Application of Fermi-hypernetted-chain theory to composite-fermion quantum Hall states

O. Ciftja

International School for Advanced Studies, Via Beirut 2-4, I-34014, Trieste, Italy
and Istituto Nazionale per la Fisica della Materia, Sezione di Trieste, Trieste, Italy

S. Fantoni

International School for Advanced Studies, Via Beirut 2-4, I-34014, Trieste, Italy;
Istituto Nazionale per la Fisica della Materia, Sezione di Trieste, Trieste, Italy;
and International Centre for Theoretical Physics, P.O. Box 586, I-34014, Trieste, Italy

The Fermi-hypernetted-chain (FHNC) theory and the effective hypernetted-chain method are applied to study the composite-fermion (CF) states of the fractional quantum Hall effect. Using this theory we compute, in the thermodynamic limit, the correlation energy, radial distribution function, and static structure factor for all unprojected CF wave functions. The unprojected excitation gaps for $\nu = 1/3, 1/5$ were obtained by adopting in the FHNC a scheme previously used to compute nuclear matter excitation spectra. The results obtained so far are consistent with Monte Carlo simulations and small-number exact diagonalizations. [S0163-1829(97)04444-5]

I. INTRODUCTION

The fractional quantum Hall effect[1] (FQHE) results from a strongly correlated incompressible fluid state[2,3] formed at special densities ρ of a two-dimensional electronic system in the extreme quantum limit of a strong perpendicular magnetic field ($B > 5$ T), low temperature ($T < 2$ K), and high mobility of electrons ($\mu > 10^5$ cm^2/V s). For fully spin-polarized (spinless) electrons, the most pronounced states occur when filling of the lowest Landau level (LLL) is $\nu = 1/m$, where $m = 1, 3, 5, \ldots$ is an odd integer.[4] The many-electron system is described by the Hamiltonian $\hat{H} = \hat{K} + \hat{V}$, where $\hat{K}$ is the kinetic energy operator

$$\hat{K} = \frac{1}{2m_e} \sum_{j=1}^{N} [-i\hbar \vec{\nabla}_j + e\vec{A}(r_j)]^2 \quad (1)$$

and

$$\hat{V} = \hat{V}_{ee} + \hat{V}_{eb} + \hat{V}_{bb} = \sum_{j<k}^{N} \frac{e^2}{\epsilon |z_j - z_k|} + \sum_{j=1}^{N} V(z_j) + \hat{V}_{bb} \quad (2)$$

is the total electron-electron, electron-neutralizing background, and background-background interaction energies, where $e > 0$ is the charge magnitude of the electron, ϵ is the dielectric constant of background, and m_e is the electron mass.

The origin of electronic states at so-called higher-order FQHE states at $\nu = p/q$ has been less clear. Jain[5,6] has proposed a remarkably simple picture to explain the origin of the FQHE by introducing the idea of a particle called a composite fermion (CF), which is an electron carrying an even number of vortices of the wave function. The fundamental property of the CF's is that they experience an effective field $B^* = B - q_e \phi_0 \rho$, where $\phi_0 = h/e$ is the quantum of the magnetic flux and q_e is an even integer. Thus the liquid of

strongly correlated electrons at B is equivalent to a liquid of weakly interacting CF's at B^*; the FQHE is regarded as the integer quantum Hall effect of composite fermions. The stable fractional filling factors obtained in this way are $\nu = p/(q_e p \pm 1)$, where $q_e = 0, 2, 4, \ldots$ is the even number of vortices attached to each electron and $p = 1, 2, 3, \ldots$ is the corresponding number of CF Landau levels. For sake of simplicity we confine the discussion below to the special filling factors $\nu = p/(q_e p + 1)$, where the corresponding CF filling factor is $\nu^* = p$.

