Liquid Crystalline States for Two-dimensional Electrons in Strong Magnetic Fields

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Background

In 1983 Laughlin introduced his famous trial wave function

$$\Psi_{1/m} = \prod_{i<j}^{N}(z_i - z_j)\prod_{\mu=1}^{m-1}(z_i - z_j - \alpha_\mu)\; e^{-\frac{1}{4l_0^2}\sum_{k=1}^{N}|z_k|^2}$$

to describe the fractional quantum Hall effect (FQHE) for odd-denominator filling factors $\nu = 1/m$.

- Since then, many attempts were done to compare the stability of these states against other known ground states (typically Wigner Crystals (WC))
- Theoretical understanding is that WC states are favorable at zero temperature for filling factors smaller than $\nu_c \simeq 1/6.5$. For larger factors the electrons are believed to form a quantum liquid state with Laughlin's wave function being an excellent choice.
- In transitional regions between QH plateaus for high LL's either a smectic or a nematic phase exists.

Liquid Crystal States

We consider translationally invariant and quasi-long-range orientational order with rotational symmetry groups C_2, C_4, and C_6. Basic requirements on how to construct Broken Rotational Symmetry (BRS) states:

- must obey Fermi statistics;
- must be translationally invariant;
- there must have a broken rotational symmetry;
- must belong to a single LL.

Construction of BRS states.

BRS wave functions can be systematically constructed by properly splitting the zeroes of he Laughlin's liquid state,

$$\Psi_{1/m} = \prod_{i<j}^{N}(z_i - z_j)\prod_{\mu=1}^{m-1}(z_i - z_j - \alpha_\mu)\exp\left(-\frac{1}{4l_0^2}\sum_{k=1}^{N}|z_k|^2\right)$$

where

$\alpha_\mu = \alpha\; e^{i2\pi(\mu-1)/(m-1)}$; α real

Monte Carlo Studies for filling factors 1/3, 1/5 and 1/7

Interaction Potentials and Monte Carlo Simulations

In order to investigate the possibility of a liquid crystal state in the lowest Landau level (LLL), we consider electrons interacting through the potentials,

- $V_1(\mathbf{r}) = e^2 \Big/ \left[\varepsilon\sqrt{r^2+\lambda^2}\right]$
- $V_2(\mathbf{r}) = e^2/\varepsilon\; \dfrac{1-e^{-\frac{r}{\lambda}}}{r}$

(λ ~ thickness of 2DES).

Since the potentials involved only depend on two-body interactions, we need to determine (by means of Monte Carlo simulations) the pair correlation function

$$g(\mathbf{r}'-\mathbf{r}'') = \frac{1}{\rho_0^2}\left\langle \sum_{i\neq j}\delta(\mathbf{r}_i-\mathbf{r}')\delta(\mathbf{r}_j-\mathbf{r}'')\right\rangle$$

and the density

$$\rho(\mathbf{r}) = \left\langle \sum_{i=1}^{N}\delta(\mathbf{r}_i-\mathbf{r})\right\rangle$$

The correlation energy in higher LL's can be obtained as

$$E_\alpha^L = \frac{1}{2}\int_0^\infty \frac{dq}{2\pi}\, q\; v(q)\left[L_L\left(\frac{q^2}{2}\right)\right]^2\left[S(q)-1\right]$$

where

$$S(q) = \int_0^{2\pi}\frac{d\theta_q}{2\pi}\, S(\mathbf{q})$$

is the angle averaged structure factor and $L_L(x)$ are the Laguerre polynomials corresponding to the Landau Level index L.

Monte Carlo Runs.

