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Signed Decomposition of Fully Fuzzy Linear Systems

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Abstract

System of linear equations is applied for solving many problems in various areas of applied sciences. Fuzzy methods constitute an important mathematical and computational tool for modeling real-world systems with uncertainties of parameters. In this paper, we discuss about fully fuzzy linear systems in the form $AX = b$ (FFLS). A novel method for finding the non-zero fuzzy solutions of these systems is proposed. We suppose that all elements of coefficient matrix A are positive and we employ parametric form linear system. Finally, Numerical examples are presented to illustrate this approach and its results are compared with other methods.

Keywords: Fuzzy numbers, Fully Fuzzy Linear Systems, Systems of Fuzzy Linear Equations, Non-zero Solutions, Decomposition Method

AMS 2000 Subject Classification Numbers: 65F05, 08A72, 11D04, 34A30

1. Introduction

Systems of linear equations are used for solving many problems in various area such as structural mechanics applications, heat transport, fluid flow, electromagnetism ....In many applications, at least one of the system's parameters and measurements are vague or imprecise and we can present them with fuzzy numbers rather than crisp numbers. Hence, it is important to develop mathematical models and numerical procedure that would appropriately treat general fuzzy system and solve them.

The system of linear equations $AX = b$ where the elements, a_{ij}, of the matrix A are crisp numbers and the elements, b_i, of b are fuzzy numbers, is called fuzzy system of linear equation (FSLE). The $n \times n$ (FSLE) has been studied by many authors (Abbasbany et al. (2006, 2006), Allahviranloo et al. (2003, 2004, 2005a, 2005b, 2006, 2006), Asady et al. (2005), Dehghan et al. (2006), Friedman et al.(1998, 2003), Ma et al. (2000), Wang et al. (2006), Xizhao et al. (2001), Zheng et al. (2006)). Friedman et al. (1998) proposed a general model for solving such fuzzy linear systems by using the embedding approach. Following Friedman et al. (1998), Allahviranloo et al. in (2003, 2004, 2005a, 2005b, 2006, 2006) and other authors in Abbasbany et al. (2006, 2006), Asady et al. (2005), Dehghan et al. (2006), Wang et al. (2006), Zheng et al. (2006) are design some numerical methods for calculating the solutions of $FSLE$.

The system of linear equations $AX = B$ where the elements, $\widetilde{a}_{ij}$, of the matrix A and the elements, $\widetilde{b}_i$, of the vector b are fuzzy numbers, is called Fully Fuzzy Linear System $FFLS$. The $n \times n$ FFLS has been studied by many authors (Buckley and Qu (1990, 1991a, 1991b), Dehghan et al. (2006, 2006), Muzzioli and Reynaerts(2006, 2007), Vroman et al. (2005, 2007a, 2007b)).

Buckley and Qu in their sequential works (1990, 1991a, 1991b) suggested different solutions for $FFLS$. Also, they found relation between these solutions. Based on their works, Muzzioli and Reynaerts in (2006, 2007) studied $FFLS$ of the form $A_1X + b_1 = A_2X + b_2$. The link between interval linear systems and fuzzy linear systems is clarified by them. Their approach contains solving of $2^{n(n+1)}$ crisp systems for all $\alpha \in [0,1]$.

Dehghan et al. (2006, 2006) have studied some methods for solving $FFLS$. They have represented fuzzy numbers in L. R. form and applied approximately operators between fuzzy numbers then found positive solutions of $FFLS$ (Dubois and Prade (1980), Zimmermann (1985)), so calculating the solutions of $FFLS$ is transformed to calculate the solutions of three crisp systems. In their approach, result of multiplying two triangular fuzzy numbers is a triangular fuzzy number which is not good approximation.

Vroman et al. in their continuous work (2005, 2007a, 2007b) suggested two method for solving $FFLS$. In (2007a) they have proposed a method to solve $FFLS$ approximately then they prove that their solution is better than Buckley and Qu's approximate solution vector X_B. Furthermore in (2005, 2007b) they have proposed an algorithm to improve their method to solve *FFLS* by parametric functions.