Let us denote the ground state of noninteracting electrons at $\nu^* = p$ by $|\Phi_p\rangle$. The corresponding wave function for the CF's is obtained by attaching q_e vortices to each electron in the state $|\Phi_p\rangle$, which amounts to correlating $|\Phi_p\rangle$ by multiplication with a Jastrow factor $\Pi_{j<k}(z_j - z_k)^{q_e}$. Thus the electronic ground state at $\nu = p/(q_e p + 1)$ is described by the trial CF wave function

$$|\Psi_{CF}\rangle = \hat{P}_{LLL} \prod_{j<k}^{N} (z_j - z_k)^{q_e} |\Phi_p\rangle, \quad (3)$$

introduced by Jain,[5] where $\hat{P}_{LLL}$ is the lowest-Landau-level projection operator. For the special case of the ground state at $\nu = 1/(q_e + 1)$, namely, for $p = 1$, the CF wave function is identical to the Laughlin wave function,[4] which is already known to be a very accurate representation of the exact ground state at $\nu = 1, 1/3, 1/5$. There is also strong evidence for the validity of the CF theory from several numerical studies performed mainly on few-electron systems.[7,8]

Exact calculations in particular are limited to systems with few electrons and extrapolation to the thermodynamic limit is not totally unambiguous. The difficulty becomes more severe as $\nu \to 1/2$. The Fermi-hypernetted-chain (FHNC) technique seems very attractive in this respect, as it treats the many-particle fermionic system exactly in the thermodynamic limit.

In this paper we apply the FHNC theory and the effective-hypernetted-chain (EFHNC) method to the unprojected CF wave functions of the FQHE corresponding to filling factors $\nu = p/(q_e p + 1)$, where $q_e = 0,2,4$ and $p = 1,2,3,\ldots$. This represents a step towards a more systematic study in which the projection onto the LLL will be taken into account. We find that FHNC theory provides a simple and powerful tool to deal with unprojected CF wave functions for both ground-state and excited-state properties. The results obtained so far are consistent with previous calculations performed on systems with a finite number of electrons.

The scheme of the paper is as follows. In Sec. II we discuss how to apply FHNC theory to the unprojected CF wave functions. In Sec. III we develop a simpler scheme that approximates the FHNC equations to compute the radial distribution function. The calculation of the quasiparticle-quasihole excitation spectrum for a specific case is presented in Sec. IV. Section V is devoted to the results and conclusions.

II. APPLICATION OF FERMI-HYPERNETTED-CHAIN THEORY FOR THE COMPOSITE-FERMION STATES

Integral equation techniques such as hypernetted-chain (HNC) theory for bosons[9,10] or Fermi-hypernetted-chain theory for fermions[11,12] allow for a realistic evaluation of the radial distribution function and related quantities for Jastrow and Jastrow-Slater wave functions. In particular, they are extremely useful when calculations must be performed strictly in the thermodynamic limit.

Generally speaking, FHNC theory is applied on Fermi systems described by a many-body wave function of the form

$$|\Psi\rangle = \prod_{i<j}^{N} f(r_{ij})|\Phi\rangle. \tag{4}$$

The ket $|\Phi\rangle$ is a Slater determinant of single-particle states $\varphi_\alpha(\vec{r})$ for fermions and $f(r_{ij}) = f(|\vec{r}_i - \vec{r}_j|)$ is the so-called dynamical correlation factor. More elaborate trial wave functions, containing, for instance, triplet and/or backflow correlations, can also be handled with the FHNC theory.

The radial distribution function $g(r_{12})$ is expressed as a sum of irreducible cluster diagrams constructed with (i) the "bosonic" bond $h(r_{ij}) = f(r_{ij})^2 - 1$ and (ii) the "statistical exchange" bond $l(\vec{r}_i, \vec{r}_j) = \hat{\rho}(\vec{r}_i, \vec{r}_j)/\rho$, where ρ is the particle density, $\hat{\rho}(\vec{r}_i, \vec{r}_j)$ is the uncorrelated one-body density matrix

$$\hat{\rho}(\vec{r}_i, \vec{r}_j) = g_s \sum_\alpha \varphi_\alpha^*(\vec{r}_i) \varphi_\alpha(\vec{r}_j), \tag{5}$$

and $g_s = 1$ is the spin degeneracy for the case of interest. In Eq. (5), the sum over α is extended over all occupied single-particle states $\varphi_\alpha(\vec{r})$.

