Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 10, Issue 1 (June 2015), pp. 544 - 551

A Super Non-dominated Point for Multi-objective Transportation Problem

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Received: March 19, 2014; Accepted: January 22, 2015

Abstract

In this paper a method to obtain a non-dominated point for the multi-objective transportation problem is presented. The superiority of this method over the other existing methods is that the presented non-dominated point is the closest solution to the ideal solution of that problem. The presented method does not need to have the ideal point and other parameters to find this solution. Also, the calculative load of this method is less than other methods in the literature.

Keywords: Multi-objective programming; Transportation problem

AMS-MSC 2010 No.: 90C29, 90C08

1. Introduction

One of the undeniable problems in the real world is that of transportation. In general terms, the transportation model presents a plan with the least expenses to transfer goods from some places to some destinations. In a lot of practical problems the transportation formulation in single-objective form cannot be appropriate, because most of the problems include several objectives like minimizing the total cost, minimizing the total time etc. The transportation model was first developed by Hitchcock (1941). To obtain all of the non-dominated solutions of a multi-objective linear transportation problem an algorithm was introduced by Isermann (1979). Such problems can be solved by programming multi-objective linear techniques like parametric method, the adjacent point method and the adjacent basic point method (see Gal (1975) and Zelenv (1974)). Martnez-Salazar (2014) solved a bi-objective transportation optimization problem using a heuristic algorithm. In Hongwei (2010) and Kundu (2013). Some methods to solve multi-objective transportation optimization problem were presented. It is an undeniable fact that among all of the non-dominated solutions the solution with the least distance from the ideal solution can be considered decision-maker, and in the second part of this paper a method is presented to calculate the closest solution to the ideal solution for multi-objective transportation problems. In the following, the convergence of the method is discussed. In the third part two examples of the proposed method and the other existing methods are solved, the optimal solution and the calculative load of the methods are also compared. The results are provided in the final part.

2. Problem Formulation

The general form of a multi-objective transportation problem is as follows:

$$
\begin{aligned}
P_1: \quad & \min Z_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}\,, \qquad k = 1,2,\dots,p,\\
& s.t.\\
& \sum_{j=1}^{n} x_{ij} = a_i\,, \qquad i = 1,\dots,m,\\
& \sum_{i=1}^{m} x_{ij} = b_j\,, \qquad j = 1,\dots,n\,,\\
& x_{ij} \geq 0 \quad i = 1,\dots,m \qquad j = 1,\dots,n,
\end{aligned}
$$

where the subscript on Z_k and the superscript on C_{ij}^{k} are used to identify the penalty criterion. Without loss of generality it may be assumed throughout this paper that $a_i > 0$ for all i, $b_j > 0$ for all j, $C_{ij}^{k} \geq 0$ for all (i, j) and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Notice that the balance condition

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

is both a necessary and sufficient condition for the existence of a feasible solution of problem P_1.

Definition 2.1 (Ehrgott (2005)**).**

x^* is an efficient solution for P_1 if there is no other x belonged to the feasible region of problem P_1 such that:$Z_i(x) \leq Z_i(x^*), i = 1,\dots,p$ and $Z_j(x) < Z_j(x^*)$ for some j. If x^* is an efficient solution for P_1 then $Z(x^*) = (Z_1(x^*),\dots,Z_p(x^*))$ is a non-dominated point for P_1.

Definition 2.2 (Ehrgott (2005)**).**

$Z^I = (Z_1^I, \dots, Z_p^I)$ is the ideal point of P_1 if $Z_k^I = \min_{x \in S} Z_k(x)$ for $k = 1, \dots, p$.

Definition 2.3.

x^* is a supper-efficient for P_1 if

1) x^* is efficient
2) $Z(x^*)$ has the least distance to the ideal point.

If x^* is a supper efficient then $Z(x^*)$ is called the super non-dominated point. In the following it is shown that a single-objective linear problem can be solved instead of a multi-objective problem P_1 and the optimal solution which is obtained from the new problem is the super-efficient solution for problem P_1.

Assume $Z_k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}$ and consider the following problem:

$$
\begin{aligned}
P_2:\quad & \min Z(x) = Z_1(x) + Z_2(x) + \cdots + Z_p(x), \\
s.t.\quad & \\
& \sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, \dots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, \dots, n, \\
& x_{ij} \geq 0, \qquad i = 1, \dots, m, \quad j = 1, \dots, n.
\end{aligned}
$$

Theorem 2.1.

