SUMMARY
Phys 2523 (University Physics II)
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- **Coulomb's Law**: $$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

where $k_e = \frac{1}{4\pi\epsilon_\circ} = 8.9875 \times 10^9$ N·m^2/C^2, and $\epsilon_\circ = 8.85 \times 10^{-12}$ C^2/(N·m^2) is the **permittivity of free space**. Generally,

$$\mathbf{F}_e(\mathbf{r}_\circ) = q_\circ \mathbf{E}(\mathbf{r}_\circ)$$

- **Electric Field** (N/C=V/m):

$$\mathbf{E}(\mathbf{r}_\circ) = k_e \sum_i \frac{q_i}{r^2}\hat{\mathbf{r}} = k_e \sum_i q_i \frac{\mathbf{r}_i - \mathbf{r}_\circ}{|\mathbf{r}_i - \mathbf{r}_\circ|^3} \quad \text{(distribution of point charges)}$$

$$\mathbf{E}(\mathbf{r}_\circ) = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}} = k_e \int q_i \frac{\mathbf{r} - \mathbf{r}_\circ}{|\mathbf{r} - \mathbf{r}_\circ|^3} dq \qquad \text{(continuous distribution)}$$

where $dq = \rho d^3 r$, and ρ is the charge density. In steady-state, the electrical force is conservative, and the electric field can be derived from a potential V:

$$\mathbf{E} = -\nabla V$$

where ∇ is the gradient operator, $\nabla = \hat{\mathbf{x}}\frac{d}{dx} + \hat{\mathbf{y}}\frac{d}{dy} + \hat{\mathbf{z}}\frac{d}{dz}$ in rectangular coordinates.

- **Potential Energy** (J): $$\Delta U = -q_\circ \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- **Electric Potential** (V=J/C): $V = U/q_\circ$; $\Delta V = \Delta U / q_\circ$

$$V(\mathbf{r}_\circ) = k_e \sum_i \frac{q_i}{r} = k_e \sum_i \frac{q_i}{|\mathbf{r}_i - \mathbf{r}_\circ|} \qquad \text{(distribution of point charges)}$$

$$V(\mathbf{r}_\circ) = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{|\mathbf{r} - \mathbf{r}_\circ|} \qquad \text{(continuous distribution)}$$

Electric charge is quantized in units of $e = 1.602 \times 10^{-19}$C. An electron volt, $eV = 1.602 \times 10^{-19}$J.

- **Electric Flux** (N·m^2/C=V·m): $$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

- **Gauss's Law**:

$$\Phi_E = \oint_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{inside}}{\epsilon_\circ} \qquad \text{(integral form)}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_\circ} \qquad \text{(differential form)}$$

- A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a magnitude $\sigma/\epsilon_\circ$, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.
5. The potential is constant everywhere inside a conductor and equal to its value at the surface.

- **Capacitance** (F):

$$C \equiv \frac{Q}{\Delta V}$$

and the energy stored in a capacitor is $U = \frac{1}{2}C(\Delta V)^2$.

- A **dielectric** material is composed of electric dipoles of magnitude $p = aq$, where a is the dipole separation. A background electric field exerts a torque $\overrightarrow{\tau} = \mathbf{p} \times \mathbf{E}$ on the dipoles. The potential energy of the system of an electric dipole in an electric field is $U = -\mathbf{p} \cdot \mathbf{E}$. The potential energy of the dipoles reduces the potential difference in a capacitor,

$$\Delta V = \frac{\Delta V_\circ}{\kappa}, \text{ and } C = \kappa C_\circ$$

where κ is the **dielectric constant**. The alignment of its dipoles results in an induced electric field (E_{ind}) that reduces the background electric field ($E_\circ$) inside the dielectric to a value $E = E_\circ - E_{ind}$. A charge density $\sigma_{ind} = \left(\frac{\kappa-1}{\kappa}\right)\sigma$ is induced on its surface. Its **dielectric strength** is the maximum electric field that can be applied to the dielectric before its insulating properties break down and it begins to conduct.

The capacitance of a **parallel-plate capacitor** is

$$C = \frac{\kappa\epsilon_\circ A}{d}$$

where A is the area of each plate, and d is the plate separation.