For a magnetic field $\vec{B}$ in the z direction, with a symmetric gauge vector potential $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$, the eigenstates of the ideal Hamiltonian

$$\hat{H}_0 = \frac{1}{2m_e}(-i\hbar\vec{\nabla} + e\vec{A})^2 \tag{6}$$

for the various Landau levels $n = 0,1,2,\ldots$ are given by

$$|n,m\rangle = \varphi_{n,m}(z) = \frac{1}{\sqrt{2^n n!}} \exp\left(\frac{zz^*}{4l_0^2}\right) \left(2l_0\frac{\partial}{\partial z}\right)^n$$

$$\times \left[\varphi_{0,m}(z)\exp\left(-\frac{zz^*}{4l_0^2}\right)\right], \tag{7}$$

where

$$\varphi_{0,m}(z) = \frac{1}{\sqrt{2^m m!}}\left(\frac{z}{l_0}\right)^m \varphi_{0,0}(z), \tag{8}$$

$$\varphi_{0,0}(z) = \frac{1}{\sqrt{2\pi l_0^2}} \exp\left(-\frac{zz^*}{4l_0^2}\right), \tag{9}$$

where φ_0 is the magnetic length and $m = 0,1,2,\ldots$ is the angular momentum quantum number. The manifold of states with energy $\hbar\omega_c(n + 1/2)$ constitutes the nth Landau level.

The first step in the application of FHNC theory is the knowledge of an orthonormal set of single-particle wave functions that fully describe the unperturbed Fermi system. To illustrate this idea, let us first consider the simple case $\nu = 1$, obtained for $p = 1$ and $q_e = 0$. The CF wave function $|\Psi_{CF}^{\nu=1}\rangle$ is in this case the Vandermonde determinant of single-particle states $\varphi_{0,m}(z)$ of Eq. (8). The density matrix for the case $\nu = 1$ is

$$\hat{\rho}_{\nu=1}(z_1, z_2) = \sum_{m=0}^{N_s-1} \varphi_{0,m}^*(z_1)\varphi_{0,m}(z_2)$$

$$= \frac{1}{2\pi l_0^2} \exp\left(-\frac{1}{4}\frac{|z_1 - z_2|^2}{l_0^2}\right)$$

$$\times \exp\left[-\frac{1}{4l_0^2}(z_1^* z_2 - z_1 z_2^*)\right], \tag{10}$$

where N_s is the degeneracy of each Landau level. We observe that $\hat{\rho}_{\nu=1}(z,z) = \rho_1$, where $\rho_1 = 1/2\pi l_0^2$ is the density that corresponds to $\nu = 1$. For the case $\nu = 1$, q_e in Eq. (3) is equal to zero and the uncorrelated radial distribution function is simply

$$g_{\nu=1}(z_1, z_2) = 1 - e^{-\pi\rho_1|z_1 - z_2|^2}. \tag{11}$$

For CF states, at filling $\nu = 1/(q_e + 1)$ (the Laughlin states) one needs the full machinery of FHNC theory. In this case only the LLL orbitals are occupied; therefore the statistical exchange term is

$$l_\nu(z_1, z_2) = \exp\left(-\frac{1}{4}\frac{|z_1 - z_2|^2}{l_0^2}\right)\exp[i\phi(z_1, z_2)], \tag{12}$$

with the phase factor $\phi(z_1, z_2)$ given by

$$\phi(z_1,z_2)=\frac{1}{2l_0^2}r_1r_2\sin(\theta_2-\theta_1)=\frac{1}{2l_0^2}(\vec{r}_2\times\vec{r}_1)_z. \quad (13)$$

It is known that the FHNC technique is intrinsically approximate because there is a set of cluster diagrams (corresponding to the so-called *elementary* diagrams) that cannot be fully included in any closed form. Several schemes have been devised to include such cluster diagrams at various levels of approximation. The simplest approximation of totally neglecting these terms [the FHNC technique at zeroth order (FHNC/0)] leads to reliable results and we have adopted it in this paper.

The full formalism of FHNC/0 theory for the Laughlin states has been reported elsewhere.[13] Here we limit ourselves to the generalization of such a scheme to the unprojected CF states.

As in a standard computation, we separate the CF pseudopotential $U(r_{12})=2q_e\ln(|z_1-z_2|)$ into short- and long-range parts

$$U_s(r_{12})=-2q_eK_0(Qr_{12}), \quad (14)$$

$$U_l(r_{12})=2q_eK_0(Qr_{12})+2q_e\ln(r_{12}). \quad (15)$$

$K_0(x)$ is the modified Bessel function and Q is a cutoff parameter of order $1/l_0$. Furthermore, all nodal and non-nodal functions are split into their short- and long-range parts so that the FHNC/0 scheme can be applied directly.

