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Thermal stresses in functionally graded hollow sphere due to non-uniform internal heat generation

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Abstract

In this article, the thermal stresses in a hollow thick sphere of functionally graded material subjected to non-uniform internal heat generation are obtained as a function of radius to an exact solution by using the theory of elasticity. Material properties and heat generation are assumed as a function of radius of sphere and Poisson's ratio as constant. The distribution of thermal stresses for different values of the powers of the module of elasticity and varying power law index of heat generation is studied. The results are illustrated numerically and graphically.

Keywords: Functionally graded material; non-uniform heat source; thermal stresses and thick hollow sphere

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1. Introduction

Functionally graded material are inhomogeneous composites having the properties that vary gradually and continuously within the material. FGMs were first developed by a group of Japanese scientists to meet the need of destructive environment of the thermal shocks and have been widely explored in various engineering applications including space technology, optics, biomedicines, etc. (1997). The analytical solution for the stresses in spheres and cylinders made of functionally graded materials are obtained by Lutz and Zimmerman (1996, 1999) under radial thermal loads, where radially graded materials with linear composition of constituent materials were considered. Obata and Noda (1994) studied the one dimensional steady thermal stresses in a functionally graded circular hollow cylinder and a hollow sphere using the perturbation method to achieve the effect of the composition of stresses to design the optimum FGM hollow circular cylinder and hollow sphere under different assumptions of temperature distribution. By the theory of laminated composites, Ootao et al. (1995) treated the theoretical analysis of a three dimensional thermal stress problem for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction in a transient state. Jabbari et al. (2002, 2003) derived the analytical solution for one dimensional and two dimensional steady state thermo elastic problem of the functionally graded circular hollow cylinder, where the material properties are expressed as functions of radius. Chen and Lin (2008) carried out the elastic analysis for a thick cylinder as well as spherical pressure vessel made of FGM with exponentially varying properties which has significant role in the stress distribution along the radial direction and useful to engineers for design. Shao and Ma (2008) carried out thermo mechanical analysis of FGM hollow cylinder subjected to mechanical loads and linearly increasing boundary temperature. Thermomechanical properties of functionally graded material are temperature independent and vary continuously in the radial direction of cylinder.

Nayak et al. (2011) presented an analysis of FGM thick spherical vessel with radially varying properties in the form of displacement, strain and stress for thermal mechanical and thermo mechanical loads and validated the method of solution and results by means of reducing it to isotropic and homogeneous material. Recently Ehsani Farshad and Ehsani Farzad (2012) analyzed the one dimensional non steady state temperature distribution in a hollow thick cylinder of FGM with non-uniform heat generation by homotopy perturbation method. Deshmukh et al. (2012) studied the thermal deflection which is built in edge in a thin hollow disc subjected to the activity of heat source which changes its place on the plate surface with time. Recently Kedar and Deshmukh (2013) studied the determination of thermal stresses in a thin clamped hollow disk under unsteady temperature field due to point heat source.

In the present work an attempt is made to study the quasi-static thermal stresses based on uncoupled thermoelasticity for FGM hollow thick sphere with non-uniform internal heat generation which is a function of the radial position. We have extended the work of Nayak et al. (2011) and non-uniform heat generation is as Farshad Ehsani and Farzad Ehsani (2012) in the form of power law function of radius and exact analytical solutions are obtained for radial and tangential stresses by using the theory of elasticity. Numerical solutions are presented for the material rich of ceramic Zirconia.

2. Formulation of Problem

Temperature distribution

Consider the FGM hollow sphere with inner radius a and outer radius b. The properties in spherical coordinate ϕ and θ direction are identical. The sphere is graded in the radial direction so that the properties of the material, modulus of elasticity, thermal expansion coefficient and thermal conductivity are the function of r. Assume that the non-uniform heat is generated within it and it is also a function of r. The following power law functions of radius in the radial direction are considered as Nayak et al. (2011) and Farshad Ehsani and Farzad Ehsani (2012)

Modulus of elasticity: $E = E_0 r^{m_1}$,

Thermal conductivity: $k = k_0 r^{n_1}$,

Coefficient of thermal Expansion: $a_t = a_{t0} r^{m_2}$,

Non- uniform heat generation: $q = q_0 r^{n_2}$, (2.1)

where, m_1, n_1, m_2 *and* n_2 are parameters and the E_0 , a_{t0} and k_0 are the material constants for the modulus of elasticity, thermal expansion coefficient, thermal conductivity and with non-uniform heat generation q (W/m^3) within the sphere and q_0 (W/m^3) is heat generation constant which is the magnitude of heat generation in homogeneous and isotropic hollow sphere.

The heat conduction equation in the steady state condition for one dimensional spherical coordinates and first kind thermal boundary condition as in Nayak et al. (2011) and introducing non-uniform heat generation term expressed in (2.1) is obtained as Ozisik (1968),

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 k_0 r^{n_1}\frac{dT}{dr}\right] + q_0 r^{n_2} = 0, \quad a \le r \le b, \quad t > 0, \tag{2.2}$$

subjected to boundary conditions

$$T = T_1 , \quad \text{at} \quad r = a, \tag{2.3}$$
$$T = T_2 , \quad \text{at} \quad r = b. \tag{2.4}$$

Problem of Thermoelasticity

The properties in spherical coordinate ϕ and θ direction are identical and u denotes the displacement in the radial direction, the strain-displacement relations are as Noda et al. (2003),

$$\varepsilon_{rr} = \frac{du}{dr}, \ \varepsilon_{\theta\theta} = \frac{u}{r}. \tag{2.5}$$

The corresponding thermo elastic stress-strain relation or Hooke's relations are

$$\sigma_{rr} = \lambda e + 2\mu\varepsilon_{rr} - (3\lambda + 2\mu)a_t T, \tag{2.6}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \lambda e + 2\mu\varepsilon_{\theta\theta} - (3\lambda + 2\mu)a_t T, \tag{2.7}$$

where, σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are the stresses in the radial and tangential direction and ε_{rr} and $\varepsilon_{\theta\theta}$ are strains in radial and tangential direction, T is the temperature change determined from the heat conduction equation (2.2), a_t is the coefficient of thermal expansion, e is the strain dilatation and λ and μ are the Lame constants related to the modulus of elasticity E and the Poisson's ratio ν as,

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad , \ \mu = \frac{E}{2(1+\nu)}. \tag{2.8}$$

The equilibrium equation in the radial direction, excluding the body force and the inertia term is,

$$r\frac{d\sigma_{rr}}{dr} + 2(\sigma_{rr} - \sigma_{\theta\theta}) = 0. \tag{2.9}$$

The stress components in terms of the displacement u are obtained by using (2.5)-(2.8) as Noda et al. (2003)

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{du}{dr} + 2\nu\frac{u}{r} - (1+\nu)a_t T\right], \tag{2.10}$$

$$\sigma_{\theta\theta} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\frac{du}{dr} + \frac{u}{r} - (1+\nu)a_t T\right]. \tag{2.11}$$

The sphere is subjected to the traction free boundary conditions

$$\sigma_{rr} = 0, \quad \text{at } r = a \text{ and } r = b. \tag{2.12}$$

The equations (2.1) to (2.12) constitute the mathematical formulation of the problem.

