(\frac{dw}{dx}\right)^{n-1} \right] \begin{bmatrix} 0 & 0 & \dfrac{dw}{dx} \\ 0 & 0 & 0 \\ \dfrac{dw}{dx} & 0 & 0 \end{bmatrix},$$

which, in turn provides the following non zero component

$$S_{zx} = \left[a + b\left(\frac{dw}{dx}\right)^{n-1} \right] \frac{dw}{dx} = S_{xz}. \tag{6}$$

The momentum equation (2) with the help of profile (5) and assumptions we made leads to

$$\frac{d}{dx} S_{xz} = -\rho g, \tag{7}$$

along with the associated boundary conditions

$$w = 0 \qquad at \qquad x = 0 \qquad (no\ slip), \tag{8}$$

and

$$S_{xz} = 0 \qquad at \qquad x = \delta \qquad (free\ surface). \tag{9}$$

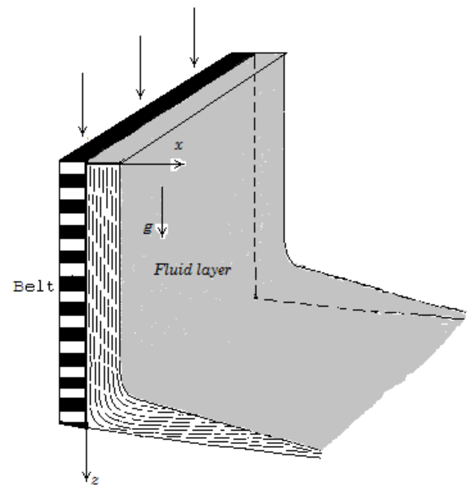

Figure 1. The geometry of the problem

Equation (7) on integration with respect to x and its subjection on boundary condition (9) gives

$$S_{xz} = \rho g(\delta - x), \tag{10}$$

which upon using equation (6) yields

$$\frac{dw}{dx} + \frac{b}{a}\left(\frac{dw}{dx}\right)^n = \frac{\rho g}{a}(\delta - x). \tag{11}$$

Introduce the following dimensionless parameters

$$w^* = \frac{w}{\sqrt{g\delta}}, \qquad x^* = \frac{x}{\delta},$$

after dropping `$*$', equation (11) and boundary condition (8) become:

$$\frac{dw}{dx} + \beta\left(\frac{dw}{dx}\right)^n = S_t(1 - x), \tag{12}$$

$$w = 0 \qquad at \qquad x = 0, \tag{13}$$

where $S_t = \dfrac{\rho g \delta^2}{a\sqrt{g\delta}}$ is the Stokes number and $\beta = \dfrac{b}{a(\frac{\delta}{\sqrt{g\delta}})^{n-1}}$ is the Sisko fluid parameter.

Equation (12) is a first order nonlinear ordinary differential equation whose exact solution seems to be impossible. In the next section, we will apply PM and ADM to solve equation (12) subject to boundary condition (13).

4. Solution of the Problem

4.1. Solution using PM

Assuming β $(0 < \beta = 1)$ to be a small parameter in equation (12), we expand $w(x)$ in a series of the form

$$w(x) = \sum_{k=0}^{\infty} \beta^k w_k(x) = w_0 + \beta w_1 + \beta^2 w_2 + \beta^3 w_3 + \ldots. \tag{14}$$

