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Coulomb energy of quasiparticle excitations in Chern–Simons composite fermion states

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Abstract

The attachment of flux tubes to electrons by a Chern–Simons (CS) singular gauge transformation of the wavefunction opened up the field theoretical description of the fractional quantum Hall effect (FQHE). Nevertheless, in Jain's composite fermion (CF) theory, quasiparticles are believed to be vortices carrying a fractional charge in addition to the winding phase of the CS flux tubes. The different structure of the wavefunction in these two cases directly affects the excitation energy gaps. By using a simple *ansatz* we were able to evaluate analytically the Coulomb excitation energies for the mean-field level CS wavefunction, thus allowing a direct comparison with corresponding numerical results obtained from Jain's CF picture. The considerable difference between the excitation energies found in these two cases demonstrates in quantitative terms the very different impact that the internal structure of the wavefunction has in these two approaches, often used interchangeably to describe the FQHE.

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1. Introduction

Two-dimensional electronic systems (2DES) subject to a strong perpendicular magnetic field display remarkable phenomena, reflecting the great importance of electronic correlations. The most important among them is the fractional quantum Hall effect (FQHE) [1,2] which results from a strongly correlated incompressible liquid state [3,4] formed at special densities of the 2DES. The basic physics of FQHE is well understood: the kinetic energy of the electrons is quenched by the strong perpendicular magnetic field and the Coulomb interaction dominates the physics of the highly degenerate partially filled Landau level.

The dominant sequence of FQHE states occurs when the filling factor of the lowest Landau level (LLL) is $\nu = p/(2mp+1)$, where $p = 1, 2, \ldots$ and $m = 1, 2, \ldots$ are integers. By definition ν is the ratio of the number of electrons to the degeneracy of each Landau level: the electron density is given by $\rho(\nu) = \nu/[2\pi l_0(B)^2]$, where $l_0(B) = \sqrt{h/(eB)}$ is the magnetic length. At these filling factors the electrons condense into a strongly correlated incompressible quantum liquid, giving rise to quantized Hall resistivity and thermally activated longitudinal resistivity.

Much of the theoretical work on FQHE is based on the study of the properties of a 2D fully spin-polarized (effectively spinless) system of N interacting electrons embedded in a uniform positive neutralizing background. The electrons with charge $-e$ ($e > 0$) and mass m_e are assumed to be confined in the xy-plane and subject to a strong perpendicular magnetic field $\mathbf{B} = (0, 0, B)$. Normally, the symmetric gauge is adopted and the vector potential is $\mathbf{A}(\mathbf{r}) = (-By/2, Bx/2, 0)$. We will consider the thermodynamic limit $N \to \infty$ and $\Omega \to \infty$, while $N/\Omega = \rho(\nu)$; where N is the number of electrons and Ω is the area of the 2D sample. The many-electron system is described by the Hamiltonian $\hat{H} = \hat{K} + \hat{V}$, where $\hat{K}$ is the kinetic energy operator.

$$\hat{K} = \frac{1}{2m_e}\sum_{j=1}^{N}[-ih\boldsymbol{\nabla}_{\mathbf{j}} + e\mathbf{A}(\mathbf{r}_j)]^2, \tag{1}$$

and $\hat{V}$ is the potential energy operator representing the

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electron–electron, electron–background and background–background interaction potentials

$$\hat{V} = \sum_{j<k}^{N} v(|\mathbf{r}_j - \mathbf{r}_k|) - \rho(\nu) \sum_{j=1}^{N} \int \mathrm{d}^2 r v(|\mathbf{r}_j - \mathbf{r}|) + \frac{\rho(\nu)^2}{2} \int \mathrm{d}^2 r_1 \int \mathrm{d}^2 r_2 v(|\mathbf{r}_1 - \mathbf{r}_2|), \quad (2)$$

where $v(|\mathbf{r}_j - \mathbf{r}_k|) = (e^2/4\pi\epsilon_0\epsilon|z_j - z_k|)$ is the Coulomb potential, $z_j = x_j + iy_j$ is the location of the jth electron in complex coordinates and ϵ is medium's dielectric constant.

