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Group Decision Making Using Comparative Linguistic Expression Based on Hesitant Intuitionistic Fuzzy Sets

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Abstract

We introduce a method for aggregation of experts' opinions given in the form of comparative linguistic expression. An algorithmic form of technique for order preference is proposed for group decision making. A simple example is given by using this method for the selection of the best alternative as well as ranking the alternatives from the best to the worst.

Keywords: Group decision making, comparative linguistic expression, hesitant fuzzy linguistic term set, intuitionistic fuzzy set

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1. Introduction

For better modelling of uncertain information, Atanassov (1986, 1999) gave the concept of intuitionistic fuzzy sets (IFS). Recently, the intuitionistic fuzzy set has been widely applied to decision making problems because it is highly useful for expressing information under a fuzzy environment (Beg and Rashid (2014a), Boran et al. (2009), De et al. (2001), Li (2005), Li et al.

(2008)). Torra (2010) introduced the hesitant fuzzy set as an extension of ordinary fuzzy sets to manage those situations in which several values are possible for a membership function. This set is defined in terms of a function that returns a set of membership values for each element in the domain. Afterwards it was also used by several other researchers for modelling decision making problems (Xia and Xu (2011), Yu et al. (2013), Zhang and Wei (2013). Often experts are restricted to providing their preferences by use of just one linguistic term, which may not reflect the exact information. To overcome this situation, the concept of the hesitant fuzzy linguistic term set (HFLTS) was introduced by Rodríguez, Martínez, and Herrera (2012). HFLTS are successfully applied in group decision making problems (Beg and Rashid (2013), Rodríguez et al. (2012), Rodríguez et al. (2013)). The use of linguistic information by experts is quite common in problems with a high degree of uncertainty and has provided reliable and successful results in different GDM problems. HFLTS provides flexibility in linguistic expressions to express preferences for decision makers; in particular it allows the use of comparative linguistic expressions.

In view of IFS, experts may feel some hesitation in non-membership values in linguistic form. Recently Beg and Rashid (2014b) introduced the concept of hesitant intuitionistic fuzzy linguistic term sets to manage both situations of hesitation: the first is possible membership linguistic terms, and the second is non-membership linguistic terms. A hesitant intuitionistic fuzzy linguistic term set (HIFLTS) presents more information about any element in that set than the ordinary fuzzy set. Our proposed method show a new linguistic GDM model. It deals with comparative linguistic expressions that are similar to those used by decision makers in real world decision making problems based on HIFLTS. It support decision makers' preference in uncertain group decision-making situations in which they require rich expressions in order to be able to express their preferences even when they hesitate among different membership and non-membership linguistic terms. This novel GDM model is based on *aggregation phase* that combines the decision makers' preferences, and on *computation phase* that obtains a solution set of alternatives. This is achieved by comparative linguistic expressions, with their transformation into linguistic intervals modeled by HIFLTS.

The remainder of the article is organized as follows. In Section 2, we give some basic concepts to understand our proposal. In Section 3, we propose a group decision-making method for comparative linguistic expressions based on HIFLTS. In Section 4, an example is given to show the practicality and feasibility of the proposed method by the ranking of alternatives. In Section 5, the conclusion of the paper is given.

2. Basic Concepts

Let X be a universe of discourse, and a fuzzy set in X is an expression A given by $A = \{\langle x, t_A(x)\rangle \,|x \in X\}$, where $t_A : X \to [0,1]$ is a membership function which characterizes the degree of membership of the element x to the set A (Zadeh (1965)). The main characteristic of fuzzy sets is that the membership function assigns to each element x in a universe of discourse X a membership degree in the interval $[0,1]$ and the non-membership degree equals one minus the membership degree, i.e. this single membership degree combines the evidence for x and the

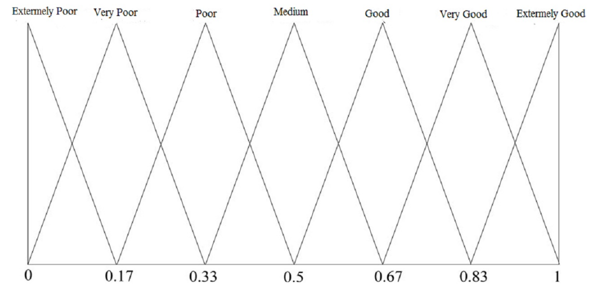

Figure 1: Set of seven terms with its semantics

evidence against x.