- **Energy Density** (J/m^3):

$$u = \frac{\epsilon E^2}{2}$$

is the energy density contained in an electric field, where $\epsilon = \kappa\epsilon_\circ$.

- **Electrical Current** (A=C/s): $$I = \frac{dQ}{dt}$$

- **Ohm's Law:** $$\mathbf{J} = \sigma \mathbf{E}$$

where $\mathbf{J} = nq\mathbf{v}_d$ is the current density, and σ is the **conductivity**. The **resistivity** $\rho = 1/\sigma = m_e/(nq^2\tau)$ in units of Ω·m, where τ is the mean time between collisions. Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature, $\rho = \rho_\circ[1 + \alpha(T - T_\circ)]$, where $\alpha = \frac{1}{\rho_\circ}\frac{\Delta\rho}{\Delta T}$ is the **temperature coefficient of resistivity**. Resistance, R can be expressed similarly.

- **Resistance** (Ω=V/A): $$R \equiv \frac{\Delta V}{I}$$

Note that this equation, $V = IR$, is also referred to as Ohm's law. In terms of the resistivity, $R = \rho l/A$, where l is the length and A is the cross-sectional area of the material.

- **Electrical Power** (W): $$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

- The **emf** ϵ of a battery is the maximum possible voltage that the battery can provide between its terminals. The terminal voltage will be $\Delta V = \epsilon - Ir$, where r is the internal resistance of the battery.

- **Combinations of Circuit Elements:**

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i} \qquad \text{(series combination)}$$

$$R_{eq} = \sum_i R_i \qquad \text{(series combination)}$$

$$C_{eq} = \sum_i C_i \qquad \text{(parallel combination)}$$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \qquad \text{(parallel combination)}$$

- **Kirchhoff's Rules:** $$\sum I_{in} = \sum I_{out} \qquad \text{(junction rule)}$$

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad \text{(loop rule)}$$

- **RC Circuits:**

Charging: $$q = q_\circ \left[1 - e^{-t/\tau}\right];\ I = I_\circ e^{-t/\tau}$$

where q is the charge on the capacitor at time t, $q_\circ = C(\Delta V)$ is the equilibrium (fully charged) value of the charge, $I_\circ = \Delta V/R$ is the current at $t = 0$, and $\tau = RC$ is the time constant of the dc circuit.

Discharging: $$q = q_\circ e^{-t/\tau};\ I = \frac{q_\circ}{RC} e^{-t/\tau}$$

where $q_\circ$ is the charge at $t = 0$.

- **Magnetic Force on Particle:**

$$\mathbf{F_B} = q\mathbf{v} \times \mathbf{B};\ |\mathbf{F_B}| = vB\sin\theta$$

- **Gyroradius:** $$r_g = \frac{mv_\perp}{|q|B}$$

where the sense of rotation for a negatively charged particle follows the right-hand rule.

- When placed in a force field, **F**, in addition to a magnetic field, **B**, a charged particle's gyrocenter will drift with a velocity:

$$\mathbf{v}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2};\ |\mathbf{v}_{drift}| = \frac{F}{|q|B}\sin\theta$$

in order that the net force on the particle be zero. For example, when the additional force is the electrical force, a charged particle will drift such that $\mathbf{F}_{net} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$. Solving for the velocity of its gyrocenter:

$$\mathbf{v}_{drift} = \frac{\mathbf{E} \times \mathbf{B}}{B^2};\ |\mathbf{v}_{drift}| = \frac{E}{B}\sin\theta$$

- **Force on a Current Segment:**

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B};\ |\mathbf{F}| = ILB\sin\theta$$

- **Torque on a Current Loop:**

$$\vec{\tau} = NI\mathbf{A} \times \mathbf{B} = \vec{\mu} \times \mathbf{B};\ \tau = NIAB\sin\theta$$

where the magnetic moment is $\vec{\mu} = NI\mathbf{A}$.