3. Solutions

Temperature distribution function

The solution of (2.2) is obtained as,

For $n_1 \neq -1$

$$T(r) = Q_1 r^{n_2-n_1+2} + C_1 r^{-n_1-1} + C_2, \tag{3.1}$$

where,

$$Q_1 = \frac{-q_0}{k_0(n_2+3)(n_2-n_1+2)}, \tag{3.2}$$

$$C_1 = -\frac{c_1}{k_0(n_1+1)}. \tag{3.3}$$

The constant of integration can be determined by using (2.3) and (2.4) in (3.1) as

$$c_1 = k_0(n_1+1)\left\{\frac{(T_1-T_2)+Q_1[b^{n_2-n_1+2} - a^{n_2-n_1+2}]}{(b^{-n_1-1} - a^{-n_1-1})}\right\}, \tag{3.4}$$

$$C_2 = T_2 + \frac{(T_1-T_2)b^{-n_1-1}}{(b^{-n_1-1} - a^{-n_1-1})} - Q_1\left\{b^{n_2-n_1+2} - \frac{\{b^{n_2-n_1+2} - a^{n_2-n_1+2}\}b^{-n_1-1}}{(b^{-n_1-1} - a^{-n_1-1})}\right\}. \tag{3.5}$$

The parameters n_1 and n_2 are chosen in such a way that the denominator is nonzero, the temperature distribution function is obtained as

$$T(r) = T_2 + \frac{(T_1-T_2)(b^{-n_1-1} - r^{-n_1-1})}{(b^{-n_1-1} - a^{-n_1-1})}$$

$$+Q_1\left\{(r^{n_2-n_1+2}-b^{n_2-n_1+2})+\frac{\{b^{n_2-n_1+2}-a^{n_2-n_1+2}\}(b^{-n_1-1}-r^{-n_1-1})}{(b^{-n_1-1}-a^{-n_1-1})}\right\}. \tag{3.6}$$

For $\boldsymbol{n_1=-1}$,

$$T(r)=Q_2r^{n_2+3}+C_3lnr+C_4\ , \tag{3.7}$$

$$Q_2=\frac{-q_0}{k_0(n_2+3)^2}\ . \tag{3.8}$$

Using the boundary condition (2.3) and (2.4) one obtains the constants

$$C_3=\frac{c_3}{k_0}\ , \tag{3.9}$$

$$c_3=k_0\left\{\frac{(T_2-T_1)-Q_2[b^{n_2+3}-a^{n_2+3}]}{ln\frac{b}{a}}\right\}, \tag{3.10}$$

$$C_4=T_2-\frac{(T_2-T_1)lnb}{ln\frac{b}{a}}-Q_2\left\{b^{n_2+3}-\frac{(b^{n_2+3}-a^{n_2+3})lnb}{ln\frac{b}{a}}\right\}, \tag{3.11}$$

$$T(r)=T_2-\frac{(T_2-T_1)ln\frac{b}{r}}{ln\frac{b}{a}}+Q_2\left\{(r^{n_2+3}-b^{n_2+3})+\frac{\{b^{n_2+3}-a^{n_2+3}\}ln\frac{b}{r}}{ln\frac{b}{a}}\right\}. \tag{3.12}$$

Solution of Thermoelastic Equations

The equilibrium equation (2.9) is converted in terms of displacement u by using the functional relations (2.1) and using equations (2.10) and (2.11) as

For $\boldsymbol{n_1\neq-1}$

$$\frac{(1-\nu)}{(1+\nu)}r^2\frac{d^2u}{dr^2}+\frac{(1-\nu)}{(1+\nu)}(m_1+2)r\frac{du}{dr}+\frac{2(1-\nu)}{(1+\nu)}\left\{\frac{(\nu m_1+\nu-1)}{(1-\nu)}\right\}u$$
$$=Lr^{m_2+n_2-n_1+3}+Mr^{m_2-n_1}+Nr^{m_2+1}$$

$$Pr^2\frac{d^2u}{dr^2}+Qr\frac{du}{dr}+Ru=Lr^{m_2+n_2-n_1+3}+Mr^{m_2-n_1}+Nr^{m_2+1}. \tag{3.13}$$

For $\boldsymbol{n_1=-1}$

$$Pr^2\frac{d^2u}{dr^2}+Qr\frac{du}{dr}+Ru=L_1r^{m_2+n_2+4}+M_1r^{m_2+1}lnr+N_1r^{m_2+1}, \tag{3.14}$$

where,

$$P=\left(\frac{1-\nu}{1+\nu}\right),\ Q=P(m_1+2),\quad R=2P\left\{\frac{(\nu m_1+\nu-1)}{(1-\nu)}\right\}, \tag{3.15}$$

$$L=a_{t0}Q_1[m_1+m_2+n_2-n_1+2]\ , \tag{3.16}$$

$$M\ =a_{t0}C_1\{m_1+m_2-n_1-1\}\ , \tag{3.17}$$

$$N=C_1a_{t0}(m_1+m_2)\ , \tag{3.18}$$

$$L_1=a_{t0}Q_1[m_1+m_2+n_2+3]\ , \tag{3.19}$$

$$M_1\ =C_3a_{t0}\{m_1+m_2-n_1-1\}\ , \tag{3.20}$$

$$N_1 = a_{t0}\{C_3 + (m_1 + m_2)C_4\}\,. \tag{3.21}$$

The general solutions of equations (3.13) and (3.14) which are non-homogeneous differential equations are obtained by adding a particular solution to the complimentary solution of homogeneous form of them. The complementary function u_c is by letting

$$u_c(r) = Xr^s. \tag{3.22}$$

Putting this in homogeneous form of (3.13) and (3.14) one gets

$$Pr^2 \frac{d^2}{dr^2}[Xr^s] + Qr\frac{d}{dr}[Xr^s] + RXr^s = 0\,,$$

$$Pr^2Xs(s-1)r^{s-2} + QrsXr^{s-1} + RXr^s = 0,$$

$$Ps(s-1) + Qs + R = 0,$$

$$Ps^2 + (Q-P)s + R = 0. \tag{3.23}$$

Equation (3.23) has two roots

$$s_{1,2} = \frac{(P-Q)\pm\sqrt{(Q-P)^2 - 4PR}}{2P}\,. \tag{3.24}$$

Thus, the complementary function for (3.13) and (3.14) are as

$$u_c(r) = X_1r^{s_1} + X_2r^{s_2}, \tag{3.25}$$

$$u_c(r) = X_3r^{s_1} + X_4r^{s_2}. \tag{3.26}$$

The particular solution $u_r(r)$ for (3.13) and (3.14) are considered in the form

$$u_p(r) = Y_1\, r^{m_2+n_2-n_1+3} + Y_2r^{m_2-n_1} + Y_3r^{m_2+1}, \tag{3.27}$$

$$u_p(r) = Y_4\, r^{m_2+n_2+4} + (Y_5 lnr + Y_6\,)\, r^{m_2+1}. \tag{3.28}$$

Substituting equation (3.27) in (3.13) and (3.28) in (3.14), and equating the coefficient of identical powers and using the values of L, M, N and $L_{1,}$ M_1 and $N_1$are obtained from (3.16) - (3.21) as