Knowing that the general CF state of the form $\nu=p/(q_ep+1)$ is described by the trial CF wave function of Eq. (3), the one-body density matrix is written as

$$\hat{\rho}_\nu(z_1,z_2)=\sum_{n=0}^{p-1}\sum_{m=0}^{N_s-1}\varphi_{n,m}^*(z_1)\varphi_{n,m}(z_2). \quad (16)$$

One can easily prove that the contribution to $\hat{\rho}_\nu(z_1,z_2)$ coming from the nth Landau level is

$$\sum_{m=0}^{N_s-1}\varphi_{n,m}^*(z_1)\varphi_{n,m}(z_2)=L_n\left(\frac{|z_1-z_2|^2}{2l_0^2}\right)$$
$$\times\sum_{m=0}^{N_s-1}\varphi_{0,m}^*(z_1)\varphi_{0,m}(z_2),$$
$$(17)$$

where $L_n(x)$ are the Laguerre polynomials of order n. After some algebra, the statistical exchange term turns out to be

$$l_\nu(z_1,z_2)=\left\{\sum_{n=0}^{p-1}L_n\left(\frac{|z_1-z_2|^2}{2l_0^2}\right)\Big/p\right\}$$
$$\times\exp\left(-\frac{|z_1-z_2|^2}{4l_0^2}\right)\exp\left[\frac{i}{2l_0^2}(\vec{r}_2\times\vec{r}_1)_z\right].$$
$$(18)$$

The systematic knowledge of the statistical exchange term $l_\nu(z_1,z_2)$ for all the fractional states $\nu=p/(q_ep+1)$ enables us to readily apply the FHNC theory to all the unprojected CF states.

The intrinsic Landau level (LL) mixing of the CF wave functions implies a projection onto the LLL. The Jastrow factor provides a good projection,[14] which is particularly effective as far as ground-state properties are concerned. However, to study other quantities, such as the excitation spectrum, the full LLL projection seems to be needed.

Unfortunately, such a projection leads to a wave function that cannot be treated directly within the FHNC formalism because the structure of a determinant of single-particle orbitals is lost. One can adopt the projection technique used by Bonesteel[15] to calculate the excitation gaps of $\nu=1/3,1/5,1/7$, which, however, is limited to Slater determinants spanning two Landau levels only. The extension of such a technique to more LL's appears to be numerically inaccessible.

A more general projection scheme, applied to few-electron systems in a spherical geometry,[8] seems to be more promising. Such a scheme introduces a many-body dependence on all single-particle orbitals, which, however, can be handled by introducing state-dependent correlations in the wave function, in close analogy to "backflow" correlations[16] of liquid ^{3}He.

III. EFFECTIVE HYPERNETTED-CHAIN METHOD

In this section, we summarize the effective hypernetted-chain (EFHNC) method,[13] also known as the Lado approximation.[17] The square modulus of the Slater determinant $|\Phi|^2$, a positive-value function, can be written in the form

$$|\Phi|^2=\exp\left[\sum_{i<j}^N w_2(r_{ij})+\sum_{i<j<k}^N w_3(\vec{r}_i,\vec{r}_j,\vec{r}_k)+\cdots\right],$$
$$(19)$$

which emphasizes the fact that the Pauli principle introduces many-body correlations between particles in analogy to the pseudopotential describing the dynamic correlations. Within the EFHNC approximation only two-body correlations[17] are retained in Eq. (19), namely,

$$|\Phi|^2\approx\exp\left[\sum_{i<j}^N w(r_{ij})\right]. \quad (20)$$

In the next step of the systematic approximation scheme, one may include triple correlation factors, etc., until the required accuracy is achieved.