Substituting series (14) into equation (12) and boundary condition (13), then equating the equal powers of β we obtain system of equations along with their boundary conditions

$$O(\varepsilon^0): \begin{array}{l} \dfrac{dw_0}{dx} = S_t(1-x), \\ w_0(0) = 0. \end{array} \tag{15}$$

$$O(\varepsilon^1): \begin{array}{l} \dfrac{dw_1}{dx} + \left(\dfrac{dw_0}{dx}\right)^n = 0, \\ w_1(0) = 0. \end{array} \tag{16}$$

$$O(\varepsilon^2): \begin{array}{l} \dfrac{dw_2}{dx} + n\left(\dfrac{dw_0}{dx}\right)^{n-1}\left(\dfrac{dw_1}{dx}\right) = 0, \\ w_2(0) = 0. \end{array} \tag{17}$$

$$O(\varepsilon^3): \begin{array}{l} \dfrac{dw_3}{dx} + n\left(\dfrac{dw_0}{dx}\right)^{n-1}\left(\dfrac{dw_2}{dx}\right) + \dfrac{n^2-n}{2}\left(\dfrac{dw_0}{dx}\right)^{n-2}\left(\dfrac{dw_1}{dx}\right)^2 = 0, \\ w_3(0) = 0. \end{array} \tag{18}$$

$$\vdots$$

Solving above system of equations subject to their corresponding boundary conditions, we obtain the following set of solutions

$$w_0(x) = \frac{S_t}{2}\left[1-(1-x)^2\right], \tag{19}$$

$$w_1(x) = -\frac{S_t^n}{n+1}\left[1-(1-x)^{n+1}\right], \tag{20}$$

$$w_2(x) = \frac{S_t^{2n-1}}{2}\left[1-(1-x)^{2n}\right], \tag{21}$$

$$w_3(x) = -\frac{nS_t^{3n-2}}{2}\left[1-(1-x)^{3n-1}\right], \tag{22}$$

$$\vdots$$

Thus, upon substituting these components ($w_0(x), w_1(x), w_2(x)$ and $w_3(x)$) from equations (19)-(22) into the series (14), we obtain

$$w(x) = \frac{S_t}{2}\left[1-(1-x)^2\right] - \frac{\beta S_t^n}{n+1}\left[1-(1-x)^{n+1}\right] + \frac{\beta^2 S_t^{2n-1}}{2}\left[1-(1-x)^{2n}\right] - \frac{\beta^3 n S_t^{3n-2}}{2}\left[1-(1-x)^{3n-1}\right] + \cdots, \tag{23}$$

which is the velocity profile for the drainage of Sisko fluid film down a vertical belt.

Remark:

If we put $\beta = 0$ in equation (23), we recover the solution for Newtonian fluid film [Van Rossum (1958)].

4.2. Solution using ADM

We rewrite equation (12) as

$$\frac{dw}{dx} = S_t(1-x) - \beta\left(\frac{dw}{dx}\right)^n. \tag{24}$$

In operator form, as suggested by [Adomian (1987)], equation (24) can be written as

$$L_x(w) = S_t(1-x) - \beta N(w), \tag{25}$$

where $L_x = \frac{d}{dx}$ is a one fold linear differential operator, which is easily invertible. The nonlinear term $\left(\frac{dw}{dx}\right)^n$ is represented by $N(w)$ i.e.,

$$N(w) = \left(\frac{dw}{dx}\right)^n.$$

Since L_x is invertible, then we defined the inverse operator L_x^{-1} as

$$L_x^{-1}(*) = \int_0^x (*)dx.$$

Applying L_x^{-1} on both sides of equation (25), we obtain

$$L_x^{-1}L_x(w) = S_t L_x^{-1}(1-x) - \beta L_x^{-1}N(w),$$

implies

$$w(x) = w(0) + \frac{S_t}{2}\left[1-(1-x)^2\right] - \beta L_x^{-1}N(w). \qquad (26)$$