In many applications, which can be modeled by a system of linear equations, system's parameters are positive and we interest to find its non-zero solutions, so it is important to propose a method to find non-zero solutions of *FFLS* , where system's parameters are positive. In this paper, we are going to find non-zero solutions of *FFLS*. We replace the original $n \times n$ FFLS by a $2n \times 4n$ parametric linear system then a numerical method for calculating the solutions is proposed.

The rest of paper is organized as follows: In Section 2, we discuss some basic definitions, results on fuzzy numbers and *FFLS*. In Section 3, the numerical procedure for finding non-zero solutions of *FFLS* is presented. The proposed algorithm is illustrated by solving some numerical examples in section 4. Conclusions are drawn in section 5.

2. Preliminaries

The basic definition of fuzzy numbers is given in (Goetschel et al.(1986)) as follow:

Definition 1: A fuzzy number is a fuzzy set $\tilde{u} : \Re \to [0,1]$ which satisfies

[1.] $\tilde{u}$ is upper semi continuous;
[2.] $\tilde{u}(x) = 0$ outside some interval $[c,d]$;
[3.] there are real numbers $a,b; c \le a \le b \le d$ for which
(i) . $\tilde{u}(x)$ is monotonic increasing on $[c,d]$;
(ii) . $\tilde{u}(x)$ is monotonic decreasing on $[b,d]$;
(iii). $\tilde{u}(x) = 1, a \le x \le b.$

The set of all fuzzy numbers is denoted by E . An alternative definition of fuzzy number is:

Definition 2: (Goetschel(1986),Kaleva(1987)) A fuzzy number $\tilde{u}$ is a pair $(\underline{u}(r), \overline{u}(r))$ of functions $\underline{u}(r), \overline{u}(r)$, $0 \le r \le 1$ which satisfy the following requirements:

(i) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function;
(ii) $\overline{\mathrm{u}}(r)$ is a bounded monotonic decreasing left continuous function;
(iii) $\underline{\mathrm{u}}(r) \le \overline{u}(r)$ $0 \le \mathrm{r} \le 1$.

A crisp number k is simply represented by $\overline{k}(r) = \underline{k}(r) = k$ $0 \le \mathrm{r} \le 1$ and is called singleton. The fuzzy number space $\{\underline{u}(r), \overline{u}(r)\}$ becomes a convex cone E which is then embedded isomorphic ally and isometric ally into a Banach space (Cong-Xing (1991, 1992)).

A fuzzy number $\tilde{a}$ can be represented by its λ -cuts $0 \le \lambda \le 1$ and

$$\sup\ \tilde{a} = Cl(\{x \mid x \in \Re \text{ and } \tilde{a}(x) > 0\}) = [\underline{a}(0), \overline{a}(0)].$$

Note that the λ-cuts of a fuzzy number are closed and bounded intervals. Also the fuzzy arithmetic based on the Zadeh extension principle can be calculated by applying interval arithmetic on the λ-cuts. There are different definitions of operations between two intervals.

For fuzzy number $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ $0 \le r \le 1$, we write (1) $\tilde{u} > 0$, if $\underline{u}(0) > 0$, (2) $\tilde{u} \ge 0$, if $\underline{u}(0) \ge 0$, (3) $\tilde{u} < 0$, if $\overline{u}(0) < 0$, and (4) $\tilde{u} \le 0$, if $\overline{u}(0) \le 0$. If $\tilde{u} \le 0$ or $\tilde{u} \ge 0$ this fuzzy number is called non-zero fuzzy number.