The EFHNC approximation greatly simplifies the calculation of the radial distribution function,[18] which is the only quantity needed to compute the interaction energy per particle. Within this approximation one has

$$|\Psi|^2=\prod_{i<j}^N f(r_{ij})^2\prod_{i<j}^N \exp[w(r_{ij})]=\prod_{i<j}^N \widetilde{f}(r_{ij})^2, \quad (21)$$

with the effective correlation factor

$$\widetilde{f}(r_{ij})=f(r_{ij})\exp[\tfrac{1}{2}w(r_{ij})]. \quad (22)$$

To construct the pseudopotential $w(r)$ for a given Slater determinant we require that the HNC evaluation of the radial

distribution function $g_{HNC}(r)$ for $|\Phi|^2$ reproduces the *exact* radial distribution function of the noninteracting system, namely,

$$g_{HNC}(r_{ij}) = g_{\text{exact}}(r_{ij}) = 1 - \frac{1}{g_s}|l_{\text{exact}}(r_{ij})|^2. \quad (23)$$

This prescription leads us to the pseudopotential

$$w(r) = \ln[g_{\text{exact}}(r)] - \frac{1}{\rho}\left[\frac{[S_{\text{exact}}(k)-1]^2}{S_{\text{exact}}(k)}\right]^{F^{-1}}, \quad (24)$$

where $S_{\text{exact}}(k) = 1 + \rho[g_{\text{exact}}(r)-1]^F$, $[f(r)]^F$ denotes the two-dimensional Fourier transform of a function $f(r)$, and $[f(k)]^{F^{-1}}$ is the two-dimensional inverse Fourier transform of $f(k)$. For a Slater determinant with p-filled Landau levels we get

$$g_{\text{exact}}(r) = 1 - \exp\left(-\frac{r^2}{2l_0^2}\right)\left\{\sum_{n=0}^{p-1} L_n\left(\frac{r^2}{2l_0^2}\right) \middle/ p\right\}^2. \quad (25)$$

With these expressions as input, Eq. (24) provides an explicit expression for the potential $w(r)$ and, consequently, due to Eqs. (21) and (22), we obtain

$$|\Psi|^2 = \prod_{i<j}^N \exp[\tilde{u}(r)], \quad (26)$$

where $\tilde{u}(r) = 2q_e\ln(r) + w(r)$. We may now employ the familiar boson HNC formalism.

Decomposing the function $w(r)$ into a short- and a long-range portion

$$w(r_{12}) = w_s(r_{12}) + w_l(r_{12}), \quad (27)$$

we may write

$$w_s(r_{12}) = \ln[g_{\text{exact}}(r_{12})] \quad (28)$$

and

$$w_l(r_{12}) = -\frac{1}{\rho}\left\{\frac{[S_{\text{exact}}(k)-1]^2}{S_{\text{exact}}(k)}\right\}^{F^{-1}}. \quad (29)$$

This decomposition achieves an analogous convenient separation for the pseudopotential $\tilde{u}(r)$,

$$\tilde{u}_s(r_{12}) = -2q_e K_0(Qr_{12}) + w_s(r_{12}), \quad (30)$$

$$\tilde{u}_l(r_{12}) = 2q_e K_0(Qr_{12}) + 2q_e\ln(r_{12}) + w_l(r_{12}). \quad (31)$$

Finally, the k-space representation of the long-range part of the pseudopotential $\tilde{u}_l(k)$ may be analytically obtained and the standard HNC theory can be employed.

IV. THE QUASIPARTICLE-QUASIHOLE EXCITATION SPECTRUM OF THE CF STATE

In this section we report a method used to compute the quasiparticle-quasihole excitations for the unprojected CF wave function within the FHNC theory. The low-energy neu-

tral excitations are obtained by promoting a single CF to the next higher CF Landau level. For instance, a low-energy band of excited states above the $\nu = 1/(q_e + 1)$ ground state is constructed by promoting a CF from the lowest pseudo-Landau level to the first excited pseudo-Landau level.

The excitation gaps can be calculated by adopting a technique introduced by Friedman and Pandharipande[19] in the context of nuclear matter. Suppose we have p Landau levels filled. The statistical exchange correlation associated with $|\Phi_p\rangle$ is $l_{q_e}(p,r_{12})$ and the interaction energy per particle $u_{q_e}(p)$ is a functional of $f(r)$ and $l_{q_e}(p,r)$. Promoting a CF from the pth LL to the $(p+1)$th one will produce a correlated wave function $|\Psi_{ph}\rangle$ that is orthogonal to $|\Psi_{CF}\rangle$ because of angular momentum conservation. The quasiparticle-quasihole excitation introduces a new statistical exchange term in the cluster diagrams of the radial distribution function, given by

$$l_{q_e}^{ph}(p,z_1,z_2) = \frac{1}{\rho}\left\{\sum_{n=0}^{p-1}\sum_{m=0}^{N_s-1}\varphi_{n,m}^*(z_1)\varphi_{n,m}(z_2)\right.$$
$$+ \frac{1}{N_s}\sum_{m=0}^{N_s-1}\varphi_{p,m}^*(z_1)\varphi_{p,m}(z_2)$$
$$\left. - \frac{1}{N_s}\sum_{m=0}^{N_s-1}\varphi_{p-1,m}^*(z_1)\varphi_{p-1,m}(z_2)\right\}. \quad (32)$$