Following the ADM [for reference see Adomian (1987), Siddiqui (2010), and Wazwaz (2009)], we decompose the unknown $w(x)$ in a series, known as decomposition series:

$$w(x) = \sum_{k=0}^{\infty} w_k(x), \qquad (27)$$

and present the expansion of nonlinear term $N(w)$ into an infinite series of Adomian polynomials A_k

$$N(w) = \sum_{k=0}^{\infty} A_k, \qquad (28)$$

where the components $w_k(x), k \geq 0$ and the Adomian polynomials A_k can easily be computed. Using equations (27) , (28) and boundary condition (13) into equation (26), one obtains

$$\sum_{k=0}^{\infty} w_k(x) = \frac{S_t}{2}\left[1-(1-x)^2\right] - \beta L_x^{-1}\left(\sum_{k=0}^{\infty} A_k\right), \qquad (29)$$

which follows

$$w_0(x) = \frac{S_t}{2}\left[1-(1-x)^2\right], \qquad (30)$$

$$w_{k+1}(x) = -\beta L_x^{-1}(A_k), \qquad k \geq 0. \qquad (31)$$

Substituting the decomposition series (27) into equation (28), then using binomial expansion, we obtain

$$A_0 = \left(w_0'(x)\right)^n, \qquad (32)$$

$$A_1 = n\left(w_0'(x)\right)^{n-1} w_1'(x), \tag{33}$$

$$A_2 = n\left(w_0'(x)\right)^{n-1} w_2'(x) + \frac{n^2-n}{2}\left(w_0'(x)\right)^{n-2}\left(w_1'(x)\right)^2, \tag{34}$$

$$\vdots,$$

where `dash' over w reprents the derivative with respect to x. The recursive relation (31), after making use of equation (30) and Adomian polynomials $A_k, k \geq 0$ from equations (32)-(34), will result the following components of $w(x)$:

$$w_1(x) = -\frac{\beta S_t^n}{n+1}\left[1-(1-x)^{n+1}\right], \tag{35}$$

$$w_2(x) = \frac{\beta^2 S_t^{2n-1}}{2}\left[1-(1-x)^{2n}\right], \tag{36}$$

$$w_3(x) = -\frac{n\beta^3 S_t^{3n-2}}{2}\left[1-(1-x)^{3n-1}\right], \tag{37}$$

$$\vdots.$$

Putting the components $w_0(x), w_1(x), w_2(x)$ and $w_3(x)$ from equations ((30), (35)-(37)) into the decomposition series (27), we get the ADM solution of equation (12) of the form:

$$w(x) = \frac{S_t}{2}\left[1-(1-x)^2\right] - \frac{\beta S_t^n}{n+1}\left[1-(1-x)^{n+1}\right] + \frac{\beta^2 S_t^{2n-1}}{2}\left[1-(1-x)^{2n}\right] - \frac{n\beta^3 S_t^{3n-2}}{2}\left[1-(1-x)^{3n-1}\right] + \cdots, \tag{38}$$

which represents the velocity profile for the drainage of Sisko fluid film down the vertical belt.

We observed that the velocity profiles (23) and (38) obtained by PM and ADM, respectively are the same. In dimensionless form, the volume flow rate Q and average film velocity $\overline{w}$ are defined by

$$Q = \overline{w} = \int_0^1 w(x)dx, \tag{39}$$

which with the help of (38) yields

$$Q = \overline{w} = \frac{S_t}{3} - \frac{\beta S_t^n}{n+2} + \frac{\beta^2 n S_t^{2n-1}}{2n+1} - \frac{\beta^3(3n-1)S_t^{3n-2}}{6} + \cdots. \tag{40}$$

For $\beta = 0$, we retrieve Newtonian case [Van Rossum (1958)]. In dimensionless form, the shear stress exerted by the belt on fluid film is given by

$$S_{XZ} = \frac{dw}{dx} + \beta\left(\frac{dw}{dx}\right)^n, \tag{41}$$

where $S_{XZ} = \dfrac{S_{xz}}{a\sqrt{\dfrac{g}{\delta}}}$. By invoking velocity profile (38) into (41), one gets