$$Y_1 = \frac{a_{t0}Q_1(m_1+m_2+n_2-n_1+2)}{(m_2+n_2-n_1+3)[P(m_2+n_2-n_1+2)+Q]+R}\,, \tag{3.29}$$

$$Y_2 = \frac{C_1a_{t0}(m_1+m_2-n_1-1)}{(m_2-n_1)[P(m_2-n_1-1)+Q]+R}\quad, C_1 = -\frac{c_1}{k_0(n_1+1)}\,, \tag{3.30}$$

$$Y_3 = \frac{C_2a_{t0}(m_1+m_2)}{(m_2+1)(Pm_2+Q)+R}\,, \tag{3.31}$$

$$Y_4 = \frac{(m_1+m_2+n_2+3)a_{t0}Q_2}{(m_2+n_2+4)\{P(m_2+n_2+3)+Q\}+R}\,, \tag{3.32}$$

$$Y_5 = \frac{C_3a_{t0}(m_1+m_2)}{(m_2+1)[Pm_2+Q]+R}\,,\quad C_3 = \frac{c_3}{k_0}\,, \tag{3.33}$$

$$Y_6 = \frac{[C_3 a_{t0} + (m_1+m_2) C_4 a_{t0}] + [(m_2+1)(m_2+1) + R] - C_3 a_{t0}(m_1+m_2)[P(2m_2+1)+Q]}{[(m_2+1)[Pm_2+Q]+R]^2}. \quad (3.34)$$

The general solution $u(r)$ of (3.13) is obtained for $\boldsymbol{n_1 \neq -1}$ as

$$u(r) = X_1 r^{s_1} + X_2 r^{s_2} + Y_1\, r^{m_2+n_2-n_1+3} + Y_2 r^{m_2-n_1} + Y_3 r^{m_2+1}. \quad (3.35)$$

The general solution $u(r)$ of (3.14) for $\boldsymbol{n_1 = -1}$ is

$$u(r) = X_3 r^{s_1} + X_4 r^{s_2} + Y_4\, r^{m_2+n_2+4} + (Y_5 lnr + Y_6)\, r^{m_2+1}. \quad (3.36)$$

Using (3.35) in (2.10) and (2.11) radial and tangential stress functions obtained

for $\boldsymbol{n_1 \neq -1}$, as:

$$\sigma_{rr} = \frac{E_0}{(1+\nu)(1-2\nu)}[\{(1-\nu)s_1 + 2\nu\}X_1 r^{m_1+s_1-1} + \{(1-\nu)s_2 + 2\nu\}X_2 r^{m_1+s_2-1} + \\ +[\{(1-\nu)(m_2+n_2-n_1) + (3-\nu)\}Y_1 - (1+\nu)a_{t0}Q_1]r^{m_1+m_2+n_2-n_1+2} + \\ +[\{(1-\nu)(m_2-n_1) + 2\nu\}Y_2 - C_1(1+\nu)a_{t0}]r^{m_1+m_2-n_1-1} + [\{(1-\nu)m_2 + \\ +(1+\nu)\}Y_3 - C_2(1+\nu)a_{t0}]r^{m_1+m_2}]. \quad (3.37)$$

$$\sigma_{\theta\theta} = \frac{E_0}{(1+\nu)(1-2\nu)}[(1+\nu s_1)X_1 r^{m_1+s_1-1} + (1+\nu s_2)X_2 r^{m_1+s_2-1} + [\{\nu(m_2+n_2 - \\ -n_1+3)+1\}Y_1 - (1+\nu)a_{t0}Q_1]r^{m_1+m_2+n_2-n_1+2} + [\{\nu(m_2-n_1)+1\}Y_2 - \\ -C_1(1+\nu)a_{t0}]r^{m_1+m_2-n_1-1} + [\{\nu(m_2+1)+1\}Y_3 - C_2(1+\nu)a_{t0}]r^{m_1+m_2}]. \quad (3.38)$$

Using (3.36) in (2.10) and (2.11) one gets

for $\boldsymbol{n_1 = -1}$, as

$$\sigma_{rr} = \frac{E_0}{(1+\nu)(1-2\nu)}[\{(1-\nu)s_1 + 2\nu\}X_3 r^{m_1+s_1-1} + \{(1-\nu)s_2 + 2\nu\}X_4 r^{m_1+s_2-1} + \\ +[\{(m_2+n_2+4) - (m_2+n_2+2)\nu\}Y_4 - (1+\nu)a_{t0}Q_2]r^{m_1+m_2+n_2+3} + \\ +[\{(1-\nu)m_2 + (1+\nu)\}Y_6 + (1-\nu)Y_5 - C_4(1+\nu)a_{t0}]r^{m_1+m_2} + \\ +[\{(1-\nu)m_2 + +(1+\nu)\}Y_5 - C_3(1+\nu)a_{t0}]r^{m_1+m_2}lnr]. \quad (3.39)$$

$$\sigma_{\theta\theta} = \frac{E_0}{(1+\nu)(1-2\nu)}[(1+\nu s_1)X_3 r^{m_1+s_1-1} + (1+\nu s_2)X_4 r^{m_1+s_2-1} + [\{\nu(m_2+n_2 + \\ +4)+1\}Y_4 - (1+\nu)a_{t0}Q_2]r^{m_1+m_2+n_2+3} + [\{\nu(m_2+1)+1\}Y_6 + \nu Y_5 - \\ -C_4(1+\nu)a_{t0}]r^{m_1+m_2} + [\{\nu(m_2+1)+1\}Y_5 - C_3(1+\nu)a_{t0}]r^{m_1+m_2}lnr]. \quad (3.40)$$

Obtaining constants X_1, X_2, X_3,and X_4 by using the condition (2.12) and then one gets the expressions for thermal stress function from (3.37) to (3.40)

for $n_1 \neq -1$ as:

$$\sigma_{rr} = \frac{E_0 a_{t0}}{A_1}$$

$$\times \left[\!\left[Q_1 \left\{ Z_1 \begin{bmatrix} (a^{m_1+s_2-1}b^{m_1+m_2+n_2-n_1+2} - b^{m_1+s_2-1}a^{m_1+m_2+n_2-n_1+2})r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1}a^{m_1+m_2+n_2-n_1+2} - a^{m_1+s_1-1}b^{m_1+m_2+n_2-n_1+2})r^{m_1+s_2-1} \\ +A_1 r^{m_1+m_2+n_2-n_1+2} \end{bmatrix} \right.\right.\right.$$

$$-A_2 Z_2 \begin{bmatrix} (a^{m_1+s_2-1}b^{m_1+m_2-n_1-1} - b^{m_1+s_2-1}a^{m_1+m_2-n_1-1})r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1}a^{m_1+m_2-n_1-1} - a^{m_1+s_1-1}b^{m_1+m_2-n_1-1})r^{m_1+s_2-1} \\ +A_1 r^{m_1+m_2-n_1-1} \end{bmatrix}$$