It has become clear in recent years that many features of the FQHE can be understood in terms of a new kind of particle called a composite fermion (CF), which is a bound state of an electron and $2m$ vortices of the many-body quantum wavefunction [5,6]. The basic property of the CFs is that they experience a reduced effective magnetic field $B^* = B(1 - 2m\nu)$. Since the degeneracy of each Landau level N_s is proportional to the magnetic field, the degeneracy N_s^* of each CF Landau level will be $N_s^* = N_s(1 - 2m\nu)$. As a result, at filling factors corresponding to the main sequence of FQHE states, $\nu = p/(2mp + 1)$ (with p an integer), the effective filling factor of CFs will be $\nu^* = p$, that is, an integer number of CF LLs. In other words, this transformation maps the FQHE of electrons onto an *integer* QHE of CFs.

There are two calculational schemes based on the intuitive physics presented earlier. The first constructs explicit wavefunctions [5] while the second method employs a Chern–Simons (CS) field theory [7] approach to investigate the CF state. Although the two schemes are based on the same physics, a precise quantitative relationship between them has not been made clear. In the CS approach proposed by Halperin, Lee and Read [7] (HLR) to describe even-denominator-filled states, the electron system is subjected to a (mathematical) singular gauge transformation which converts it into a new system of fermions interacting with a CS magnetic-like field $\mathbf{b}(\mathbf{r})$ in addition to the original magnetic field. The transformation can be thought of as the binding of $2m$ fictitious magnetic flux quanta to an electron. If Ψ_ν is a solution of the Schrödinger equation $\hat{H}\Psi_\nu = E\Psi_\nu$ it will be related by

$$\Psi_\nu = \prod_{j<k}^{N} \frac{(z_j - z_k)^{2m}}{|z_j - z_k|^{2m}} \Psi', \quad (3)$$

to the wavefunction Ψ' which is a solution of the Schrödinger equation $\hat{H}'\Psi' = E\Psi'$, with Hamiltonian $\hat{H}' = \hat{K}' + \hat{V}$ and transformed kinetic energy operator,

$$\hat{K}' = \frac{1}{2m_e} \sum_{j=1}^{N} \{ -ih\nabla_j + e[\mathbf{A}(\mathbf{r}_j) - \mathbf{a}(\mathbf{r}_j)] \}^2, \quad (4)$$

where $\mathbf{a}(\mathbf{r})$ is the CS vector potential, $\mathbf{b}(\mathbf{r}) = \nabla \times \mathbf{a}(\mathbf{r}) = 2m\phi_0\rho(\mathbf{r})\mathbf{e}_z$ is the fictitious CS magnetic field, $\rho(\mathbf{r})$ is the local particle density, $\phi_0 = h/e$ is the magnetic field flux quantum and $\mathbf{e}_z$ is a unit vector perpendicular to the plane.

In a translationally invariant system, the mean-field Hamiltonian may be obtained by replacing $\mathbf{b}(\mathbf{r})$ by its mean value $\langle \mathbf{b} \rangle = 2m\phi_0\rho(\nu)\mathbf{e}_z$, where $\rho(\nu)$ is the uniform electronic density. This reduces the magnetic field to $B^* = B(1 - 2m\nu)$, thus increasing the effective filling factor ν^* for the CFs. When $\nu^* = \rho(\nu)\phi_0/B^* = p$ with p an integer, the corresponding electron filling factor is $\nu = \rho(\nu)\phi_0/B = p/(2mp + 1)$, which is precisely the Jain's series of FQHE states. In other words, at the mean-field level, the system can be described as a system of fermions subject to a reduced magnetic field $B^* = B/(2mp + 1)$. Therefore the CS field theory solution (at the mean-field level) will be the following CS CF wavefunction.

$$\Psi_\nu = \prod_{j<k}^{N} \frac{(z_j - z_k)^{2m}}{|z_j - z_k|^{2m}} \Phi_p(B^*), \quad (5)$$

where $\Phi_p(B^*)$ is the Slater determinant of p filled CF Landau levels evaluated at the magnetic field shown in the argument.