$$\begin{aligned} S_{XZ} &= S_t(1-x) - \beta S_t^n(1-x)^n + n\beta^2 S_t^{2n-1}(1-x)^{2n-1} \\ &\quad - \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2}(1-x)^{3n-2} + \beta\Big[S_t(1-x) - \beta S_t^n(1-x)^n \\ &\quad + n\beta^2 S_t^{2n-1}(1-x)^{2n-1} - \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2}(1-x)^{3n-2} + \cdots\Big]^n + \cdots, \end{aligned} \tag{42}$$

which at $x = 0$ provides shear stress exerted by the belt on fluid film at the belt surface:

$$\begin{aligned} S_{XZ}|_{x=0} &= S_t - \beta S_t^n + n\beta^2 S_t^{2n-1} - \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2} \\ &\quad + \beta\left[S_t - \beta S_t^n + n\beta^2 S_t^{2n-1} - \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2} + \cdots\right]^n + \cdots. \end{aligned} \tag{43}$$

In dimensionless form, the force exerted by fluid film on the belt surface during drainage of Sisko fluid film is defined by

$$F_z = -\int_0^1 S_{XZ}|_{x=0}\, dx. \tag{44}$$

Equation (44), after making use of (43) yields

$$\begin{aligned} F_z &= -S_t + \beta S_t^n - n\beta^2 S_t^{2n-1} + \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2} \\ &\quad - \beta\left[S_t - \beta S_t^n + n\beta^2 S_t^{2n-1} - \frac{n(3n-1)}{2}\beta^3 S_t^{3n-2} + \cdots\right]^n - \cdots, \end{aligned} \tag{45}$$

which provides the force exerted by the fluid film on the belt surface.

The vorticity vector Ω (in dimensionless form) is calculated as

$$\overline{\Omega} = -\Big[S_t(1-x) - \beta S_t^n(1-x)^n + n\beta^2 S_t^{2n-1}(1-x)^{2n-1}$$

$$-\frac{n(3n-1)}{2}\beta^3 S_t^{3n-2}(1-x)^{3n-2}+\cdots\Bigg]\mathbf{j}, \tag{46}$$

in which $\mathbf{j}$ is the unit vector in the y -direction.

5. Results and Discussion

In the previous section, we have obtained the physical quantities like velocity profile, shear stress exerted by the belt on fluid film, volume flow rate, force exerted by the fluid film on the belt surface and the vorticity vector for the drainage of Sisko fluid film down a vertical belt. In this section, we shall give some physical insight to these physical quantities via Tables 1-3 (velocity profile, shear stress and vorticity vector) and graphs 2-5 (velocity profile and volume flow rate).

Table 1 represents the velocity distribution of Sisko fluid film (38), draining from a vertical belt for fluid behaviour index n, Sisko fluid parameter β and Stokes number S_t. From this table we depicts that, as we proceed in our domain the velocity $w(x)$ increases, i.e., $\forall\ x\epsilon[0,1]$ the velocity increases. This increase in velocity is more for Newtonian fluid film as compared to Sisko fluid film. Shear thickening fluid film showed more increase in velocity as compared to the shear thinning fluid film.

Table 2 shows the shear stress distribution of Sisko fluid film (42), draining from a vertical belt for fluid behaviour index n, Sisko fluid parameter β and Stokes number S_t. This table shows that shear stress is positive which indicates the fact that the it is applied from a region of lower velocity to a higher one. We delineate that shear stress S_{XZ} being the function of x decreases, i.e., decreases $\forall\ x\epsilon[0,1]$. Furthermore, we also noted that the decrease in shear stress for Sisko fluid film is more as compared to Newtonian fluid film and belt exert less shear stress on the Sisko fluid film than that of the Newtonian fluid film. Shear thinning fluid film bears more shear stress as compared to the shear thickening fluid film.