$$\left. -A_3 Z_3 \begin{bmatrix} (a^{m_1+s_2-1}b^{m_1+m_2} - b^{m_1+s_2-1}a^{m_1+m_2})r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1}a^{m_1+m_2} - a^{m_1+s_1-1}b^{m_1+m_2})r^{m_1+s_2-1} \\ +A_1 r^{m_1+m_2} \end{bmatrix} \right\}$$

$$-A_4 Z_2 \begin{bmatrix} (a^{m_1+s_2-1}b^{m_1+m_2-n_1-1} - b^{m_1+s_2-1}a^{m_1+m_2-n_1-1})r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1}a^{m_1+m_2-n_1-1} - a^{m_1+s_1-1}b^{m_1+m_2-n_1-1})r^{m_1+s_2-1} \\ +A_1 r^{m_1+m_2-n_1-1} \end{bmatrix}$$

$$\left. +A_5 Z_3 \begin{bmatrix} (a^{m_1+s_2-1}b^{m_1+m_2} - b^{m_1+s_2-1}a^{m_1+m_2})r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1}a^{m_1+m_2} - a^{m_1+s_1-1}b^{m_1+m_2})r^{m_1+s_2-1} \\ +A_1 r^{m_1+m_2} \end{bmatrix} \right]\!\!\right] \tag{3.41}$$

$$\sigma_{\theta\theta} = \frac{E_0 a_{t0}}{A_1}$$

$$\times \left[\!\left[Q_1 \left\{ Z_1 \left[\frac{(\upsilon s_1 + 1) r^{m_1+s_1-1}}{[(1-\upsilon)s_1 + 2\upsilon]} (a^{m_1+s_2-1}b^{m_1+m_2+n_2-n_1+2} - b^{m_1+s_2-1}a^{m_1+m_2+n_2-n_1+2}) \right.\right.\right.\right.$$

$$\left. + \frac{(\upsilon s_2 + 1) r^{m_1+s_2-1}}{[(1-\upsilon)s_2 + 2\upsilon]} (b^{m_1+s_1-1}a^{m_1+m_2+n_2-n_1+2} - a^{m_1+s_1-1}b^{m_1+m_2+n_2-n_1+2}) \right]$$

$$+ Z_4 A_1 r^{m_1+m_2+n_2-n_1+2}$$

$$-A_2 \left\langle Z_2 \left[\frac{(\upsilon s_1 + 1) r^{m_1+s_1-1}}{[(1-\upsilon)s_1 + 2\upsilon]} (a^{m_1+s_2-1}b^{m_1+m_2-n_1-1} - b^{m_1+s_2-1}a^{m_1+m_2-n_1-1}) \right.\right.$$

$$\left. + \frac{(\upsilon s_2 + 1) r^{m_1+s_2-1}}{[(1-\upsilon)s_2 + 2\upsilon]} (b^{m_1+s_1-1}a^{m_1+m_2+n_2-n_1+2} - a^{m_1+s_1-1}b^{m_1+m_2+n_2-n_1+2}) \right]$$

$$\left. +Z_5 A_1 r^{m_1+m_2-n_1-1} \right\rangle$$

$$-A_3 \left\langle Z_3 \left[\frac{(\upsilon s_1 + 1) r^{m_1+s_1-1}}{[(1-\upsilon)s_1 + 2\upsilon]} (a^{m_1+s_2-1}b^{m_1+m_2} - b^{m_1+s_2-1}a^{m_1+m_2}) \right.\right.$$

$$+\frac{(\upsilon s_2+1)r^{m_1+s_2-1}}{[(1-\upsilon)s_2+2\upsilon]}(b^{m_1+s_1-1}a^{m_1+m_2}-a^{m_1+s_1-1}b^{m_1+m_2})\Big]+Z_6A_1r^{m_1+m_2}\Big\}$$

$$-A_4\Big\{Z_2\Big[\frac{(\upsilon s_1+1)r^{m_1+s_1-1}}{[(1-\upsilon)s_1+2\upsilon]}(a^{m_1+s_2-1}b^{m_1+m_2-n_1-1}-b^{m_1+s_2-1}a^{m_1+m_2-n_1-1})$$

$$+\frac{(\upsilon s_2+1)r^{m_1+s_2-1}}{[(1-\upsilon)s_2+2\upsilon]}(b^{m_1+s_1-1}a^{m_1+m_2-n_1-1}-a^{m_1+s_1-1}b^{m_1+m_2-n_1-1})\Big]+Z_5A_1r^{m_1+m_2-n_1-1}\Big\}$$

$$+A_5\Big\{Z_3\Big[\frac{(\upsilon s_1+1)r^{m_1+s_1-1}}{[(1-\upsilon)s_1+2\upsilon]}(a^{m_1+s_2-1}b^{m_1+m_2}-b^{m_1+s_2-1}a^{m_1+m_2})$$

$$+\frac{(\upsilon s_2+1)r^{m_1+s_2-1}}{[(1-\upsilon)s_2+2\upsilon]}(b^{m_1+s_1-1}a^{m_1+m_2-n_1-1}-a^{m_1+s_1-1}b^{m_1+m_2-n_1-1})\Big]+Z_6A_1r^{m_1+m_2}\Big\}\Big]\Big] \tag{3.42}$$

where,

$$A_1=(a^{m_1+s_1-1}b^{m_1+s_2-1}-b^{m_1+s_1-1}a^{m_1+s_2-1}),\quad A_2=\frac{(b^{n_2-n_1+2}-a^{n_2-n_1+2})}{(b^{-n_1-1}-a^{-n_1-1})},$$

$$A_3=\left[b^{n_2-n_1+2}-\frac{(b^{n_2-n_1+2}-a^{n_2-n_1+2})b^{-n_1-1}}{(b^{-n_1-1}-a^{-n_1-1})}\right],\quad A_4=\frac{(T_1-T_2)}{(b^{-n_1-1}-a^{-n_1-1})},$$

$$A_5=\left[T_2+\frac{(T_1-T_2)b^{-n_1-1}}{(b^{-n_1-1}-a^{-n_1-1})}\right],\quad A_6=\frac{(b^{n_2+3}-a^{n_2+3})}{ln\frac{b}{a}},\ A_7=\left[b^{n_2+3}-\frac{(b^{n_2+3}-a^{n_2+3})lnb}{ln\frac{b}{a}}\right],$$

$$A_8=\frac{(T_2-T_1)}{ln\frac{b}{a}},\quad A_9=\left[T_2-\frac{(T_2-T_1)lnb}{ln\frac{b}{a}}\right].$$