Such singular gauge transformation of the wavefunction corresponds to attaching $2m$ magnetic flux tubes to each electron. Flux tubes are neither charged, nor low-energy excitations, so in reality it is not clear how electrons can bind to them. In the CF theory, the quasiparticle is believed to be an electron bound to $2m$ *vortices*, instead of $2m$ flux tubes. Differently from a flux tube, a vortex brings a zero into the wavefunction, is therefore charged and, as a result, the attachment of vortices (instead of flux tubes) brings a significantly different physics. In fact, attachment of vortices forms the basis of the Jain's CF wavefunction theory [5]. Under general circumstances, however, Jain's CF wavefunction also needs a complete LLL projection in order to generate results directly comparable with experiments. The LLL projection is quite far from being a trivial operation and after projection it is not clear how to associate electrons and vortices in an unambiguous way. Nevertheless, both projected and unprojected Jain's CF wavefunctions describe binding of vortices to electrons and a significant amount of work concerning both groundstate and excited state properties of these wavefunctions has been reported in the literature [8–11].

On the other hand, based on field-theoretical grounds [12] the attachment of flux tubes, as in the CS CF wavefunction, precedes the attachment of vortices. Although it is expected that the attachment of flux tubes to the wavefunction does not capture the complete physics obtained by attachment of vortices, a detailed quantitative comparison of the two processes is highly desirable from a theoretical point of view and has not yet been realized. Since the nature of quasiparticles in the FQHE crucially depends on whether flux tubes or vortices are attached to electrons, a study of the low-energy excitations and excited state properties on both cases seems to be quite appealing on its own.

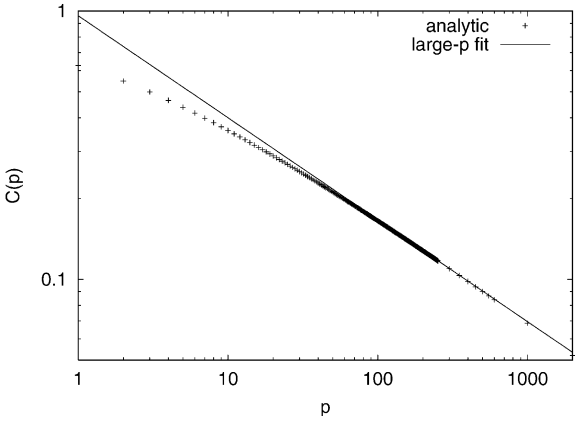

Fig. 1. The characteristic integral $C(p)$ (Eq. (11)) plotted in a logarithmic scale as a function of p. The fit corresponds to $C(p) \propto p^{-0.38}$.

The intent of this work is to provide a comparison between the excitation energies of flux-bound CFs described by the mean-field level CS CF wavefunction of Eq. (5) and those obtained for wavefunctions where vortices instead of fluxes are attached. We provide analytical expressions for the Coulomb excitation energy gaps corresponding to a specific CS CF wavefunction that incorporates the idea of flux binding to electrons and discuss the results in the context of the HLR theory. We compare our analytic results with corresponding numerical results obtained using Jain's picture of CFs where vortices bind to electrons. We show in quantitative terms how the excitation energy gaps are affected by the internal structure of the wavefunction.

2. Coulomb excitation energies

The excitation energy gap is defined as the energy to add one quasiparticle and one quasihole far away from each other (we will neglect exciton effects) to the FQHE ground-state at the given filling factor. In the CF language, the excitation energy gap corresponds to the quasiparticle–quasihole excitation obtained by promoting a single CF from the uppermost filled CF Landau level to the next higher CF Landau level.