We present the vorticity vector distribution of Sisko fluid film (46), draining from a vertical belt for fluid behaviour index n, Sisko fluid parameter β and Stokes number S_t through table 3. This table shows that vorticity vector is negative, which indicates that fluid film has clockwise rotational effects. Being function of x, the vorticity effect decreases $\forall\ x\epsilon[0,1]$, i.e., maximum near the belt and minimum near the free surface. Newtonian fluid film show more clockwise rotation when we compared it with the Sisko fluid film. Furthermore, comparison of the shear thinning fluid film and the shear thickening fluid film showed that the shear thickening fluid film has greater rotational effects.

Table 1. Velocity distribution of thin film flow of Sisko fluid when $S_t = 0.6$

	Newtonian fluid film	Shear thinning fluid film	Shear thickening fluid film
	($\beta = 0$)	($n = 0.7, \beta = 0.4$)	($n = 1.3, \beta = 0.4$)
x	$w(x)$	$w(x)$	$w(x)$
0.0	0.00	0.000000	0.000000
0.1	0.06	0.036629	0.042050
0.2	0.11	0.068916	0.080104
0.3	0.15	0.096909	0.114087
0.4	0.19	0.120662	0.143917
0.5	0.22	0.140241	0.169509
0.6	0.25	0.155721	0.190771
0.7	0.27	0.167201	0.207597
0.8	0.29	0.174814	0.219866
0.9	0.30	0.178752	0.227426
1.0	0.31	0.179374	0.230046

Table 2. Shear stress distribution of thin film flow of Sisko fluid when $S_t = 0.6$

	Newtonian fluid film	Shear thinning fluid film	Shear thickening fluid film
	($\beta = 0$)	($n = 0.7, \beta = 0.4$)	($n = 1.3, \beta = 0.4$)
x	S_{XZ}	S_{XZ}	S_{XZ}
0.0	0.60	0.594372	0.577928
0.1	0.54	0.534223	0.522448
0.2	0.48	0.474051	0.466415
0.3	0.42	0.413847	0.409841
0.4	0.36	0.353601	0.352738
0.5	0.30	0.293295	0.295118
0.6	0.24	0.232894	0.236998
0.7	0.18	0.172332	0.178397
0.8	0.12	0.111436	0.119338
0.9	0.06	0.049528	0.059854
1.0	0.00	0.000000	0.000000

Table 3: Vorticity vector distribution of thin film flow of Sisko fluid when $S_t = 0.6$

	Newtonian fluid film	Shear thinning fluid film	Shear thickening fluid film
	($\beta = 0$)	($n = 0.7, \beta = 0.4$)	($n = 1.3, \beta = 0.4$)
x	$\overline{\Omega}$	$\overline{\Omega}$	$\overline{\Omega}$
0.0	-0.60	-0.388141	-0.943462
0.1	-0.54	-0.344508	-0.834565
0.2	-0.48	-0.301317	-0.728243
0.3	-0.42	-0.258631	-0.624625
0.4	-0.36	-0.216535	-0.523869
0.5	-0.30	-0.175144	-0.426169
0.6	-0.24	-0.134621	-0.331783
0.7	-0.18	-0.095214	-0.241065
0.8	-0.12	-0.057354	-0.154551
0.9	-0.06	-0.021933	-0.073202
1.0	0.00	0.000000	0.000000

Figures 2-4 are drawn to observe the effects of parameters n, β and S_t on velocity profile of the Sisko fluid film. Figure 2 shows that the velocity increases with the increase in fluid behaviour index n. It is observed that the velocity of the thin film is strongly dependent upon the Sisko fluid parameter β, which can be seen in Figure 3. The decrease in velocity with increasing β is evident. We also see that there is considerable decrease in magnitude of thin film velocity from Newtonian fluid film to Sisko fluid film. Figure 4 depicts that the velocity increases with increasing S_t.

In order to observe the effect of β on volume flow rate, figure 5 is drawn. It delineates that for any particular value of β, the volume flow rate increases with increasing S_t and decreases with the increase in β.