For arbitrary $\tilde{u} = (\underline{u}(r), \overline{u}(r)), \tilde{v} = (\underline{v}(r), \overline{v}(r))$ and $k > 0$ we define addition $\tilde{u} + \tilde{v}$, subtraction $\tilde{u} - \tilde{v}$, multiplication $\tilde{u}.\tilde{v}$ and scalar product on by k as

Addition: $$\underline{u+v}(r) = \underline{u}(r) + \underline{v}(r), \quad \overline{u+v}(r) = \overline{u}(r) + \overline{v}(r). \tag{1}$$

Subtraction: $$\underline{u-v}(r) = \underline{u}(r) - \overline{v}(r), \quad \overline{u-v}(r) = \overline{u}(r) - \underline{v}(r). \tag{2}$$

Multiplication: $$\begin{aligned} \underline{uv}(r) &= \min\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)\}, \\ \overline{uv}(r) &= \max\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)\}. \end{aligned} \tag{3}$$

Multiplication of two fuzzy numbers for two important cases in more detail is as follows:

Case 1: $\tilde{u} \ge 0$ and $\tilde{v} \ge 0$ $\quad \underline{uv}(r) = \underline{u}(r)\underline{v}(r), \quad \overline{uv}(r) = \overline{u}(r)\overline{v}(r).$

Case 2: $\tilde{u} \ge 0$ and $\tilde{v} \le 0$ $\quad \underline{uv}(r) = \overline{u}(r)\underline{v}(r), \quad \overline{uv}(r) = \underline{u}(r)\overline{v}(r).$

Scalar product:

$$k\tilde{u} = \begin{cases} (k\underline{u}(r), k\overline{u}(r)), & k \ge 0, \\ (k\overline{u}(r), k\underline{u}(r)), & k < 0. \end{cases} \tag{4}$$

Note that, distributive law for fuzzy number's multiplication is invalid. Also cancellation law does not hold in fuzzy numbers arithmetic i.e. if $\tilde{u}, \tilde{v}$ and $\tilde{w}$ are fuzzy numbers where $\tilde{u} + \tilde{v} = \tilde{w}$, if $\tilde{v}$ is non- singleton then $\tilde{u} \ne \tilde{w} - \tilde{v}$.

Definition 3: The $n \times n$ linear system of equations

$$\begin{cases} \tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n = \tilde{b}_1, \\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n = \tilde{b}_2, \\ \qquad \vdots \\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n = \tilde{b}_n, \end{cases} \tag{5}$$

where the elements, $\tilde{a}_{ij}$, $1 \le i, j \le n$, of the coefficient matrix A and the elements, $\tilde{b}_i$, of the vector are fuzzy number is called a fully fuzzy linear system of equations $FFLS$.

If A is a crisp matrix, fuzzy linear system $AX = b$ is called Fuzzy System of Linear Equations $FSLE$.

Definition 4: For any $(FFLS)$ $AX = b$ and for all $\lambda \in [0,1]$,

$$\begin{cases} \tilde{a}_{11}^{\lambda}\tilde{x}_1^{\lambda} + \tilde{a}_{12}^{\lambda}\tilde{x}_2^{\lambda} + \cdots + \tilde{a}_{1n}^{\lambda}\tilde{x}_n^{\lambda} = \tilde{b}_1^{\lambda}, \\ \tilde{a}_{21}^{\lambda}\tilde{x}_1^{\lambda} + \tilde{a}_{21}^{\lambda}\tilde{x}_2^{\lambda} + \cdots + \tilde{a}_{2n}^{\lambda}\tilde{x}_n^{\lambda} = \tilde{b}_2^{\lambda}, \\ \quad\quad\quad \vdots \\ \tilde{a}_{n1}^{\lambda}\tilde{x}_1^{\lambda} + \tilde{a}_{n2}^{\lambda}\tilde{x}_2^{\lambda} + \cdots + \tilde{a}_{nn}^{\lambda}\tilde{x}_n^{\lambda} = \tilde{b}_n^{\lambda}, \end{cases} \tag{6}$$

where $\tilde{a}_{ij}^{\lambda}, \tilde{x}_j^{\lambda}$ and $\tilde{b}_i^{\lambda}$, $1 \le i, j \le n$, $\lambda \in [0,1]$ are λ-cut sets of fuzzy numbers $\tilde{a}_{ij}, \tilde{x}_j$ and $\tilde{b}_i$ $1 \le i, j \le n$, respectively, is called λ-cut system of linear system and represented by $A^{\lambda} X^{\lambda} = b^{\lambda}$, $0 \le \lambda \le 1$.