For $n=-1$

$$\sigma_{rr}=\frac{E_0a_{t0}}{A_1}\times\Bigg[\!\!\Bigg[Q_2\Bigg\{Z_7\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2+n_2+3}-b^{m_1+s_2-1}a^{m_1+m_2+n_2+3})r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2+n_2+3}-a^{m_1+s_1-1}b^{m_1+m_2+n_2+3})r^{m_1+s_2-1}\\+A_1r^{m_1+m_2+n_2+3}\end{bmatrix}$$

$$-A_6\Big\langle Z_8\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2}-b^{m_1+s_2-1}a^{m_1+m_2})r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2}-a^{m_1+s_1-1}b^{m_1+m_2})r^{m_1+s_2-1}+A_1r^{m_1+m_2}\end{bmatrix}$$

$$+Z_3\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2}lnb-b^{m_1+s_2-1}a^{m_1+m_2}lna)r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2}lna-a^{m_1+s_1-1}b^{m_1+m_2}lnb)r^{m_1+s_2-1}\\+A_1r^{m_1+m_2}lnr\end{bmatrix}\Big\rangle$$

$$-A_7Z_3\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2}-b^{m_1+s_2-1}a^{m_1+m_2})r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2}-a^{m_1+s_1-1}b^{m_1+m_2})r^{m_1+s_2-1}+A_1r^{m_1+m_2}\end{bmatrix}\Bigg\}$$

$$+A_8\Big\langle Z_8\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2}-b^{m_1+s_2-1}a^{m_1+m_2})r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2}-a^{m_1+s_1-1}b^{m_1+m_2})r^{m_1+s_2-1}+A_1r^{m_1+m_2}\end{bmatrix}$$

$$+Z_3\begin{bmatrix}(a^{m_1+s_2-1}b^{m_1+m_2}lnb-b^{m_1+s_2-1}a^{m_1+m_2}lna)r^{m_1+s_1-1}\\+(b^{m_1+s_1-1}a^{m_1+m_2}lna-a^{m_1+s_1-1}b^{m_1+m_2}lnb)r^{m_1+s_2-1}\\+A_1r^{m_1+m_2}lnr\end{bmatrix}\Big\rangle$$

$$+A_9 Z_3 \left[\begin{matrix} (a^{m_1+s_2-1} b^{m_1+m_2} - b^{m_1+s_2-1} a^{m_1+m_2}) r^{m_1+s_1-1} \\ +(b^{m_1+s_1-1} a^{m_1+m_2} - a^{m_1+s_1-1} b^{m_1+m_2}) r^{m_1+s_2-1} + A_1 r^{m_1+m_2} \end{matrix} \right] \Big]\Big], \quad (3.43)$$

where,

$$Z_1 = \frac{-2(m_2+n_2-n_1+2)}{(1-\nu)(m_2+n_2-n_1+3)(m_1+m_2+n_2-n_1+4)+2(\nu m_1+\nu-1)},$$

$$Z_2 = \frac{2(1-m_2+n_1)}{(1-\nu)(m_2-n_1)(m_1+m_2-n_1+1)+2(\nu m_1+\nu-1)},$$

$$Z_3 = \frac{-2m_2}{(1-\nu)(m_2+1)(m_1+m_2+2)+2(\nu m_1+\nu-1)} ,\mathrm{l}$$

$$Z_4 = \frac{-[(m_1+m_2+n_2-n_1+3)(m_2+n_2-n_1+3)-(m_1+1)]}{(1-\nu)(m_2+n_2-n_1+3)(m_1+m_2+n_2-n_1+4)+2(\nu m_1+\nu-1)},$$

$$Z_5 = \frac{-[(m_2-n_1)(m_1+m_2-n_1)-(m_1+1)]}{(1-\nu)(m_2-n_1)(m_1+m_2-n_1+1)+2(\nu m_1+\nu-1)},$$

$$Z_6 = \frac{-[(m_2+1)(m_1+m_2)+(m_2-n_1)]}{(1-\nu)(m_2+1)(m_1+m_2+2)+2(\nu m_1+\nu-1)},$$

$$Z_7 = \frac{-2(m_2+n_2+3)}{(1-\nu)(m_2+n_2+4)(m_1+m_2+n_2+5)+2(\nu m_1+\nu-1)}.$$

$$\sigma_{\theta\theta} =$$

$$\frac{E_0 a_{t0}}{A_1} \Bigg[\Bigg[Q_2 \Bigg\{ Z_7 \left[\begin{matrix} \frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2+n_2+3} - b^{m_1+s_2-1} a^{m_1+m_2+n_2+3}) \\ + \frac{(\nu s_2+1) r^{m_1+s_2-1}}{[(1-\nu)s_2+2\nu]} (b^{m_1+s_1-1} a^{m_1+m_2+n_2+3} - a^{m_1+s_1-1} b^{m_1+m_2+n_2+3}) \end{matrix} \right]$$

$$+ Z_9 A_1 r^{m_1+m_2+n_2+3}$$

$$-A_6 \langle Z_8 \left[\frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} - b^{m_1+s_2-1} a^{m_1+m_2}) \right] + Z_{10} A_1 r^{m_1+m_2}$$

$$Z_3 \Big[\frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} lnb - b^{m_1+s_2-1} a^{m_1+m_2} lna)$$

$$+ \frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} lnb - b^{m_1+s_2-1} a^{m_1+m_2} lna) \Big] + Z_{11} A_1 r^{m_1+m_2} lnr \rangle$$

$$-A_7 \langle Z_3 \Big[\frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} lnb - b^{m_1+s_2-1} a^{m_1+m_2} lna)$$

$$+ \frac{(\nu s_2+1) r^{m_1+s_2-1}}{[(1-\nu)s_2+2\nu]} (b^{m_1+s_1-1} a^{m_1+m_2} - a^{m_1+s_1-1} b^{m_1+m_2}) \Big] + Z_{11} A_1 r^{m_1+m_2} \Big\}$$

$$+A_8 \left\{ Z_8 \left[\begin{matrix} \frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} - b^{m_1+s_2-1} a^{m_1+m_2}) \\ + \frac{(\nu s_2+1) r^{m_1+s_2-1}}{[(1-\nu)s_2+2\nu]} (b^{m_1+s_1-1} a^{m_1+m_2} - a^{m_1+s_1-1} b^{m_1+m_2}) \end{matrix} \right] \right. + Z_{10} A_1 r^{m_1+m_2}$$

$$Z_3 \left[+ \begin{matrix} \frac{(\nu s_1+1) r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]} (a^{m_1+s_2-1} b^{m_1+m_2} lnb - b^{m_1+s_2-1} a^{m_1+m_2} lna) \\ \frac{(\nu s_2+1) r^{m_1+s_2-1}}{[(1-\nu)s_2+2\nu]} (b^{m_1+s_1-1} a^{m_1+m_2} lna - a^{m_1+s_1-1} b^{m_1+m_2} lnb) \end{matrix} \right] + Z_{11} A_1 r^{m_1+m_2} lnr$$

$$+A_9\left\{\begin{array}{c} Z_3\left[\begin{array}{l}\frac{(\nu s_1+1)r^{m_1+s_1-1}}{[(1-\nu)s_1+2\nu]}\left(a^{m_1+s_2-1}b^{m_1+m_2}-b^{m_1+s_2-1}a^{m_1+m_2}\right)\\ +\frac{(\nu s_2+1)r^{m_1+s_2-1}}{[(1-\nu)s_2+2\nu]}\left(b^{m_1+s_1-1}a^{m_1+m_2}-a^{m_1+s_1-1}b^{m_1+m_2}\right)\end{array}\right]\\ +Z_{11}A_1r^{m_1+m_2}\end{array}\right\}\Bigg]\Bigg], \tag{3.44}$$

where

$$Z_8=\frac{2\left[(1-\nu)m_2^2-(1+\nu)m_1\right]}{[(1-\nu)(m_2-n_1)(m_1+m_2-n_1+1)+2(\nu m_1+\nu-1)]^2},$$

$$Z_9=\frac{[(m_1+1)-(m_2+n_2+4)(m_1+m_2+n_2+4)]}{(1-\nu)(m_2+n_2+4)(m_1+m_2+n_2+5)+2(\nu m_1+\nu-1)},$$

$$Z_{10}=\frac{-m_1[m_2(1+3\upsilon)+(1+\upsilon)m_1+4]}{[(1-\nu)(m_2-n_1)(m_1+m_2-n_1+1)+2(\nu m_1+\nu-1)]^2},$$

$$Z_{11}=\frac{-\left[m_2^2+m_1m_2+2m_2\right]}{(1-\nu)(m_2+1)(m_1+m_2+2)+2(\nu m_1+\nu-1)}.$$