The excited state wavefunction constructed using CFs (whether we have flux tubes or vortices bound to electrons) is not entirely in the LLL. Such intrinsic Landau level mixing in the wavefunction brings in some kinetic energy contribution into the excitation energy gap. In general, the energy gap can be written as $\Delta(\nu) = E_g(\nu) + \Delta_{KE}(\nu)$, where $E_g(\nu)$ is due to Coulomb correlation effects and $\Delta_{KE}(\nu) = \mathcal{O}(\hbar\omega_c)$ is related to the kinetic energy associated with the Landau level mixing. In the present work, we consider only intra-LL excitations consistent with the (generally used) assumption $h\omega_c \to \infty$. In this case only the Coulomb correlation energies are significant and $\Delta(\nu) \cong E_g(\nu)$ [9].

Exact calculations using Jain's CF approach have only been possible in systems with a small number of electrons [8]. However, for flux-bound electrons as in the CS CF picture, we point out that calculations can be readily performed by adopting a technique firstly introduced by Friedman and Pandharipande [13] in the context of nuclear matter. Suppose we are studying the FQHE state with filling $\nu = p/(2mp + 1)$ having p CF Landau levels filled. The uppermost filled CF Landau level is the one with quantum index $(p - 1)$. The promotion of a CF from the CF Landau level with quantum index $(p - 1)$ to the one with quantum index p produces a correlated wavefunction Ψ_ν^{ph} that describes the quasiparticle–quasihole excitation. Instead of promoting a single CF from the uppermost filled CF Landau level to the next empty one, we choose to promote a fraction $\Delta N = xN (\ll N_s^*)$ of such CFs. By consequence the quasiparticle–quasihole excitation energy will be given by:

$$E_g(\nu) = \lim_{\Delta N \to 1} \frac{\partial}{\partial \Delta N} \Delta U(x, N), \tag{6}$$

where the change on the total correlation energy due to this process is $\Delta U(x, N)$ and x is the small fraction of displaced CFs. Eq. (6) may be rewritten as

$$E_g(\nu) = \lim_{x \to 0} \frac{\partial}{\partial x} \left[\frac{\Delta U(x, N)}{N} \right],$$
$$\frac{\Delta U(x, N)}{N} = \frac{1}{N} \frac{\langle \Psi_\nu^{ph}(x) | \hat{V} | \Psi_\nu^{ph}(x) \rangle}{\langle \Psi_\nu^{ph}(x) | \Psi_\nu^{ph}(x) \rangle} - \frac{1}{N} \frac{\langle \Psi_\nu | \hat{V} | \Psi_\nu \rangle}{\langle \Psi_\nu | \Psi_\nu \rangle}, \tag{7}$$

where $\Psi_\nu^{ph}(x)$ denotes the quasiparticle–quasihole wavefunction for xN quasiparticle–quasihole excitations (by construction $\Psi_\nu^{ph}(x = 0) \equiv \Psi_\nu$). Only the first term in Eq. (7) is dependent on the fraction x, therefore one can write the quasiparticle–quasihole excitation energy as

$$E_g(\nu) = \lim_{x \to 0} \frac{\partial}{\partial x} u_\nu(x), \tag{8}$$

where $u_\nu(x)$ is the correlation energy per particle corresponding to the quasiparticle–quasihole wavefunction,

$$u_\nu(x) = \frac{1}{N} \frac{\langle \Psi_\nu^{ph}(x) | \hat{V} | \Psi_\nu^{ph}(x) \rangle}{\langle \Psi_\nu^{ph}(x) | \Psi_\nu^{ph}(x) \rangle}. \tag{9}$$

The mathematical details of the calculation of $u_\nu(x)$ are given in Appendix A (see Eq. (A10)). The final result is that the quasiparticle–quasihole Coulomb energy gap corresponding to the CS CF wavefunction (Eq. (5)) is given by:

$$E_g(\nu) = \frac{e^2}{4\pi\epsilon_0\epsilon l_0(B)} \frac{C(p)}{\sqrt{2mp + 1}}, \tag{10}$$