An interval $[a,b]$ is non zero if $[a,b] \subseteq [0,+\infty)$ or $[a,b] \subseteq (-\infty,0]$. Some authors investigate $(FFLS)$ using this fact that λ-cut of fuzzy numbers is interval. For more information, see (Muzzioli and Reynaerts (2006, 2007)). When interval arithmetic is used to solve $FFLS$, the problem of finding the solution of $A^{\lambda} X^{\lambda} = b^{\lambda}$ convert to a multi objective nonlinear optimal problem which for solvability needs to satisfy certain conditions. Hence this approach is not suitable.

Definition 5: A fuzzy number vector $(\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^t$ given by $\tilde{x}_i = (\underline{x}_i(r), \overline{x}_i(r))$ $1 \le i \le n, 0 \le r \le 1$ is called a solution of $(FFLS)$, if

$$\underline{\sum_{i=1}^{n} a_{ij} x_j(r)} = \sum_{i=1}^{n} \underline{a_{ij} x_j(r)} = \underline{b}_i(r), \quad \overline{\sum_{i=1}^{n} a_{ij} x_j(r)} = \sum_{i=1}^{n} \overline{a_{ij} x_j(r)} = \overline{b}_i(r). \tag{7}$$

We define non-zero solution of $(FFLS)$ as follows:

Definition 6: A solution vector $(\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^t$ of $(FFLS)$ is nonzero if $\tilde{x}_i \ge 0$ or $\tilde{x} \le 0$, $i = 1, 2, \cdots, n$.

Necessary and sufficient condition for the existence of a Non-zero fuzzy solution of $(FFLS)$ is:

Theorem: If $(FFLS)$ $AX = b$ has a solution which it is a fuzzy number, then $AX = b$ will have a nonzero fuzzy number solution if and only if 0-cut system of linear system represented by $A^0 X^0 = b^0$ has nonzero solution.

Proof: Let $(\widetilde{x}_1, \widetilde{x}_2, \cdots, \widetilde{x}_n)^t$ is fuzzy number solution of $AX = b$. $AX = b$ Has nonzero solution, if and only if, $\widetilde{x}_i \geq 0$ or $\widetilde{x} \leq 0$, $i = 1,2,\cdots,n,$ if and only if

$$\sup p\,\widetilde{a} = Cl(\{x \mid x \in \Re \text{ and } \widetilde{a}(x) > 0\}) = [\underline{a}(0), \overline{a}(0)]$$

are non- zero if and only if $A^0 X^0 = b^0$ has non- zero solution. In fact, many simultaneous system of linear equation have non-zero solution.

3. Non-Zero Solution of $(FFLS)$

In this section, we are going to find the solution of $(FFLS)$. To do that, if $(FFLS)$ have fuzzy non-zero solution, we replace original $n \times n$ $(FFLS)$ by $2n \times 2n$ parametric system then by proposed algorithm it is founded.