4. Validation

If, in the expression for radial stress and tangential stress one substitutes m_1, m_2, n_1 and n_2 equal to zero, one gets the expression for radial stress and tangential stress in an isotropic and homogeneous hollow sphere with uniform volumetric heat generation. This fact can be used for validation of the problem.

When one substitutes $m_1=m_2=n_1=n_2=0$ in equation (2.1), E, a_t, k and q become E_0, α_{t0}, k_0 and q_0 which are modulus of elasticity, coefficient of thermal expansion, thermal conductivity and constant volumetric heat generation respectively for an isotropic and homogeneous material. From equations (3.4), (3.5), (3.24), (3.29) - (2.34), (3.41) and (3.42) one obtains,

$s_1=1$, $s_2=-2$ and

$$\sigma_{rr}=\frac{2E_0a_tQ_1}{5(1-\nu)\left(\frac{b^3}{a^3}-1\right)}\left[(b^2-a^2)\left(\frac{b^3}{r^3}-1\right)-(b^2-r^2)\left(\frac{b^3}{a^3}-1\right)\right]$$
$$-\frac{E_0a_t(T_2-T_1)}{(1-\nu)}\left[\frac{\left(\frac{b^3}{r^3}-1\right)}{\left(\frac{b^3}{a^3}-1\right)}-\frac{\left(\frac{b}{r}-1\right)}{\left(\frac{b}{a}-1\right)}\right], \tag{4.1}$$

$$\sigma_{\theta\theta}=-\frac{2E_0a_tQ_1}{5(1-\nu)\left(\frac{b^3}{a^3}-1\right)}\left[(b^2-a^2)\left(\frac{b^3}{2r^3}+1\right)-(b^2-2r^2)\left(\frac{b^3}{a^3}-1\right)\right]$$
$$+\frac{E_0a_t(T_2-T_1)}{(1-\nu)}\left[\frac{\left(\frac{b^3}{2r^3}+1\right)}{\left(\frac{b^3}{a^3}-1\right)}+\frac{\left(\frac{b}{2r}-1\right)}{\left(\frac{b}{a}-1\right)}\right]. \tag{4.2}$$

If one puts $Q_1 = 0$ in equations (4.1) and (4.2), one verifies that the results obtained will be the expressions for the radial and the tangential stresses for an isotropic and homogeneous hollow thick sphere without heat generation. The results obtained for thermal stresses with non-uniform heat generation are validated with the results of Nayak et al. (2011) by putting Q_1 and Q_2 equal to zero in the expressions (3.41) to (3.44) respectively.

5. Special case and numerical calculations

To construct the mathematical thermoelastic model of a FG thick hollow sphere one considers a thermal gradient through its radial direction. In numerical representative results are presented for rich of ceramic (Zirconia) material. The FG hollow thick vessel of single constituent with

Inner radius:	$a = 1m$,
Outer radius:	$b = 1.2m,$
Poisson's ratio is:	$\nu = 0.3.$,
Inner surface of the hollow sphere is fixed at:	$T_1 = 10^0C$,
Outer surface is kept at:	$T_2 = 0^0C$,

$q_0 = 500W/m^3.$

The thermo mechanical properties of ceramic Zirconia are,

$E_0 = 151GPa$,

$a_{t0} = 10 \times 10^{-6}/^0C$,

$k_0 = 2.09W/mK$.

Graphical illustrations

For graphical illustrations of this problem, one considers following two cases.

Case 1:

In the first part, the power index for the modulus of elasticity, coefficient of thermal expansion, heat conduction coefficient and heat generation functional are assumed to be identical $m_1 = m_2 = n_1 = n_2$. For $m_1 = m_2 = n_1 = n_2 \neq -1$, one uses the temperature distribution obtained by (3.6) and thermal stress components which obtained in equations (3.41) and (3.42). While for $m_1 = m_2 = n_1 = n_2 = -1$, the temperature distribution and thermal stress components are determined by using the expressions (3.12), (3.43) and (3.44) respectively. **The temperature distribution and thermal stresses with non-uniform heat generation are represented graphically and discussed as a particular case with variation in power index parameter as $\boldsymbol{m_1 = m_2 = n_1 = n_2 = 0, 1, 2, 3, -1, -2}$.**

Figure 1 represents the variation in the temperature with radius in the presence of non-uniform heat source within the sphere. The temperature increases as power law decreases for

$m = 3,2,1,0,-2$ but it is interesting to note that the temperature shows small variation for the parameter $m = -1$ when one compares results with the other parameter values.

Figure 2 represents the radial stress distribution with heat generation within it. The radial stress is zero on the surfaces, due to assumed mechanical boundary conditions. The radial stress is compressive throughout the sphere. It is observed that the compression shifts towards the outer surface as parameters decreases as 3, 2, 1 and -2. For $m = 0$ the variation can be observed as the case of the homogeneous and isotropic material.

It is interesting to note that the radial stress is tensile in nature for $m = -1$ as shown in a **Figure 3**.

Figure 4 gives the variation in tangential stress with radius in the presence of non-uniform source of heat inside the sphere. The stress is decreases from inner to outer surface. The curve associated some values of a parameter the variation of tangential is almost uniform across the radius.