Definition 1. (Rodríguez et al. (2012)) Let S be a linguistic term set and $S = \{s_0, ..., s_g$ An HFLTS H_S is an ordered finite subset of the consecutive linguistic terms of S.

Let S be a linguistic term set, $S = \{s_0, ..., s_g\}$ Then we define the empty HFLTS and the full HFLTS for a linguistic variable ϑ as follows.

1) empty HFLTS: $H_S(\vartheta) = \{\ \}$,

2) full HFLTS: $H_S(\vartheta) = S$.

Any other HFLTS is formed with at least one linguistic term in S.

Example 1.

Let S be a linguistic term set (Fig. 1). Then $S = \{s_0:$ Extremely Poor $(EP), s_1:$ Very Poor (VP), $s_2:$ Poor $(P), s_3:$ Medium $(M), s_4:$ Good $(G), s_5:$ Very Good $(VG), s_6:$ Extremely Good $(EG)\}$.

Definition 2. Let Sbe an ordered finite set of linguistic terms, $S = \{s_0, ..., s_g\}$, A is an ordered finite subset of the consecutive linguistic terms of S. Then the 'max' and 'min' operators on set A are defined as:

1) $\max(A) = \max(s_i) = s_j,\ s_i \in A$ *and* $s_i \leq s_j \qquad \forall i$;

2) $\min(A) = \ \min(s_i) = s_j,\ s_i \in A$ *and* $s_i \geq s_j \qquad \forall i$.

Atanassov (1986) generalized the concept of the fuzzy set and introduced the concept of intuitionistic fuzzy sets as follows.

Definition 3. (Atanassov (1986)) Let $X = \{x_1, x_2, \ldots\}$ be a universe of discourse. An intuitionistic fuzzy set in X is an expression A given by $A = \{(x_i,\ t_A(x_i),\ f_A(x_i))|x_i \in X\}$, where $t_A : X \to [0, 1], f_A : X \to [0, 1]$ with the condition $0 \leq t_A(x_i) + f_A(x_i) \leq 1$, for all x_i *in* X. The numbers $t_A(x_i)$ and $f_A(x_i)$ represent the degree of membership and the degree of non-membership of the element x_i in the set A, respectively.

For convenience the element $(x_i, t_A(x_i), f_A(x_i))$ of intuitionistic fuzzy sets is known as an intuitionistic fuzzy number and it is denoted as $x_i = (t_A(x_i), f_A(x_i))$.

For each intuitionistic fuzzy set A in X, if $\pi_A(x) = 1 - t_A(x) - f_A(x)$, then $\pi_A(x)$ is called the degree of indeterminacy of x to A.

Definition 4. (Xu and Yager (2006)) Let $a = (a_1, a_1')$ be an intuitionistic fuzzy number If $Sc(a) = (a_1 - a_1')$, then $Sc(a)$ is called a score of a, where $Sc(a) \in [-1, 1]$.

Definition 5. (Xu and Yager (2006)) Let $a = (a_1, a_1')$ be an intuitionistic fuzzy number. If $Ac(a) = (a_1 + a_1')$, then $Ac(a)$ is called an accuracy of a, where $Ac(a) \in [0, 1]$.

Next we introduce the concept of hesitant intuitionistic fuzzy linguistic term set (HIFLTS).

Definition 6. (Beg and Rashid (2014b)) A hesitant intuitionistic fuzzy linguistic term set on X are functions h and h' that, when applied to X, return ordered finite subsets of the consecutive linguistic term set, $S = \{s_0, ..., s_g\}$, which can be represented as the following mathematical symbol:

$$E = \{(x, h(x), h'(x))|x \in X\},$$

where $h(x)$ and $h'(x)$ are subsets of the consecutive linguistic terms of S, denoting the possible membership degrees and non-membership degrees of the element $x \in X$ to the set E with the conditions that $\max(h(x)) + \min(h'(x)) \leq s_g$ and $\min(h(x)) + \max(h'(x)) \leq s_g$.

For convenience, $(h(x),\ h'(x))$ denotes a hesitant intuitionistic fuzzy linguistic term element (HIFLTE). Any other HIFLTS is formed with at least one linguistic term in S.

Atanassov (1999) introduced the concept of envelope for HFLTS. Beg and Rashid (2014b) further modified this concept for HIFLTS.