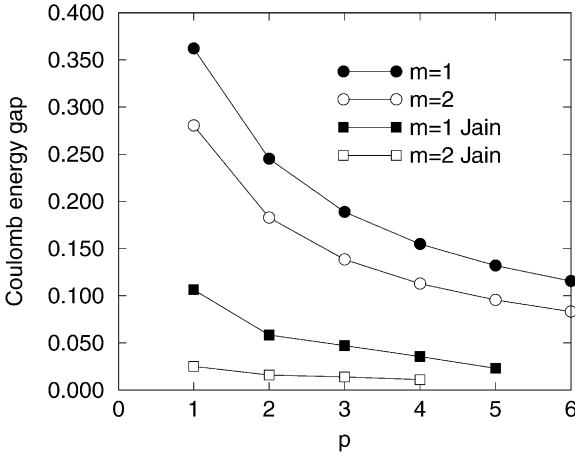

Fig. 2. Exact excitation energy gaps, $E_g(\nu)$ corresponding to the flux-bound CS CF wavefunction Ψ_ν obtained at the mean-field level of the CS field theory for filling factors $\nu = p/(2mp + 1)$ as function of p for $m = 1$ and 2 and $p \geq 1$, see Eq. (10). The results are compared to corresponding values for energy gaps [8] obtained for the LLL projected Jain's CF wavefunction, where vortices instead of flux tubes are bound to electrons. All energies are in units of $e^2/[4\pi\epsilon_0\epsilon l_0(B)]$. For large p: $E_g(\nu) \propto p^{-0.88}$.

where the m-independent $C(p)$ is given by

$$C(p) = \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t^2/2} L_{p-1}^1\left(\frac{t^2}{2}\right)\left[L_{p-1}\left(\frac{t^2}{2}\right) - L_p\left(\frac{t^2}{2}\right)\right], \tag{11}$$

and $L_n^\alpha(x)$ are associated Laguerre polynomials [14].

In what follows, we calculated exactly the integrals appearing in Eq. (11) for increasing values of p up to 2000. The dependence of $C(p)$ on p is shown in Fig. 1. In the large-p limit $C(p) \propto p^{-0.38}$.

In Fig. 2, we show the dependence of $E_g(\nu)$ on p for filling factors $\nu = p/(2mp + 1)$ for $m = 1$ and 2 with the excitation energy values expressed in units of $e^2/[4\pi\epsilon_0\epsilon l_0(B)]$. A striking feature of the results is the fact that the excitation energy gaps decrease very slowly as we approach the even-denominator-filled states $\nu = 1/2$ ($m = 1$) and $\nu = 1/4$ ($m = 2$) where, as expected, they vanish. To give a quantitative estimate of how the energy excitation gap is affected when we switch from flux attachment to vortex attachment, we compare our analytic results for flux-bound CFs to Monte Carlo results [8,15,16,] based on Jain's CF wavefunctions where vortex attachment is considered. One notes that the energy gaps for flux-bound CFs are considerably above the energy gaps obtained in the vortex-bound case. Our analytic results for flux-bound CFs are based on a mean-field wavefunction, so we have to expect an overestimation of the energy gaps since we are neglecting fluctuations, however, the large difference was not completely anticipated.

The wavefunction Ψ_ν obtained at the mean-field level of the CS theory (Eq. (5)) has a high occupation of higher LLs (is not projected) and has no special short-distance correlations when compared to Jain's CF wavefunction [5], obtained by throwing away the denominators in Eq. (5) and performing a projection onto the LLL [8,17]. In particular, the mean-field CS CF formalism does not lead to screening of the quasiparticles' or quasiholes' charge, differently from the fractionalization of charge which occurs beyond mean-field. Nevertheless both Jain's and the mean-field CS wavefunctions describe at some level CFs, and these have been generally assumed to be equivalent to a first degree of approximation. The results presented in this paper show how these assumptions can be misleading in some circumstances.