Results were obtained with

- 2×10^6-4×10^7 Monte Carlo Steps (MCS) for averaging
- systems of 200-400 electrons

Soft Charge Density Waves

We consider the classical distribution function

$$|\Psi_{1/m}|^2 \propto e^{-\beta V}$$

where

$$-\beta V = 2\sum_{i<j}^{N}\left[\ln|z_i - z_j| + \sum_{\mu=1}^{m-1}\ln|z_i - z_j - \alpha_\mu|\right] - \frac{1}{2}\sum_{i=1}^{N}|z_i|^2$$

The potential generated by the addition of some charge $\delta\rho(\mathbf{r})$ will cause a redistribution of the particles inducing a density change

$$\rho_{ind}(\mathbf{r}) = \int d^2r'\rho_0\left[g(\mathbf{r}-\mathbf{r}')-1\right]\delta\rho(\mathbf{r}')$$

generating a total potential

$$\beta\phi(\mathbf{k}) = \frac{4\pi S(\mathbf{k})}{k^2}\left[1+\sum_{\mu=1}^{m-1}e^{i\alpha_\mu \mathbf{k}}\right]\delta\rho(\mathbf{k})$$

assuming a small fluctuation from the uniform state

$$\rho(\mathbf{r}) = \rho_0 + \rho_1\cos(\mathbf{q}\cdot\mathbf{r})\;;\;\rho_1 \ll \rho_0$$

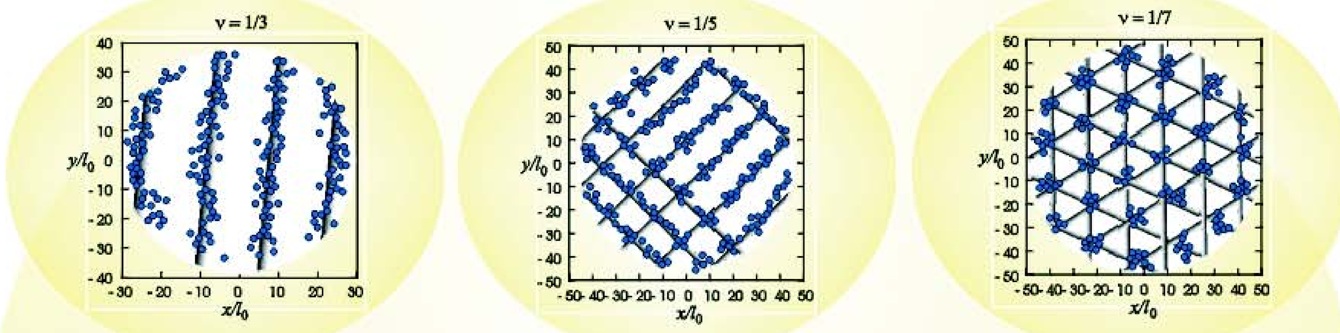
we obtain an "excess energy" per unit area

$$\frac{\beta u^{exc}}{\rho_1} = \frac{1}{2}\frac{2\pi S(\mathbf{q})}{q^2}\left[1+\sum_{\mu=1}^{m-1}e^{i\alpha_\mu \mathbf{q}}\right]$$

which becomes negative for a variety of wave vectors.

Results

Underlying charge density wave in liquid crystalline states

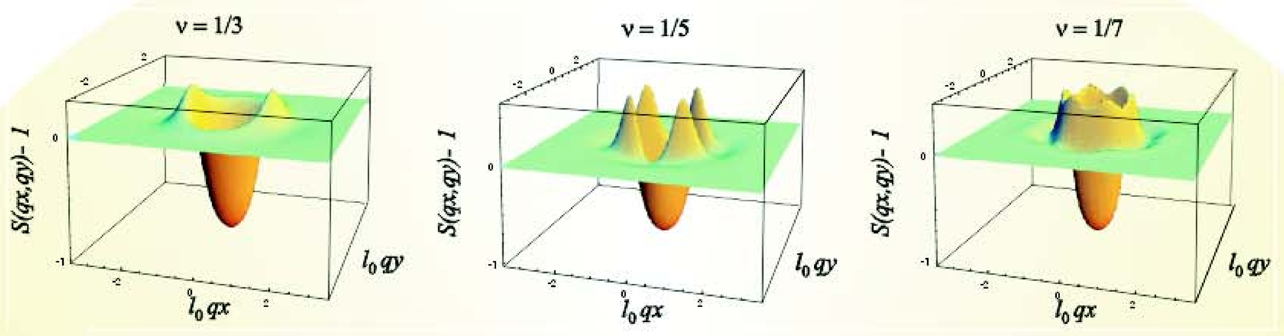


Electron configurations for a nematic (ν=1/3, α=7, left panel), tetratic (ν=1/5, α=8, center panel) and hexatic (ν=1/7, α=10, right panel).