- **Magnetic Field Due to Currents** (T):

Long, straight wire: $$B = \frac{\mu_\circ I}{2\pi r}$$

Center of a current loop: $$B = \frac{N\mu_\circ I}{2R}$$

Center of a partial current loop: $$B = \frac{N\mu_\circ I\theta}{4\pi R}$$

Interior of a solenoid: $$B = \mu_\circ nI = \frac{\mu_\circ NI}{\ell}$$

where $\mu_\circ = 4\pi \times 10^{-7}$ T·m/A, N is the number of turns, and the direction of **B** is found using the right-hand rule.

- **Ampere's Law:** $$\oint \mathbf{B} \cdot \overrightarrow{d\ell} = \mu_\circ I_{enclosed}$$

- **Biot-SavartLaw:** $$d\mathbf{B} = \frac{\mu_\circ I}{4\pi} \frac{\overrightarrow{d\ell} \times \hat{\mathbf{r}}}{r^2}$$

- **Energy Density** (J/m^2): $$u = \frac{B^2}{2\mu_\circ}$$

- **Magnetic Flux** (Wb=T·m^2): $$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

- **Faraday's Law** (V): $$\epsilon \ = -N\frac{d\Phi_B}{dt}$$

Motional Emf: $$\epsilon \ = vBL$$

Electrical generator: $$\epsilon = NBA\omega \sin \omega t = \epsilon_{max} \sin \omega t$$

where $\omega = 2\pi f$ is the angular frequency, and $\epsilon_{max} = NBA\omega$

- **Lenz's law** is stated as follows: The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. This statement is a consequence of the law of conservation of energy.

- **Inductance** (H):

Self Inductance:
$$L = \frac{N\Phi_B}{I}$$
$$\epsilon = -L\frac{\Delta I}{\Delta t}$$

Energy stored in inductor:
$$U = \frac{1}{2}LI^2$$

Mutual Inductance:
$$M = \frac{N_S\Phi_S}{I_P}$$
$$\epsilon = -M\frac{\Delta I_P}{\Delta t}$$

Transformers:
$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

where the subscript "P" refers to the primary coil and subscript "S" refers to the secondary coil.

- **AC Circuits:**

The output of an AC generator is sinusoidal, the voltage varies as

$$V = V_{\max}\sin\omega t$$

where $\omega = 2\pi f$ is the angular frequency, and f is the linear frequency. The **rms current and voltage** are

$$I_{\mathrm{rms}} = \frac{I_{\max}}{\sqrt{2}}\ ; \quad V_{\mathrm{rms}} = \frac{V_{\max}}{\sqrt{2}}$$

The voltage and current across a resistor are in phase

$$V_{\mathrm{R,rms}} = I_{\mathrm{rms}}R$$

The voltage across a capacitor lags the current by 90°, and

$$V_{\mathrm{C,rms}} = I_{\mathrm{rms}}X_{\mathrm{C}}$$

where the $X_{\mathrm{C}} = \frac{1}{\omega C}$ is the **capacitive reactance**.

The voltage across an inductor leads the current by 90°, and

$$V_{\mathrm{L,rms}} = I_{\mathrm{rms}}X_{\mathrm{L}}$$

where the $X_{\mathrm{C}} = \omega L$ is the **inductive reactance**.

- **RLC Series Circuit:**

Ohm's law for the LRC series circuit reads

$$V_{\max} = I_{\max} Z$$

where $Z = \sqrt{R^2 + (X_{\rm L} - X_{\rm C})^2}$ is the **impedence** of the circuit. The phase angle ϕ between the current and the voltage obeys

$$\tan\phi = \frac{X_{\rm L} - X_{\rm C}}{R}$$

The power delivered by the generator is dissipated in the resistor; there is no power loss in the ideal capacitor or inductor.

$$P_{\rm ave} = I_{\rm rms}^2 R = I_{\rm rms} V_{\rm rms} \cos\phi$$

where the voltage across the resistor is $V_{\rm R} = V_{\rm rms}\cos\phi$.