3. Asymptotic behavior of the gap

As the filling factor of the main sequence of FQHE states approaches the even denominator fractions 1/2 and 1/4 (obtained as $p \to \infty$ and $m = 1$ and 2, respectively) it is expected that the quasiparticle–quasihole excitation gap should vanish. From the behavior of $C(p)$ shown in Fig. 1 and Eq. (10), it is evident that this is, indeed, the case: our analysis of the exact CS excitation energies shows that $E_g(\nu)$ decays as $p^{-0.88}$ to a very good approximation for $50 \lesssim p \lesssim 2000$.

These results can be compared to the HLR theory at both mean-field [7] and beyond [18]. At the mean-field level, the HLR theory [7] predicts

$$E_g\left(\nu = \frac{p}{2mp+1}\right) \sim \frac{1}{(2mp+1)}\,\frac{1}{4\pi\epsilon_0}\,\frac{e^2}{\epsilon l_0(B)}, \tag{12}$$

as $p \to \infty$. This behavior is consistent with the fact that within this mean-field theory the effective mass m^* is unrenormalized by the fluctuations of the CS gauge field (see Eq. (4)) and should be p-independent. Since our approach neglects *all* fluctuation effects, the resulting large p energy gaps should decay *at most* as fast as $1/p$. Our results are consistent with this argument.

When interactions beyond the mean-field and/or RPA level are included and fluctuations in the transverse CS gauge field are considered, a considerable renormalization of the effective mass is to be expected. In fact, using a fermionic CS approach, Stern and Halperin [18] considered these fluctuations and found a divergent renormalization of m^* as $p \to \infty$. Therefore in this limit, the low energy excitations vanish faster as

$$E_g \approx \frac{\pi}{2m\sqrt{m}}\,\frac{1}{(2mp+1)\ln(2mp+1)}\,\frac{1}{4\pi\epsilon_0}\,\frac{e^2}{\epsilon l_0(B)}. \tag{13}$$

At present time, a fully exact calculation of energy gaps, starting from a microscopic wavefunction that incorporates effects beyond mean-field theory appears to be a very difficult task.

Quite recently, a sophisticated calculation for the FQHE energy gaps has been performed by Shankar [19] using the

Hamiltonian theory of CFs that he and Murthy developed [20]. In their Hamiltonian approach one firstly performs a $2m$-fold flux attachment by a CS transformation, and then attaches the necessary number of zeros to transform the fluxes into full vortices [20]. In this approach, the Hamiltonian contains complicated expressions for the charge and other operators, while the wavefunction remains quite simple (this treatment seems complimentary to Jain's approach, where the Hamiltonian is simple, but the wavefunction is very complicated because of the LLL projection operator). The formalism was then used to compute activation gaps for FQHE states and analytic expressions were derived for all fractions of the form $\nu = p/(2mp + 1)$. Based on this approach various quantities of interest were calculated to a reasonable accuracy: in particular it was shown that for the Zhang–Das Sarma [21] (ZDS) interaction potential, $v(r) = e^2/(\epsilon\sqrt{r^2 + \lambda^2})$, the gap energies were accurate to within 10–20% as compared to Monte Carlo work [22] for $\lambda > 1$. Favorable electronic correlations in the LLL are build up only through vortex attachment, as a result the energy gaps derived from the Hamiltonian theory are in better agreement and directly comparable to experimental and/or Monte Carlo results.

On the other hand, we limited ourselves to a mean-field model with a specific choice of the wavefunction incorporating attachment of $2m$-fold flux tubes to electrons. We were thus able to provide analytical results strictly valid for the specific choice of the mean-field derived CS CF wavefunction, without intending to directly compare the obtained energy gap values to realistic calculations where $2m$-fold vortices are attached to the electrons and full projection into the LLL is performed. The objective of this calculation was to demonstrate the significant effect that the lack of vortex attachment has on the excitation properties of the system. This is important due to the generalized impression that most of the physics of the FQHE is contained in the simple CS mean-field model.