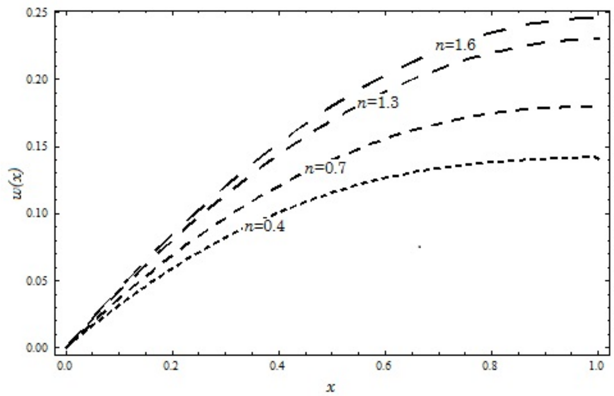

Figure 2. Profiles of the Sisko fluid film velocity, for various values of fluid behaviour index n when $S_t = 0.6$ and $\beta = 0.4$.

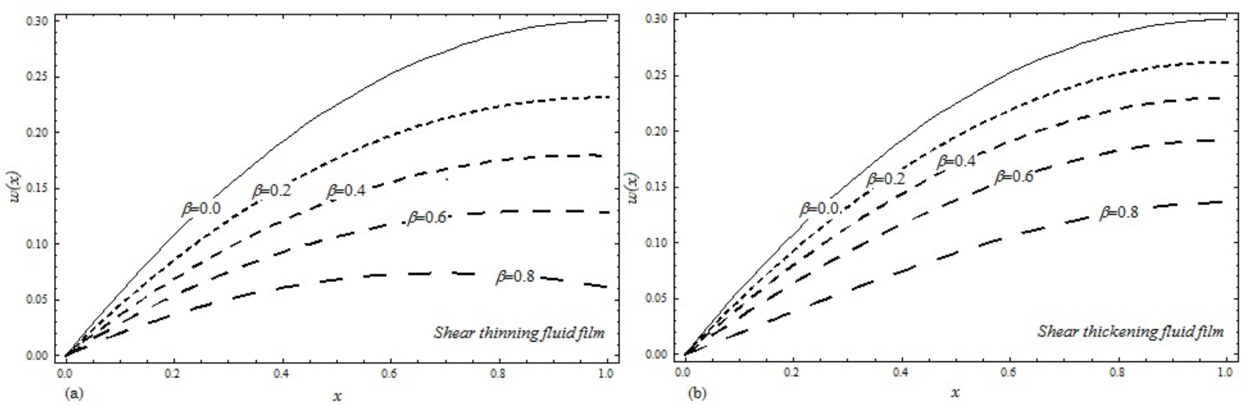

Figure 3. Profiles of the velocity, for various values of Sisko fluid parameter β when $S_t = 0.6$, (a) $n = 0.7$ and (b) $n = 1.3$