In calculating the energy per particle

$$u_{q_e}^{ph}(p) = \frac{1}{N}\frac{\langle\Psi_{ph}|\hat{V}|\Psi_{ph}\rangle}{\langle\Psi_{ph}|\Psi_{ph}\rangle}$$

corresponding to $|\Psi_{ph}\rangle$, the quasiparticle-quasihole exchange term $l_{q_e}^{ph}(p,z_1,z_2)$ must occur only once in any FHNC cluster diagram, so that the excitation energy $\Delta_{q_e}(p) = u_{q_e}^{ph}(p) - u_{q_e}(p)$ is of the order of $1/N_s$, as it should be.

The calculation of $\Delta_{q_e}(p)$ can be done by introducing a ''mixed'' statistical exchange correlation

$$l_{q_e}(x,p,r_{12}) = l_{q_e}(p,r_{12}) + x[l_{q_e}(p+1,r_{12}) - l_{q_e}(p,r_{12})], \quad (33)$$

where the fraction x of CF's removed from the pth LL and placed to the next higher $(p+1)$th level is considered as a smallness parameter. The derivative with respect to x of the excitation energy $\Delta_{q_e}^{ph}(x,p)$ gives the quasiparticle-quasihole gap for a general CF state $\nu = p/(q_e p + 1)$, namely,

$$\Delta_{q_e}(p) = \frac{\partial}{\partial x}\{u_{q_e}^{ph}[l_{q_e}(x,p,r),f(r)] - u_{q_e}^{ph}[l_{q_e}(p,r),f(r)]\}. \quad (34)$$

The calculation of $u_{q_e}^{ph}[l_{q_e}(x,p,r),f(r)]$ is done in the same way as the calculation for $u_{q_e}^{ph}[l_{q_e}(p,r),f(r)]$, namely, employing the same FHNC code.

TABLE I. Interaction energies per particle $u(\nu)$ expressed in units $e^2/\epsilon l_0$, computed using unprojected CF wave functions, for fillings $\nu = p/(2p+1)$. The values in the second and third columns refer to the FHNC/0 and EFHNC/0 approximations, in the fourth column we report the estimates of Jain and Kamilla (Ref. 20) obtained using projected CF wave functions in the spherical geometry, while in the fifth column we show the results of Ref. 21 using unprojected CF wave functions.

ν	FHNC/0	EFHNC/0	Ref. 20	Ref. 21
1/3	−0.40257	−0.4056	−0.409828(27)	−0.4092(7)
2/5	−0.43054	−0.4309	−0.432804(62)	−0.4489(1)
3/7	−0.44510	−0.4452	−0.442281(62)	−0.4644(2)
4/9	−0.45300	−0.4531	−0.447442(115)	−0.4734(15)
5/11	−0.45796	−0.4580	−0.450797(175)	
6/13	−0.46137	−0.4614		
7/15	−0.46386	−0.46389		
8/17	−0.46576	−0.46578		

V. RESULTS AND CONCLUSIONS

In this section we report the results obtained for the ground-state interaction energy per particle, radial distribution function, and quasiparticle-quasihole excitation spectrum for several unprojected CF wave functions. The ground-state interaction energy per particle is computed from

$$u_{q_e}(p) = \frac{1}{N} \frac{\langle \Psi_{CF} | \hat{V} | \Psi_{CF} \rangle}{\langle \Psi_{CF} | \Psi_{CF} \rangle} = \frac{\rho}{2} \int v(r)[g(r)-1] d^2r,$$

(35)

where $v(r) = e^2/\epsilon r$. The radial distribution function $g(r)$ has been calculated by using the FHNC/0 and EFHNC/0 approximations as described in the previous sections. Tables I and II present the ground-state energies per particle of the two series of FQHE states $\nu = 1/3, 2/5, 3/7, \ldots$ and $\nu = 1/5, 2/9, 3/13$. One can see that the results obtained with the two schemes are almost identical. At $\nu = 1/2$ both FHNC/0 and EFHNC/0 give an interaction energy per particle of $-0.479 e^2/\epsilon l_0$.