4. Conclusions

In summary, by using a simple ansatz we were able to calculate analytically the excitation energy gaps for a mean-field CS CF wavefunction where $2m$-fold flux tubes are attached to electrons. This provides a direct quantitative comparison with corresponding numerical results for energy gaps where vortices, and not flux tubes, are bound to electrons as in Jain's CF wavefunction case. Based on these analytical results, we can directly estimate in quantitative terms the sizeable effect that the internal structure of CFs has on FQHE energy gaps, when switching from flux attachment to vortex attachment. The large discrepancy in the excitation energies thus obtained clearly shows the limitations of the commonly held belief that most of the important physics of the FQHE is contained in the simple mean-field approximation of the CS CF picture.

Acknowledgements

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Appendix A

For convenience we list here the calculation of the Coulomb correlation energy per electron $u_\nu(x)$ (Eq. (9)) which is given by

$$u_\nu(x) = \frac{1}{N}\frac{\langle \Psi_\nu^{ph}(x)|\hat{V}|\Psi_\nu^{ph}(x)\rangle}{\langle \Psi_\nu^{ph}(x)|\Psi_\nu^{ph}(x)\rangle} = \frac{\rho(\nu)}{2}\int \mathrm{d}^2 r_{12}[g_\nu(x, r_{12}) - 1]v(r_{12}), \tag{A1}$$

where the radial distribution function $g_\nu(x, r_{12})$ is defined in terms of $\Psi_\nu^{ph}(x)$ by

$$g_\nu(x, r_{12}) = \frac{N(N-1)}{\rho(\nu)^2}\frac{\int \mathrm{d}^2 r_3 \cdots \mathrm{d}^2 r_N |\Psi_\nu^{ph}(x)|^2}{\int \mathrm{d}^2 r_1 \cdots \mathrm{d}^2 r_N |\Psi_\nu^{ph}(x)|^2}. \tag{A2}$$

In the thermodynamic limit, the above quantity depends on the 2D interparticle distance $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ and the quasiparticle–quasihole fraction parameter x. In our case, one observes that $|\Psi_\nu^{ph}(x)|^2 = |\Phi_p(x, B^*)|^2$, where the latter term corresponds to a squared Slater determinant representing $(p-1)$ fully filled CF Landau levels, the pth CF Landau level (with CF Landau level index $(p-1)$) filled except for $\Delta N = xN(\ll N_s^*)$ CF holes, and the $(p+1)$th CF Landau level (with CF Landau level index p) having the ΔN CFs that were removed from the underlying level. As a standard result, it follows that for fully spin-polarized electrons (CFs)

$$g_\nu(x, r_{12}) = 1 - |l_p(x, r_{12})|^2, \tag{A3}$$

where the 'statistical exchange' factor is computed from $l_p(x, r_{12}) = \hat{\rho}_p(x, \mathbf{r}_1, \mathbf{r}_2)/\rho(\nu)$. The (reduced) one-body density matrix $\hat{\rho}_p(x, \mathbf{r}_1, \mathbf{r}_2)$ which corresponds to the dynamically uncorrelated state $\Phi_p(x, B^*)$ is given by

$$\hat{\rho}_p(x, z_1, z_2) = \sum_{n=0}^{p-1}\sum_{l=0}^{N_s^*-1}\varphi_{n,l}^*(z_1)\cdot\varphi_{n,l}(z_2) + \frac{\Delta N}{N_s^*}\left[\sum_{l=0}^{N_s^*-1}\varphi_{p,l}^*(z_1)\cdot\varphi_{p,l}(z_2) - \sum_{l=0}^{N_s^*-1}\varphi_{(p-1),l}^*(z_1)\cdot\varphi_{(p-1),l}(z_2)\right], \tag{A4}$$

where $\Delta N/N_s^* = xp$ is the average occupancy of a quantum state in the partially filled CF Landau level with index p. The symmetric gauge single-particle eigenstates for magnetic

field B^* are

$$\varphi_{n,l}(z) = \frac{1}{\sqrt{2^n n!}} \exp\left[\frac{zz^*}{4l_0(B^*)^2} \right] \times \left(2l_0(B^*)\frac{\partial}{\partial z}\right)^n \left\{ \varphi_{0,l}(z) \exp\left[-\frac{zz^*}{4l_0(B^*)^2} \right] \right\}, \quad \text{(A5)}$$