Every optimal solution for problem P_2 is an efficient solution for problem P_1.

Proof:

Assume that x^* is an optimal solution of P_2, but not the efficient solution of P_1, so, there is a feasible solution (the feasible region of both problems is identical) like y that dominates x^*, that is, $Z_i(y) \leq Z_i(x^*), i = 1, \dots, p$ and $Z_j(y) < Z_j(x^*)$ for some j, adding the side of the inequalities there is: $Z_1(y) + \cdots + Z_p(y) < Z_1(x^*) + \cdots + Z_p(x^*)$, which is in contrast with x^* to optimal for P_2. So, x^* is an efficient point for P_1.

Definition 2.4.

If $Z^I = (Z_1^I, \dots, Z_p^I)$ is the ideal solution of problem P_1, the distance of each point of the objective space from the ideal point (with the norm of L_1) is considered as following:

$$d := \sum_{i=1}^{p} \left| Z_i - Z_i^I \right|.$$

Theorem 2.2.

The problem of finding the super-efficient solution (the closest point to the ideal point for the multi-objective transportation problem) for the problem P_1 is equal to finding the optimal solution for the problem P_2.

Proof:

If S is the feasible region of problem P_1, the problem of the closest solution to the ideal solution is considered as following:

$$\begin{aligned} min\ \ d &= \sum_{i=1}^{p} \left| Z_i - Z_i^I \right|. \\ s.t. \quad & x \in S \end{aligned}$$

Because Z^I is the ideal point of P_1, then for each $x \in S$ and for all i,

$$Z_i^I \leq Z_i(x) \rightarrow \left| Z_i - Z_i^I \right| = Z_i - Z_i^I$$
$$\rightarrow d = \sum_{i=1}^{p} (Z_i - Z_i^I) = \sum_{i=1}^{p} Z_i - \sum_{i=1}^{p} Z_i^I .$$

Since the vector Z^I is specified and obtained, then it can be assumed that

$$\alpha = -\sum_{i=1}^{p} Z_i^I$$

is a fixed value. So, for each $x \in S$ there is

$$\left\{ \min_{x \in S}\ d = \sum_{i=1}^{p} \left| Z_i - Z_i^I \right| = \sum_{i=1}^{p} Z_i + \alpha \right\} = \left\{ \min_{x \in S}\ d = \sum_{i=1}^{p} Z_i \right\}.$$

This is the favorite result and the proof is complete.

3. Numerical example

Example 1. Consider the following two-objective transportation problem:

$$
\begin{aligned}
&min\ Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + \\
&\qquad 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} \\
&min\ Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + \\
&\qquad 6x_{31} + 2x_{32} + 5x_{33} + x_{34} \\
&s.t \quad \sum_{j=1}^{4} x_{1j} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17 \\
&\qquad \sum_{i=1}^{3} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14, \quad \sum_{i=1}^{3} x_{i4} = 16 \\
&\qquad x_{ij} \geq 0, \ \ i = 1.2.3, \quad j = 1,2,3,4.
\end{aligned} \tag{1}
$$

Lushu et al. (2000) computed (163,190) (integer solution) or (163.33, 190.83) as the optimal compromise value of the objective vector (Z_1, Z_2). Ringuest et al. (1978). Computed (156, 200) as the most preferred value of the objective vector (Z_1, Z_2). Bit et al. (1992) obtained (160, 195) (integer solution) or (160.8591, 193.9260) as the optimal compromise value of the objective vector (Z_1, Z_2).