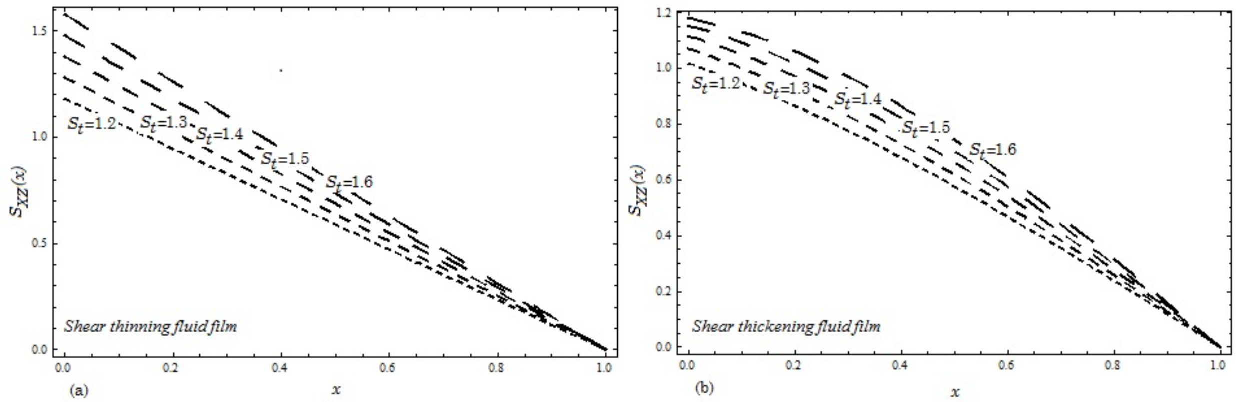

Figure 4. Profiles of the velocity, for various values of Stokes number S_t when $\beta = 0.4$, (a) $n = 0.7$ and (b) $n = 1.3$.

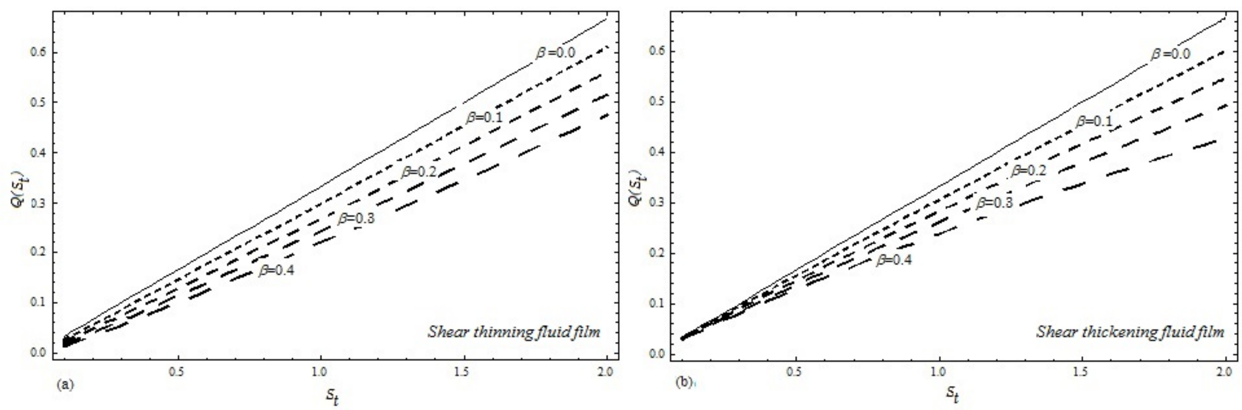

Figure 5. Profiles of the flow rate, for various values of Sisko fluid parameter β when (a) $n = 0.7$ and (b) $n = 1.3$

6. Concluding Remarks

In the present work, the drainage of a Sisko fluid film down the vertical belt is modeled. The governing nonlinear inhomogeneous ordinary differential equation is solved using PM and ADM. The results obtained (using PM and ADM) are found identical. Though, PM is well known and widely used approximate method which relies on the existence of a relatively small parameter but ADM is very simple and convenient method. ADM can be used to get any higher order solution recursively and with great accuracy. ADM does not require linearization, perturbation or any other such restrictive assumptions. The influences of fluid behaviour index n, Sisko fluid parameter β and Stokes number S_t on velocity of the Sisko fluid film, shear stress, volume flow rate and vorticity vector have been observed. We concluded that :

- Velocity of the Sisko fluid film increases with the increase in n and S_t and decreases with increasing β. The drainage of Sisko fluid film is slower as compared to the Newtonian fluid film.

- Shear stress experienced by the Sisko fluid film decreases in the domain and Sisko fluid film experienced less magnitude of shear stress as compared to the Newtonian fluid film.

- Volume flow rate of the Sisko fluid film increases with increase in the n and S_t and decreases with increasing β.

- Vorticity effect is maximum near the belt and minimum near the free surface and the Newtonian fluid film has more vorticity effect as compared to the Sisko fluid film.

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