Let $AX = b$ be $(FFLS)$. Consider i^{th} equation of this system:

$$\sum_{i=1}^{n} \tilde{a}_{ij}\tilde{x}_j = \tilde{b}_i, \quad j = 1,2,\cdots,n. \tag{8}$$

Now, let $AX = b$ has non-zero solution, we define

$$J = \{j \mid 1 \leq j \leq n \text{ and } \tilde{x}_j \geq 0\}. \tag{9}$$

Hence Eq. (8) can be transformed to

$$\sum_{i=1}^{n} \tilde{a}_{ij}\tilde{x}_j = \sum_{j \in J} \tilde{a}_{ij}\tilde{x}_j + \sum_{j \notin J} \tilde{a}_{ij}\tilde{x}_j = \tilde{b}_i, \quad i = 1,2,\cdots,n. \tag{10}$$

We define two n-vectors $\widetilde{Y} = (\widetilde{y}_1, \widetilde{y}_2, \cdots, \widetilde{y}_n)^t$ and $\widetilde{Z} = (\widetilde{z}_1, \widetilde{z}_2, \cdots, \widetilde{z}_n)^t$ as follows:

$$\tilde{z}_j = \begin{cases} \tilde{x}_j, & j \in J, \\ 0, & j \notin J, \end{cases} \quad \tilde{y}_j = \begin{cases} \tilde{x}_j, & j \notin J, \\ 0, & j \in J. \end{cases} \tag{11}$$

This is obvious that

$$\tilde{z}_j + \tilde{y}_j = \tilde{x}_j, \qquad 1 \leq j \leq n. \tag{12}$$

By replacing (11) and (12) in (10)

$$\begin{aligned}\sum_{i=1}^{n}\tilde{a}_{ij}\tilde{x}_j &= \sum_{j\in J}\tilde{a}_{ij}\tilde{x}_j + \sum_{j\notin J}\tilde{a}_{ij}\tilde{x}_j = \sum_{j\in J}\tilde{a}_{ij}\tilde{z}_j + \sum_{j\notin J}\tilde{a}_{ij}\tilde{y}_j \\ &= \sum_{i=1}^{n}\tilde{a}_{ij}\tilde{z}_j + \sum_{i=1}^{n}\tilde{a}_{ij}\tilde{y}_j = \tilde{b}_i, \quad i = 1,2,\cdots,n.\end{aligned} \tag{13}$$

By applying (1)-(4) and (7) and replacing in (14) we have:

$$\underline{b_i}(r) = \sum_{j=1}^{n}\underline{a_{ij}z_j}(r) + \sum_{j=1}^{n}\underline{a_{ij}y_j}(r), \quad i = 1,2,\cdots,n, \tag{14}$$

and

$$\overline{b_i}(r) = \sum_{j=1}^{n}\overline{a_{ij}z_j}(r) + \sum_{j=1}^{n}\overline{a_{ij}y_j}(r), \quad i = 1,2,\cdots,n. \tag{15}$$

Since $\tilde{z}_j \geq 0, \tilde{y}_j \leq 0, \tilde{a}_{ij} \geq 0$, $1 \leq \text{i, j} \leq \text{n}$ by (3) we have

$$\begin{aligned}&\underline{a_{ij}z_j}(r) = \underline{a_{ij}}(r)\underline{z_j}(r), \quad \underline{a_{ij}y_j}(r) = \overline{a_{ij}}(r)\underline{y_j}(r), \\ &\overline{a_{ij}z_j}(r) = \overline{a_{ij}}(r)\overline{z_j}(r), \quad \overline{a_{ij}y_j}(r) = \underline{a_{ij}}(r)\overline{y_j}(r).\end{aligned} \tag{16}$$

By replacing (16) in (14) and (15),

$$\underline{b_i}(r) = \sum_{j=1}^{n}\underline{a_{ij}}(r)\underline{z_j}(r) + \sum_{j=1}^{n}\overline{a_{ij}}(r)\underline{y_j}(r), \quad i = 1,2,\cdots,n, \tag{17}$$

and

$$\overline{b_i}(r) = \sum_{j=1}^{n}\overline{a_{ij}}(r)\overline{z_j}(r) + \sum_{j=1}^{n}\underline{a_{ij}}(r)\overline{y_j}(r), \quad i = 1,2,\cdots,n. \tag{18}$$