For solving the problem above with the presented method this paper, we have to solve the following problem:

$$
\begin{aligned}
&min\ Z = Z_1 + Z_2 = 5x_{11} + 6x_{12} + 10x_{13} + 10x_{14} + 6x_{21} + 17x_{22} + 12x_{23} + 14x_{24} + \\
&\qquad 14x_{31} + 11x_{32} + 9x_{33} + 7x_{34} \\
&s.t \quad \sum_{j=1}^{4} x_{1j} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17 \\
&\qquad \sum_{i=1}^{3} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14, \quad \sum_{i=1}^{3} x_{i4} = 16 \\
&\qquad x_{ij} \geq 0, \ \ i = 1.2.3, \quad j = 1,2,3,4.
\end{aligned}
$$

The optimal solution x^* is obtained:

$$x^* = (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34})$$

$$x_{11} = x_{14} = x_{22} = x_{24} = x_{31} = x_{32} = 0$$

$$
\begin{aligned}
&x_{12} = 3 \qquad x_{21} = 11 \qquad x_{33} = 1 \\
&x_{13} = 5 \qquad x_{23} = 8 \qquad x_{34} = 16.
\end{aligned}
$$

By Theorem 2.1, $Z(x^*) = (Z_1(x^*), Z_2(x^*)) = (176,175)$ is a non-dominated point for the problem (1). By Definition 2.2, obtained $Z^I = (143,167)$ as the ideal point of the problem (1).

The calculated non-dominated points of the problem (1) by various other methods and their distance from the ideal point are presented in the following table:

Table 1. Result of Analysis for Example 1.

Method	Lusha	Ringues	Bit	Our method
$Z = (Z_1, Z_2)$	(163,190)	(156,200)	(160,195)	(176,175)
$d = \sum_{i=1}^{2} \lvert Z_i - Z_i^I \rvert$	43	46	45	41

Table 1 shows that the obtained solution by the presented method has the minimum from the ideal point.

Example 2.

In the following transportation objective problem obtained in Bit (1992), all of the non-dominated points and their distances to the ideal point have been collected in a table using a method known as Figueria.

$$
\begin{aligned}
&\min Z_1 = 5x_{11} + 9x_{12} + 9x_{13} + 4x_{21} + 6x_{22} + 2x_{23} + 2x_{31} + x_{32} + x_{33} \\
&\min Z_2 = 4x_{11} + 7x_{12} + 3x_{13} + 2x_{21} + x_{22} + 5x_{23} + 7x_{31} + 6x_{32} + 6x_{33} \\
&s.t. \\
&\quad x_{11} + x_{12} + x_{13} = 15, \qquad x_{11} + x_{21} + x_{31} = 10 \\
&\quad x_{21} + x_{22} + x_{23} = 20, \qquad x_{12} + x_{22} + x_{32} = 4 \\
&\quad x_{31} + x_{32} + x_{33} = 10, \qquad x_{13} + x_{23} + x_{33} = 31 \\
&\quad x_{ij} \geq 0 \qquad i, j = 1,2,3
\end{aligned}
$$

Solving both problems individually, the ideal solution was obtained (145,179). All of the non-dominated points of the problem which obtained by the Figueria method and the distance of these points to the ideal point are shown in the following table using the d meter.

Regarding the Table 2, points of $\{x_1, x_2, x_3, x_4, x_5\}$ are the non-dominated points which have the least distance to the ideal point. To obtain each of the non-dominated points using the Figueria method, complicated calculations are required, while using the proposed method solving a transportation linear programming problem which its target function is the sum of two given target functions, i.e. $Z_1 + Z_2$ and its feasible region is the feasible region of the two-objective problem, the solution of $x_5 = (161{,}199)$ is introduced as the optimized solution which is included in the points having the least distance to the ideal point. More surprising fact is that if the distance from the ideal point is being considered with the norm of L_2 the x_5 is the only point having the least distance from the ideal point.

Table 2. Result of Analysis for Example 2

Point	$Z_1(x_i)$	$Z_2(x_i)$	$d = \sum_{i=1}^{2} \lvert Z_i - Z_i^I \rvert$
x_1	145	215	36
x_2	149	211	36
x_3	153	207	36
x_4	157	203	36
x_5	161	199	36
x_6	167	195	38
x_7	173	191	40
x_8	179	187	42
x_9	185	183	39
x_{10}	191	179	46

4. Conclusion

As in the case of a linear transportation programming problem a non-dominated solution for the multi-objective transportation problem which has the least distance to the ideal point, can be obtained. Even through the method of solution may be relatively complicated and has a great deal of calculative load. The ideal solution itself to problem (2.2) and any other parameters may not be needed. This represents an advantage.

Acknowledgment

The authors are grateful to the referrers for their helpful comments on some parts of the paper.

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