Our results are in good agreement with the most recent estimates by Jain and Kamilla[20] obtained with projected CF wave functions for rather large systems in the standard spherical geometry and agree within a few percent with the unprojected estimates of Kamilla and Jain.[21]

There are two approximations in our calculations. One concerns the neglect of elementary diagrams, the other is the missing LLL projection of the CF wave function. The inclusion of elementary diagrams can be easily performed within the EFHNC scheme. We have done that by using the scaling approximation,[22] finding that the ground-state interaction energy is lowered by $\sim 1\%$. As far as the LLL projection is

TABLE II. Same as in Table I for fillings $\nu = p/(4p+1)$.

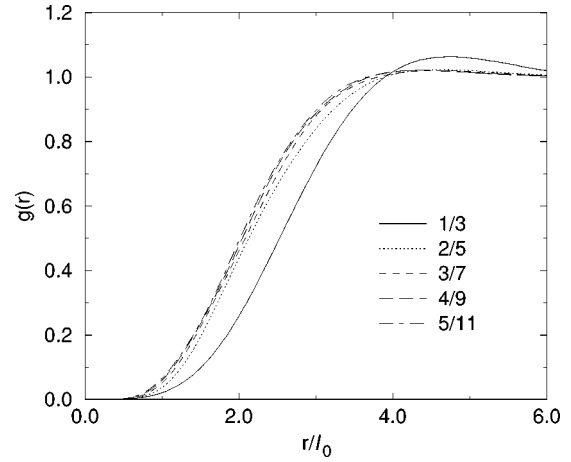| ν	FHNC/0	EFHNC/0	Ref. 20
1/5	−0.32281	−0.3243	−0.327499(5)
2/9	−0.33743	−0.33748	−0.342782(35)
3/13	−0.34380	−0.34384	−0.348349(19)

FIG. 1. Radial distribution function $g(r)$ computed using the FHNC/0 theory for the series of FQHE states $\nu = 1/3, 2/5, 3/7, 4/9, 5/11$, for the unprojected CF wave functions

concerned, our results show that its absence has little influence on the ground-state properties of the system and slightly increases with ν approaching 1/2. The radial distribution function $g(r)$ for all fractional Hall states $\nu = 1/3, 2/5, 3/7, \ldots$ and $\nu = 1/5, 2/9, 3/13$, obtained using unprojected CF wave functions, is plotted in Figs. 1 and 2, respectively.

The excitation gaps for Laughlin states obtained using the FHNC theory, for the unprojected CF wave functions, are given in Table III. They are compared with the corresponding results of Bonesteel[15] from a variational Monte Carlo simulation for 42 electrons for the unprojected and projected cases and the extrapolated exact diagonalization results of Fano et al.[23] As expected, the lack of projection of the CF excited state onto the LLL leads to an underestimation of the excitation gap by a factor of about 2.

Within the EFHNC method, one can calculate analytically the small-r behavior for different filling factors. As an explicit example we took the case of $\nu = 1/3$ and $\nu = 2/5$, which have the same Jastrow factor but different Slater determinant functions. One has that $g(r \to 0) \approx \exp[\widetilde{u}_s(r \to 0)]$.
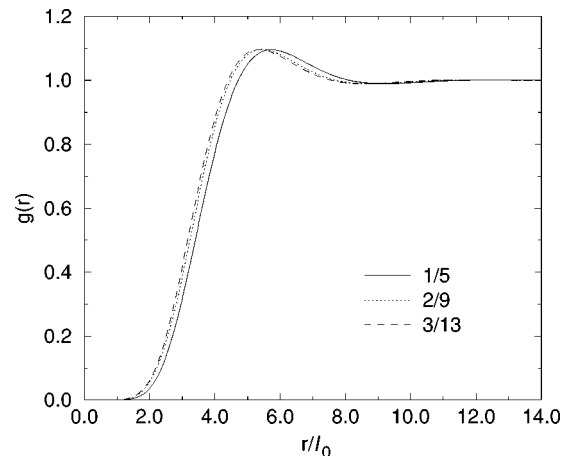

FIG. 2. Radial distribution function $g(r)$ for the states $\nu = 1/5, 2/9, 3/13$, computed from the FHNC/0 theory for the unprojected CF wave functions.