where

$$\varphi_{0,l}(z) = \frac{1}{\sqrt{2^l l! 2\pi l_0(B^*)^2}} \left[\frac{z}{l_0(B^*)} \right]^l \exp\left[-\frac{zz^*}{4l_0(B^*)^2} \right], \quad \text{(A6)}$$

where $n = 0, 1, \ldots$ denotes the index of various CF Landau levels and $l = 0, 1, \ldots$ $(N_s^* - 1)$ is their angular momentum quantum number. Noting that

$$\sum_{l=0}^{N_s^*-1} \varphi_{n,l}^*(z_1)\varphi_{n,l}(z_2) = L_n\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) \cdot \sum_{l=0}^{N_s^*-1} \varphi_{0,l}^*(z_1)\varphi_{0,l}(z_2), \quad \text{(A7)}$$

one finds that

$$\hat{\rho}_p(x, z_1, z_2) = \sum_{l=0}^{N_s^*-1} \varphi_{0,l}^*(z_1)\varphi_{0,l}(z_2) \left\{ L_{p-1}^1\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) + xp\left[L_p\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) - L_{p-1}\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) \right] \right\}, \quad \text{(A8)}$$

where $L_n(x) = (e^x/n!)(\mathrm{d}^n/\mathrm{d}x^n)(x^n e^{-x})$ are the Laguerre polynomials of order $n = 0, 1, \ldots$ and $L_n^k(x) = (-1)^k(\mathrm{d}^k/\mathrm{d}x^k) \times [L_{n+k}(x)]$ are the generalized Laguerre polynomials of order $n = 0, 1, \ldots$ and degree $k = 0, 1, \ldots$

In the thermodynamic limit [23], both density and filling factor $\nu = N/N_s$ are kept constant as the number of electrons N and the Landau level degeneracy N_s go to infinity. Since the degeneracy N_s^* of each CF Landau level is directly proportional to N_s then also $(N_s^* - 1)$ goes to infinity in this limit. At this point the summation over l in Eq. (A8) is extended from 0 to ∞ and one obtains the one-body density matrix $\hat{\rho}_p(x, z_1, z_2)$ that no longer depends on N_s^*. The statistical exchange term $l_p(x, z_1, z_2) = \hat{\rho}(x, z_1, z_2)/\rho(\nu)$ is a complex quantity, but since the radial distribution function is found by squaring it (see Eq. (A3)), the phase factor of $l_p(x, z_1, z_2)$ vanishes and we obtain

$$g_\nu(x, r_{12}) = 1 - \exp\left[-\frac{r_{12}^2}{2l_0(B^*)^2} \right] \left\{ \frac{1}{p} L_{p-1}^1\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) + x\left[L_p\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) - L_{p-1}\left(\frac{r_{12}^2}{2l_0(B^*)^2} \right) \right] \right\}^2. \quad \text{(A9)}$$

The calculation of $u_\nu(x)$ follows from Eq. (A1). After writing $\rho(\nu) = p/[2\pi l_0(B^*)^2]$, we introduce the dimensionless variable $t = r_{12}/l_0(B^*)$, and by rescaling the magnetic length $l_0(B^*)^2 = l_0(B)^2(2mp + 1)$ we obtain

$$u_\nu(x) = -\frac{p}{2\sqrt{2mp+1}} \int_0^\infty \mathrm{d}t \exp\left(-\frac{t^2}{2} \right) \left\{ \frac{1}{p} L_{p-1}^1\left(\frac{t^2}{2} \right) + x\left[L_p\left(\frac{t^2}{2} \right) - L_{p-1}\left(\frac{t^2}{2} \right) \right] \right\}^2 \frac{1}{4\pi\epsilon_0} \frac{e^2}{\epsilon l_0(B)}. \quad \text{(A10)}$$

The results in Eqs. (10) and (11) follow immediately.

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