Definition 7. (Beg and Rashid (2014b)) The envelope of an HIFLTS A, is defined as:

$$env(A) = \{(x_{i,}[\min(h(x_i)), \max(h(x_i))], [\min(h'(x_i)), \max(h'(x_i))])]|x_i \in A)\}.$$

For convenience, $env(A(x)) = ([\min(h(x)), \max(h(x))], [\min(h'(x)), \max(h'(x))])$ is an envelope of hesitant intuitionistic fuzzy linguistic term element (EHIFLTE).

Consequently, an envelope of HIFLTS gives the complete information of HIFLTS. If we know the envelope of HIFLTS and linguistic term set then we can write HIFLTS.

Remark 1: (Herrera and Martinez (2000), Martinez and Herrera (2012))

(1) The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $\{0, \ldots, g\}$ which indicates the index of the closest linguistic term in S.

(2) Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. We define the function $\Delta : [0, g] \to \langle S \rangle$ given by

$$\Delta(\beta) = (s_i, \alpha), \text{ with} i = round(\beta) \text{ and } \alpha = \beta - i.$$

(3) Δ is a bijective function and $\Delta^{-1} : \langle S \rangle \to [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$.

Remark 2: (Rodriguez et al. (2013)) The convention between a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation, $s_i \in S \Rightarrow (s_i, 0)$.

3. Group Decision Making using HIFLTS

In general, group decision-making problems include uncertain imprecise data and information. These are new and essentially general type of matrices, called index matrices, and their extensions as intuitionistic fuzzy index matrices, extended intuitionistic fuzzy index matrices, temporal intuitionistic fuzzy index matrices, etc. (Atanassov (2014)). In our proposed scheme, fuzzy decision matrices are used to represent the opinions of decision makers. Now we give steps for the group decision making model for comparative linguistic expression based on HIFLTS.

Let $\tilde{X}^l = [(H^l_{S_{ij}}, H'^l_{S_{ij}})]_{m\times m}$ be a fuzzy decision matrix for the group decision making (GDM) problem and the following notations are used to depict the considered problems:

$M = \{m_1, m_2, \ldots, m_K\}$ is the set of the decision makers or experts involved in the group decision making process;

$P = \{P_1, P_2, \ldots, P_m\}$ is the set of the considered alternatives.

Preference of alternative P_i on the alternative P_j is denoted as HIFLTE $(H^l_{S_{ij}}, H'^l_{S_{ij}})$ for the decision maker l where $1 \leq l \leq K$.

A. Transformation of the linguistic expression into linguistic intervals

Using the envelope of HIFLTE, we transform all the fuzzy preference matrix to such a form that the entry of each matrix is denoted as the envelope of HIFLTE.

Let $\tilde{X}^l = [(env(H^l_{S_{ij}}), env(H'^l_{S_{ij}}))]_{m\times m}$ where $env(H^l_{S_{ij}}) = [\min(H^l_{S_{ij}}), \max(H^l_{S_{ij}})]$ be a fuzzy preference matrix for the GDM problem.

B. Choice of an aggregation operator for linguistic intervals

We calculate the one preference matrix $\tilde{X}$ by aggregating the opinions of DMs ($\tilde{X}^1$, $\tilde{X}^2$, $\cdots$, $\tilde{X}^K$);

$\tilde{X} = [x_{ij}]$, where

$$\begin{aligned} x_{ij} \;=\; & \Bigg(\left[\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\min(H^l_{S_{ij}})).\right), \Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\max(H^l_{S_{ij}}))\right)\right], \\ & \left[\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\min(H'^l_{S_{ij}}))\right), \Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\max(H'^l_{S_{ij}}))\right)\right]\Bigg). \end{aligned}$$

C. Intuitionistic linguistic interval for each alternative

We develop the hesitant intuitionistic fuzzy linguistic interval for each alternative P_i.

$$P_i = (P_i^+, P_i^-)$$

where

$$P_i^+ = \Big[\Delta\left(\frac{1}{m-1}\sum_{i=1}^{m}\Delta^{-1}\left(\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\min(H_{S_{ij}}^{l})).\right)\right)\right),$$
$$\Delta\left(\frac{1}{m-1}\sum_{i=1}^{m}\Delta^{-1}\left(\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\max(H_{S_{ij}}^{l}))\right)\right)\right)\Big].$$

and

$$P_i^- = \Big[\Delta\left(\frac{1}{m-1}\sum_{i=1}^{m}\Delta^{-1}\left(\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\min(H_{S_{ij}}^{\prime l}))\right)\right)\right),$$
$$\Delta\left(\frac{1}{m-1}\sum_{i=1}^{m}\Delta^{-1}\left(\Delta\left(\frac{1}{K}\sum_{l=1}^{K}\Delta^{-1}(\max(H_{S_{ij}}^{\prime l}))\right)\right)\right)\Big].$$