- **Resonance in a Series RLC Circuit:**

According to Ohm's law, the rms current is maximum when the impedence is a minimum. The minimum value of the impedence is R and occurs when $X_{\rm L} = X_{\rm C}$. This occurs at the **resonant frequency**:

$$f_\circ = \frac{1}{2\pi\sqrt{LC}}$$

- **Maxwell's Equations:**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_\circ}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

$$\nabla \times \mathbf{B} = \mu_\circ \mathbf{j} + \mu_\circ \epsilon_\circ \frac{d\mathbf{E}}{dt}$$

- **Electromagnetic Waves/Light:**

$$c = \frac{1}{\sqrt{\mu_\circ \epsilon_\circ}} = 2.9979... \times 10^8 \mathrm{m/s}$$

$$v = f\lambda = \frac{1}{\sqrt{\mu\epsilon}}$$

Average energy density:

$$\bar{u} = \frac{1}{2}\epsilon E_{max}^2 + \frac{1}{2\mu}B_{max}^2 = \epsilon E_{rms}^2 = \frac{1}{\mu}B_{rms}^2$$

where $E = vB$, $E_{rms} = E_{max}/\sqrt{2}$ and $B_{rms} = B_{max}/\sqrt{2}$

Intensity (Poynting vector) (W/m^2):

$$\mathbf{S} = \frac{1}{\mu}\mathbf{E} \times \mathbf{B};\ |\mathbf{S}| = vu,\ \bar{S} = v\bar{u}$$

Doppler effect: $$f_{obs} = f_{source}\left(1 \pm \frac{v_{rel}}{c}\right)$$

Reflection: $$\theta_i = \theta_r$$

Index of refraction: $$n = \frac{c}{v} \geq 1;\ \lambda_n = \frac{\lambda}{n}$$

Snell's Law of Refraction: $$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Total Internal Reflection: $$\sin\theta_c = \frac{n_2}{n_1},\ \text{where } (n_1 > n_2)$$

Single-slit diffraction: $$\sin\theta_{dark} = \frac{m\lambda}{W},\ m = \pm 1, \pm 2, ...$$

Double-slit diffraction: $$\sin\theta_{bright} = \frac{m\lambda}{d},\ m = 0, \pm 1, \pm 2, ...$$

$$\sin\theta_{dark} = \left(m + \frac{1}{2}\right)\frac{\lambda}{d},\ m = 0, \pm 1, \pm 2, ...$$

Diffraction grating: $$\sin\theta_{bright} = \frac{m\lambda}{d},\ m = 0, \pm 1, \pm 2, ...$$

Additional properties: Interference, Fermat's Principle, Dispersion, Huygens' Principle

- **Quantum Effects:** (h = Planck's constant = $6.626... \times 10^{-34}$ J·s)

  Photon:
  $$E = hf,\ p = \frac{h}{\lambda} = \frac{hf}{c}$$

  Photoelectric effect:
  $$hf = KE_{max} + W_\circ,\ \text{where } W_\circ = \text{ work function}$$

  deBroglie wavelength:
  $$\lambda = \frac{h}{p}$$

  Heisenberg Uncertainty Principle:
  $$\Delta p_x \Delta x \geq \frac{h}{4\pi},\ \Delta E \Delta t \geq \frac{h}{4\pi}$$

  Bohr model (1 valence electron):
  $$r_n = \frac{r_1 n^2}{Z_{eff}},\ \text{where } r_1 = 5.29 \times 10^{-11}\ \text{m} \ = \ \text{Bohr radius}$$
  $$E_n = -(13.6\ \text{eV})\frac{Z_{eff}^2}{n^2}$$

Quantum numbers:

Principal Quantum Number:

$n = 1,\ 2,\ 3,\ ... \quad (E_n = -(13.6\ \text{eV})\frac{Z_{eff}^2}{n^2})$

Orbital Quantum Number:

$l = 0,\ 1,\ 2,\ ...,\ (n-1) \quad (L = \sqrt{l(l+1)}\frac{h}{2\pi})$

Magnetic Quantum Number:

$m_l = -l,\ ...-2,\ -1,\ 0,\ 1,\ 2,\ ...,\ l \quad (L_z = m_l \frac{h}{2\pi})$

Spin Quantum Number:

$m_s = \pm\frac{1}{2}$

The **Pauli Exclusion Principle** states that no two electrons in an atom may have the same set of quantum numbers.