If C_1 and C_2 are parametric $n \times n$ matrices by elements

$$(C_1)_{ij} = \underline{a_{ij}}(r) \quad (C_2)_{ij} = \overline{a_{ij}}(r) \tag{19}$$

and if Z_1, Z_2, Y_1, Y_2, B_1 and B_2 are parametric *n*-vectors by elements

$$\begin{aligned}&(Z_1)_j = \underline{z_j}(r), \quad (Z_2)_j = \overline{z_j(r)}, \\ &(Y_1)_j = \underline{y_j}(r), \quad (Y_2)_j = \overline{y_j}(r), \\ &(B_1)_j = \underline{b_j}(r), \quad (B_2)_j = \overline{b_j}(r),\end{aligned} \tag{20}$$

then, $n \times n$ Matrix representation of $(FFLS)\ AX = b$ converts to

$$\begin{pmatrix} C_1 & 0 & C_2 & 0 \\ 0 & C_2 & 0 & C_1 \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, \tag{21}$$

where this coefficient matrix represent in $2n \times 4n$ matrix form. But in fact, by definition of Y and Z, $2n$ element of variable matrix is zero and hence $2n$ column of coefficient matrix are omitted. Hence, we replace $n \times n$ $(FFLS)$ by $2n \times 2n$ system of linear parametric equations. If we solve this system, its solution is non-zero solution of $(FFLS)$.

Proposed algorithm to find FFLS's non-zero solution is as follow:

Non-Zero Solution of $(FFLS)$ s Algorithm:
Suppose $AX = b$ is a $(FFLS)$ where have a fuzzy number solution.

1. Solve $A^0 X^0 = b^0$ system. If this system has non-zero solution then go to 2 else go to 6.
2. Transform $AX = b$ to (21) system.
3. Omit 2n columns of coefficient matrix.
4. Solve $2n \times 2n$ parametric system (21).
5. This solution is a non-zero solution of $(FFLS)$ $AX = b$ go to 7.
6. This system has a zero solution and this algorithm cannot solve it.
7. End.

4. Examples

Example 1. Dehghan(2006) Consider the fully fuzzy linear system $AX = b$, where

$$A = \begin{pmatrix} (4+r, 6-r) & (5+r, 8-2r) \\ (6+r, 7) & (4, 5-r) \end{pmatrix} \text{ and } b = \begin{pmatrix} (40+10r, 67-17r) \\ (43+5r, 55-7r) \end{pmatrix}.$$

Dehghan et al. in (2006) have solved this system and its solution vector has presented as $\tilde{x}_1 = (\frac{43}{11} + \frac{1}{11}r, 4)$ and $\tilde{x}_2 = (\frac{54}{11} + \frac{1}{11}r, \frac{21}{4} - \frac{1}{4}r)$, which are triangular fuzzy numbers.

We solve this system by our algorithm as follows:
This system has a positive solution hence $\tilde{z}_1 = \tilde{x}_1, \tilde{z}_2 = \tilde{x}_2, \tilde{y}_1 = 0, \tilde{y} = 0.$ Since

$$C_1 = \begin{pmatrix} 4+r & 5+r \\ 6+r & 4 \end{pmatrix} \text{ and } C_2 = \begin{pmatrix} 6-r & 8-2r \\ 7 & 5-r \end{pmatrix} \text{ and}$$

$$\begin{pmatrix} C_1 & 0 & C_2 & 0 \\ 0 & C_2 & 0 & C_1 \end{pmatrix} = \begin{pmatrix} 4+r & 5+r & 0 & 0 & 6-r & 8-2r & 0 & 0 \\ 6+r & 4 & 0 & 0 & 7 & 5-r & 0 & 0 \\ 0 & 0 & 6-r & 8-2r & 0 & 0 & 4+r & 5+r \\ 0 & 0 & 7 & 5-r & 0 & 0 & 6+r & 4 \end{pmatrix}.$$