D. Building preference relation

We calculate the preference matrix X based on intuitionistic fuzzy numbers;

$$X = [P_{ij}]_{m\times m},$$

where

$$P_{ij} = \left(P_{ij}^+, P_{ij}^-\right)$$

and

$$P_{ij}^+ = \frac{\max(0, \Delta^{-1}\max(P_i^+) - \Delta^{-1}\min(P_j^+)) - \max(0, \Delta^{-1}\min(P_i^+) - \Delta^{-1}\max(P_j^+))}{\left(\Delta^{-1}\max(P_i^+) - \Delta^{-1}\min(P_i^+) + \Delta^{-1}\max(P_j^+) - \Delta^{-1}\min(P_j^+)\right)\times 2},$$
$$P_{ij}^- = \frac{\max(0, \Delta^{-1}\max(P_i^-) - \Delta^{-1}\min(P_j^-)) - \max(0, \Delta^{-1}\min(P_i^-) - \Delta^{-1}\max(P_j^-))}{\left(\Delta^{-1}\max(P_i^-) - \Delta^{-1}\min(P_i^-) + \Delta^{-1}\max(P_j^-) - \Delta^{-1}\min(P_j^-)\right)\times 2}.$$

E. Overall preference of each alternative

The overall preference of each alternative P_i on all the other alternatives in form of intuitionistic fuzzy numbers

$$P_i = (P_i^+, P_i^-)$$

where

$$P_i^+ = \frac{\sum_{j=1}^{m} P_{ij}^+}{m}$$

and

$$P_i^- = \frac{\sum_{j=1}^{m} P_{ij}^-}{m}.$$

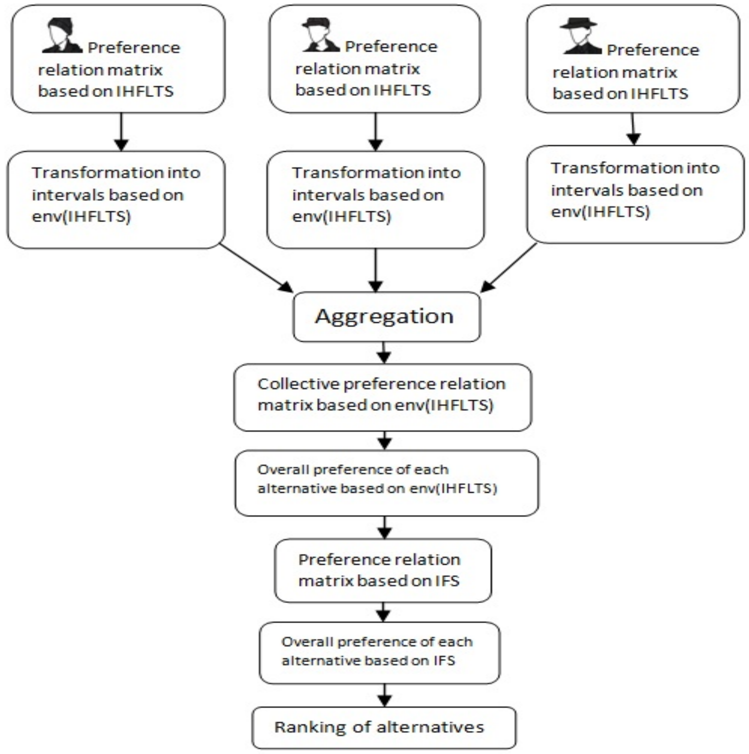

Figure 2: Flow chart of GDM based on HIFLTS

F. Rank the alternatives

Rank the alternatives from best to worst by sorting the score of P_i from the largest to the smallest. If the score of any two or more than two P_i are same, then to rank these alternatives from best to worst we sort the accuracy of P_i from the largest to the smallest.

A flow chart of the proposed method of GDM model with comparative linguistic expressions based on HIFLTS is shown in Figure 2.