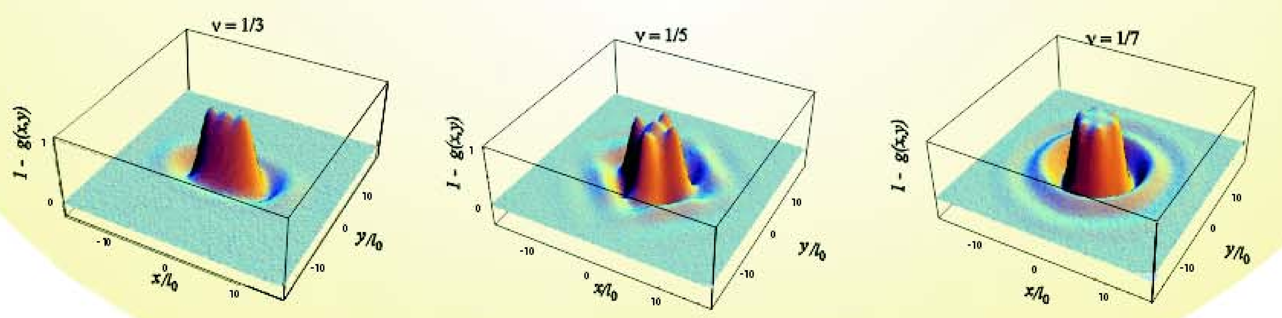
The CDW's have 1, 2 and 3 different directors for BRS states of the groups C_2, C_4, and C_6 respectively. The "excess energy" becomes negative for wave vectors in these directions for each filling factor.

Results

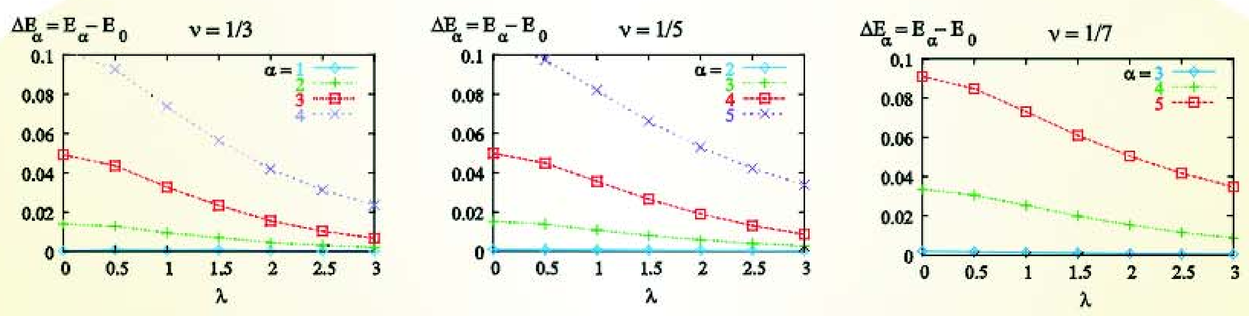
Structure Factor



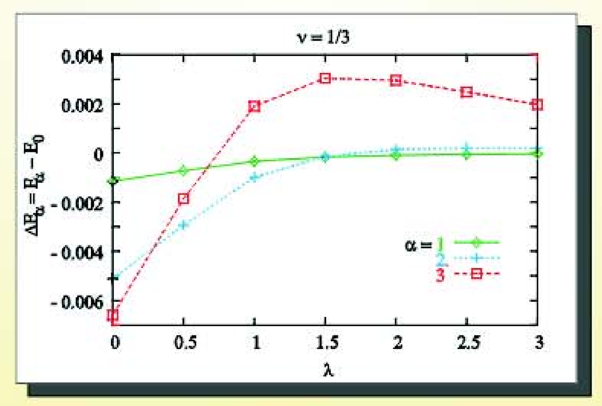
Pair Correlation Function



Correlation Energy in the LLL



Correlation Energy in the second excited LL for filling factor ν=1/3



References:

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