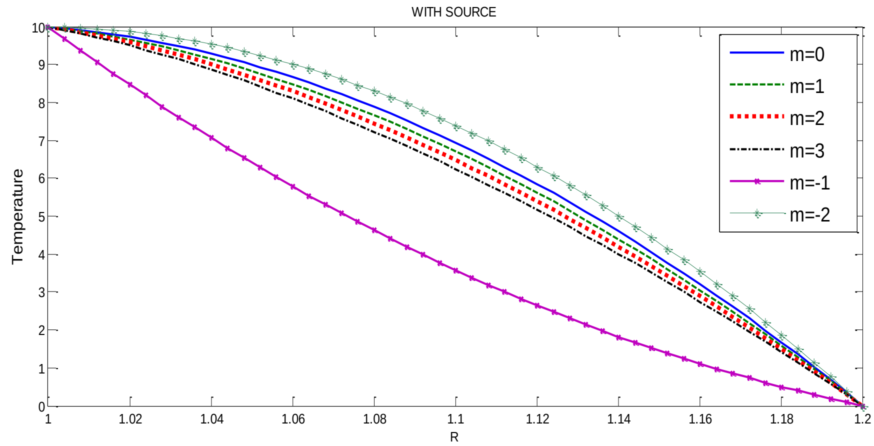

Figure 1. Temperature distribution with radius for $m = 0,1,2,3,-1,-2$

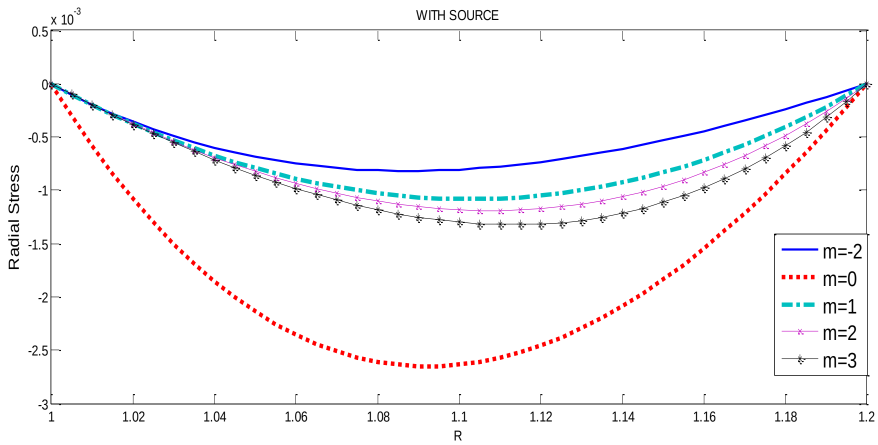

Figure 2. Radial stress with radius for $m = 0,1,2,3,-2$

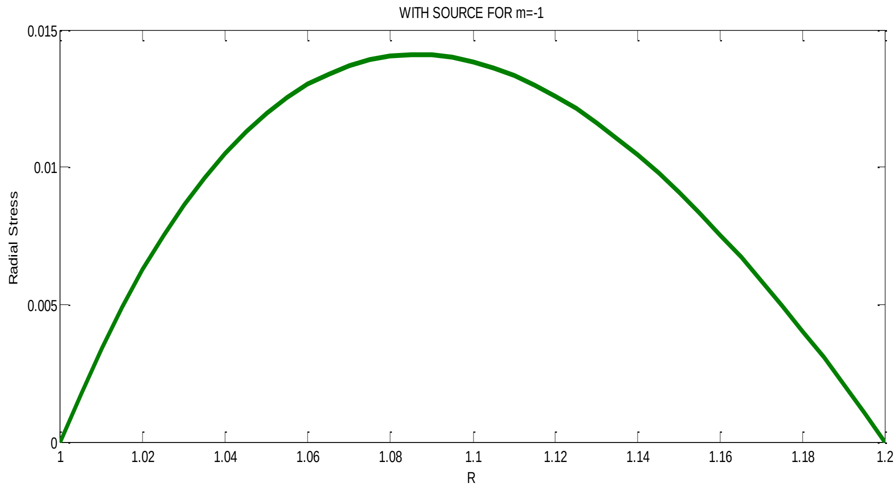

Figure 3. Radial stress distribution with radius for $m = -1$

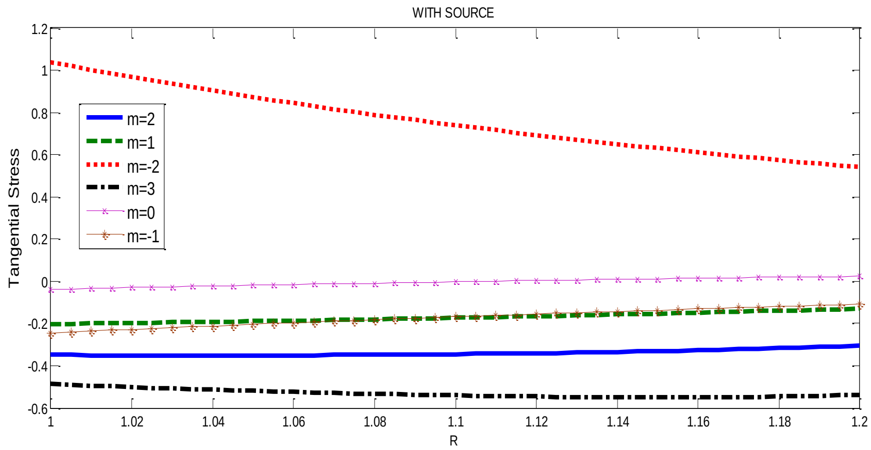

Figure 4. Tangential stress distribution with radius

Case 2:

In second part of the analysis, the temperature distribution and thermal stresses are represented graphically with variation in the power index parameter of heat generation n_2 for the fixed values of the other power index parameters of modulus of elasticity, coefficient of thermal expansion, heat conduction coefficient m_1, m_2, n_1.

Figures 5, 6 and 7 represents the radial stress distribution along the radial direction with variation in the power law index of heat generation functional. In **Figure 5** stress function is plotted with radius for $m_1 = m_2 = n_1 = 2$ with varying the value of n_2. For $n_2 = 5$ and -5 the stress is compressive within the sphere and as the values of n_2 increase the nature of stress changes. For greater values of n_2 the stress become tensile for the region up to radius about $r \approx 1.13$ and remaining part is compressive. **Figure 6** shows the variation in radial stress with $m_1 = m_2 = n_1 = -2$ and with varying the value of n_2. It gives the same result with small variation in the values of stress. For $m_1 = m_2 = n_1 = -1$, the result is totally different which is shown in **Figure 7**. When one increases the value of heat generation parameter n_2 , the radial stresses change their nature from compression to tensile. In **Figure 8** and **Figure 9** the tangential s tress distribution is plotted with the radial direction with respect to the change in the power index of heat generation parameter. The nature of the stresses varies with the variation in the value of parameter n_2.