TABLE III. Energy gaps for $\nu = 1/3, 1/5$, computed using the FHNC theory for the unprojected CF wave function, are given in the second column. They are compared with the results of Bonesteel (Ref. 15) and Fano *et al.* (Ref. 23). The excitation gap energies are all expressed in units of $e^2/\epsilon l_0$.

ν	Δ_{unp}(FHNC)	Δ_{unp} (Ref. 15)	Δ_{proj} (Ref. 15)	Δ (Ref. 23)
1/3	0.04	0.048(2)	0.106(3)	0.1036(2)
1/5	0.01	0.014(2)	0.025(3)	0.0244(3)

Using Eq. (30) and the formula $\lim_{r \to 0} K_0(Qr) = -\ln(Qr/2) - \gamma$, where $\gamma = 0.5772 \ldots$ is the so-called Euler constant, we get

$$g(r \to 0) \approx \exp\left[2q_e \ln\left(\frac{Qr}{2}\right) + 2q_e \gamma + w_s(r \to 0) \right]. \quad (36)$$

For filling $\nu = 1/3$, $w_s(r \to 0) \approx 2\ln(r/l_0) - \ln(2)$, implying

$$\ln[g(r \to 0)] \sim (2q_e + 2)\ln\left(\frac{r}{l_0}\right). \quad (37)$$

Therefore, the leading term in the small-r behavior of $g(r)$ is $(r/l_0)^6$ for $\nu = 1/3$.

The same calculation for filling $\nu = 2/5$ gives $w_s(r \to 0) \approx 2\ln(r/l_0)$. It turns out that also the state $\nu = 2/5$ has the same small-r behavior in $g(r)$ as the case $\nu = 1/3$. One can prove that $(r/l_0)^6$ is the leading term of $g(r)$ at small r also for the successive fillings $\nu = 3/7, \ldots$.

The log-log plot of the small-r behavior of $g(r)$ obtained numerically from our EFHNC/0 calculations for the cases $\nu = 1/3$ and $\nu = 2/5$ is shown in Fig. 3. The explanation of the anomaly lies in the missing projection onto the LLL. The absence of such a projection mainly affects the small-r behavior of the radial distribution function.

Such a leading term is the one fixed by the Laughlin part of the wave function and is not modified by the inclusion of higher Landau levels through the Slater determinant part. Therefore, we expect that the main effect of projection onto the LLL is the correction of such a tendency.
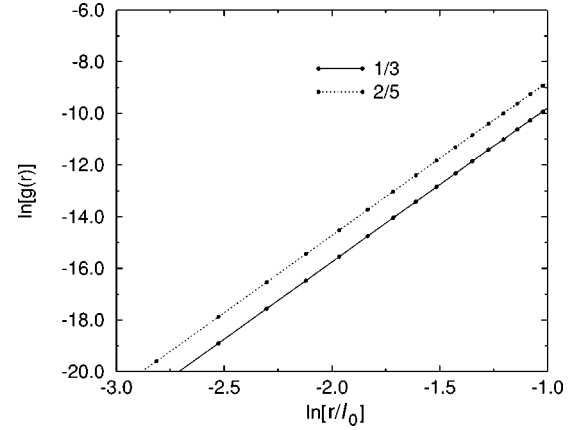

FIG. 3. Small-r behavior for fillings $\nu = 1/3$ (circled) and $\nu = 2/5$ (squared). $\ln[g(r)]$ versus $\ln[r/l_0]$ is plotted. One observes the same small-r behavior $g(r) \sim (r/l_0)^6$ for both cases.

In this paper we applied the FHNC/0 and EFHNC/0 approaches to study, in the thermodynamic limit, the unprojected CF states of the fractional quantum Hall effect. Such a treatment for ground-state properties seems to give consistent results compared with Monte Carlo simulations and small-number diagonalizations. We find that the effect of spuriousness from higher Landau levels is small in the calculation of ground-state properties, whereas it is not at all negligible for excited-state properties, such as quasiparticle-quasihole energy gaps. Within the EFHNC/0 method we proved that the radial distribution function for the unprojected CF's at fillings with $p > 1$ does not have the correct small-r behavior.

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