Since $\tilde{y}_1 = \tilde{y}_2 = 0$, coefficient matrix is transformed to:

$$\begin{pmatrix} 4+r & 5+r & 0 & 0 \\ 6+r & 4 & 0 & 0 \\ 0 & 0 & 6-r & 8-2r \\ 0 & 0 & 7 & 5-r \end{pmatrix}.$$

Our solutions vector is:

$$\tilde{x}_1 = \left(\frac{-5r^2 - 28r - 55}{-r^2 - 7r - 14}, \frac{-3r^3 + 4r^2 + 189r - 630}{-r^3 + 3r^2 + 44r - 156}\right)$$

And

$$\tilde{x}_2 = \left(\frac{-5r^2 - 37r - 68}{-r^2 - 7r - 14}, \frac{7r^2 + 22r - 139}{r^2 + 3r - 26}\right).$$

Example 2. Consider the fully fuzzy linear system $AX = b$, where

$$A = \begin{pmatrix} (1+r, 3-r) & (2r^2, 4-2r^2) \\ (2r, 5-3r) & (1+3r^2, 6-2r) \end{pmatrix} \text{ and } b = \begin{pmatrix} (2r^3 + 10r - 12, 2r^3 - 12r^2 - 6r + 20) \\ (3r^3 - 3r^2 + 23r - 19, -2r^2 - 16r + 30) \end{pmatrix}.$$

Dehghan's approach, cannot find this systems solution, because:

1. Coefficients of system have various L.R. form.
2. Solution of systems is not positive.

If we solve 0-cut of fully fuzzy linear system, we have $\tilde{z}_1 = \tilde{x}_1, \tilde{z}_2 = 0, \tilde{y}_1 = 0, \tilde{y}_2 = \tilde{x}_2$. And

$$C_1 = \begin{pmatrix} 1+r & 2r^2 \\ 2r & 1+3r^2 \end{pmatrix} \text{ and } C_2 = \begin{pmatrix} 3-r & 4-2r^2 \\ 5-3r & 6-2r \end{pmatrix}.$$

With (21), our coefficients matrix is

$$\begin{pmatrix} C_1 & 0 & C_2 & 0 \\ 0 & C_2 & 0 & C_1 \end{pmatrix} = \begin{pmatrix} 1+r & 2r^2 & 0 & 0 & 3-r & 4-2r^2 & 0 & 0 \\ 2r & 1+3r^2 & 0 & 0 & 5-3r & 6-2r & 0 & 0 \\ 0 & 0 & 3-r & 4-2r^2 & 0 & 0 & 1+r & 2r^2 \\ 0 & 0 & 5-3r & 6-2r & 0 & 0 & 2r & 1+3r^2 \end{pmatrix}.$$

Since $z_2 = y_1 = 0$, coefficient matrix is transformed to:

$$\begin{pmatrix} 1+r & 0 & 3-r & 0 \\ 2r & 0 & 5-3r & 0 \\ 0 & 4-2r^2 & 0 & 2r^2 \\ 0 & 6-2r & 0 & 1+3r^2 \end{pmatrix}.$$

Our solution vector is $\tilde{x}_1 = (-4+2r,-2r), \tilde{x}_2 = (1+r,5-r)$.

5. Conclusion

In this paper, we propose a novel method for solving the fully fuzzy linear system $AX = b$ where all elements of coefficient matrix are positive .Its non-zero solution is found by replacing $n \times n$ original system by $2n \times 2n$ parametric form system. We first, determine positive and negative solutions of this system by solving 0-cut system and then replace $n \times n$ fuzzy coefficient matrix by $2n \times 4n$ parametric coefficient matrix. Finally, we omit 2n columns of this matrix and solve this $2n \times 2n$ system. We compare our approach by Dehghan's (2006, 2006) approach. In case which $FFLS$ has negative solution, Dehghan's approach cannot obtain the solution but the proposed approach find it easily. Furthermore, the result of proposed method is more precise and stable than other methods.

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