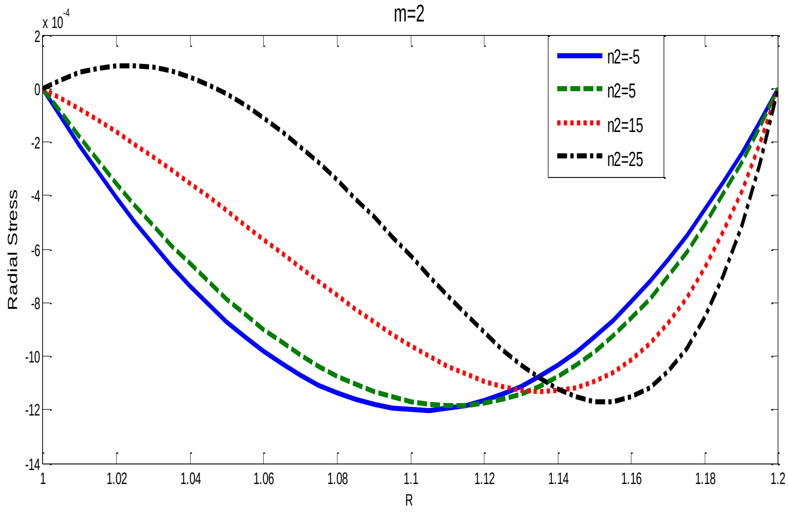

Figure 5. Radial stresses with varying heat generation parameter

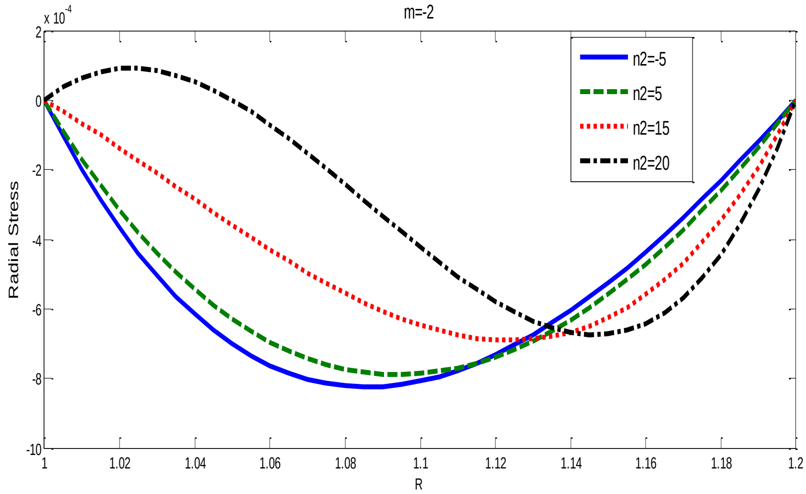

Figure 6. Radial stresses with varying heat generation parameter

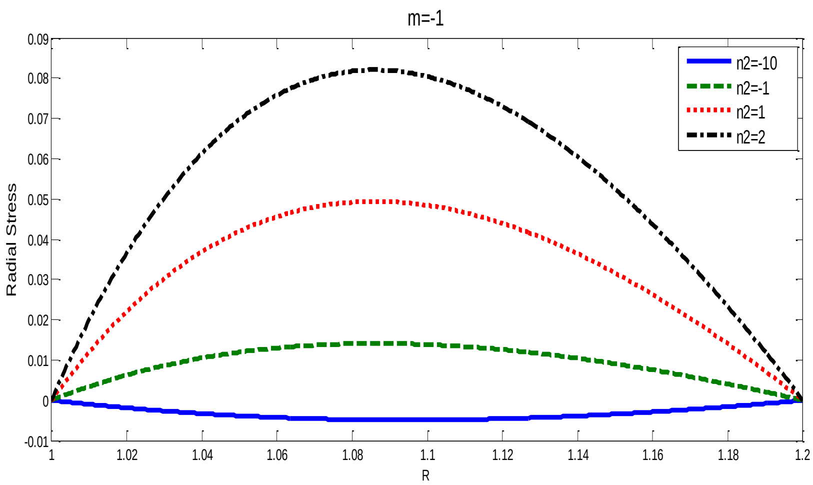

Figure 7. Radial stress with varying heat generation parameter for $m = -1$

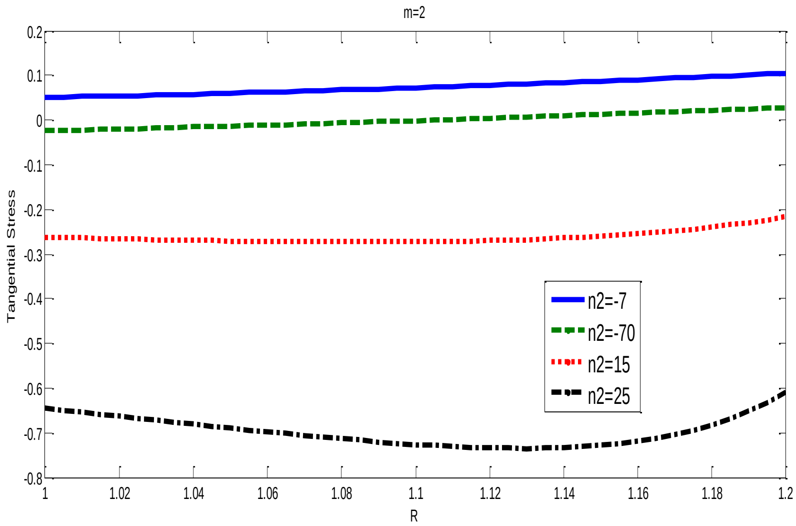

Figure 8: Tangential stress distribution with radius for varying n_2

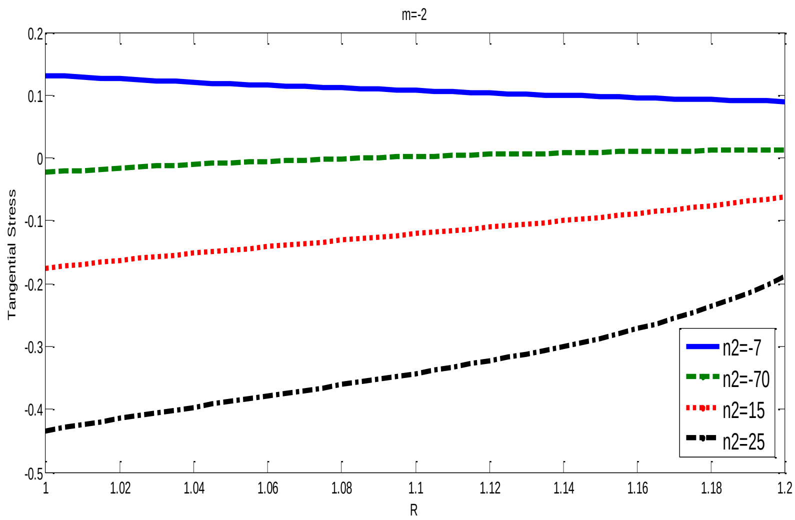

Figure 9. Tangential stress distribution with radius for varying n_2

6. Conclusion

In this article exact analytical solutions are obtained for the temperature distribution and thermal stresses with a non-uniform internal heat generation when the inner and outer surface of the FGM hollow sphere is maintained at a constant temperature. As a special case, mathematical model is constructed for hollow sphere of Zirconia with material properties as specified in the numerical calculations.

In this study the attempt is made to observe the variation in the thermal stresses in presence of variable heat source which varying from the inner to the outer surface. It is observed that in the presence of the source in the present form the temperature increases as the power law index decreases. In presence of the heat source the radial stress is compressive inside sphere as per the earlier results as Nayak et al. (2011) but in our study it is found that the compression shifts towards the outer surface. It is Interesting to observe that for $m = -1$ the nature of radial stress becomes tensile which is a new finding. The tangential stress is decreases from inner to outer surface.

When the power index of the heat source function is varied keeping the power indices of material properties fixed, the radial stress switches from compression to tensile for $n_2 > 5$ in the region $r < 1.13$ while for other part it remains under compression. The nature of the tangential stress changes with change in the power index parameter in source function.
In this article the temperature distribution and thermal stresses in an FGM hollow sphere is obtained with non-uniform heat source inside it. The effect of change in power index parameter of heat source on thermal stress keeping other parameters fixed is also discussed and compared with the results of Nayak et al. (2011). The results can be generalized for other parameter values.

This is a novel approach to study the thermal stresses in an FGM hollow sphere with non-uniform heat generation within the sphere and the results presented in this problem are new and not discussed previously in the open literature.

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