4. Illustrative example

Atanassov et al. (2014) presented the intercriteria decision making analysis based on the apparatus of index matrices, intuitionistic fuzzy sets and applied in different areas of science and practice. In this section, we use the method proposed in Section 3 to get the most desirable alternative. A university committee, composed of three members of the Board of Directors, wants to decide the best teacher award and there are four candidates (teachers) for this award: John (P_1), Adam (P_2), Amin (P_3), and Noshad (P_4). Committee members give their assessment in the comparative linguistic terms. The four possible alternatives P_i $(i = 1, 2, 3, 4)$ are to be evaluated using the HIFLTS by three decision makers m_K $(K = 1, 2, 3)$ and also transform this information in linguistic intervals as listed in Tables 1-3.

Table 1. Comparative preference ($\tilde{X}^1$) with respect to decision maker 1 (m_1).

	P_1	P_2	P_3	P_4
P_1		[G,VG],[EP,VP]	[VP,P],[M,G]	[VP,P],[M,G]
P_2	[VP,P],[M,G]		[M,G],[EP,P]	[G,VG],[VP,P]
P_3	[G,VG],[EP,P]	[M,G],[VP,P]		[VP,P],[P,G]
P_4	[VG,EG],[EP,VP]	[VP,P],[M,G]	[VP,P],[M,G]	

Table 2. Comparative preference ($\tilde{X}^2$) with respect to decision maker 2 (m_2).

	P_1	P_2	P_3	P_4
P_1		[VG,EG],[EP,VP]	[EP,VP],[M,G]	[M,G],[VP,P]
P_2	[EP,VP],[P,M]		[G,VG],[EP,VP]	[VG,EG],[EP,EP]
P_3	[M,G],[EP,VP]	[VP,P],[M,G]		[EP,VP],[P,M]
P_4	[VG,EG],[EP,EP]	[M,G],[EP,P]	[VP,P],[P,G]	

Table 3. Comparative preference ($\tilde{X}^3$) with respect to decision maker 3 (m_3).

	P_1	P_2	P_3	P_4
P_1		[VG,EG],[EP,EP]	[M,G],[VP,P]	[EP,VP],[M,G]
P_2	[M,G],[VP,M]		[VG,EG],[EP,EP]	[M,G],[VP,M]
P_3	[VP,P],[P,G]	[VG,EG],[EP,EP]		[EP,VP],[M,G]
P_4	[G,VG],[VP,P]	[G,VG],[EP,VP]	[EP,P],[P,M]	

The collective comparative preference matrix ($\tilde{X}$) is constructed by utilizing Tables 1-3 in Table 4.

Table 4. Collective comparative preference

	P_1
P_1	
P_2	[(VP,0.33),(P,0.33)],[(P,0),(M,0.33)]
P_3	[(M,-0.33),(G,-0.33)],[(VP,-0.33),(P,0.33)]
P_4	[(VG,-0.33),(EG,-0.33)],[(EP,0.33),(VP,0)]
	P_2
P_1	[(VG,-0.33),(EG,-0.33)],[(EP,0),(VP,-0.33)]
P_2	
P_3	[(M,0),(G,0)],[(VP,0.33),(P,0)]
P_4	[(M,-0.33),(G,-0.33)],[(VP,0),(P,0.33)]
	P_3
P_1	[(VP,0.33),(P,0.33)],[(P,0.33),(M,0)]
P_2	[(G,0),(VG,0)],[(EP,0),(VP,0)]
P_3	
P_4	[(VP,-0.33),(P,0)],[(P,0.33),(G,-0.33)]
	P_4
P_1	[(VP,0.33),(P,0.33)],[(P,0.33),(M,0.33)]
P_2	[(G,0),(VG,0)],[(VP,-0.33),(P,-0.33)]
P_3	[(EP,0.33),(VP,0.33)],[(P,0.33),(G,-0.33)]
P_4	

Intuitionistic linguistic intervals for each alternative are developed in Table 5.

Table 5. Intuitionistic linguistic intervals for alternatives

P_1	[(P,0.44),(M,0.44)],[(VP,-0.44),(P,0.44)]
P_2	[(M,0.11),(G,0.11)],[(VP,-0.11),(P,0)]
P_3	[(P,0),(M,0)],[(VP,0.44),(M,-0.33)]
P_4	[(M,-0.33),(G,-0.22)],[(VP,0.22),(P,0.33)]

Intuitionistic preference relation is developed here.

	P_1	P_2	P_3	P_4
P_1		(0.0833,0.3889)	(0.3611,0.2368)	(0.1842,0.3056)
P_2	(0.4167,0.1111)		(0.5000,0.1190)	(0.3421,0.175)
P_3	(0.1389,0.2631)	(0.0000,0.381)		(0.0789,0.3095)
P_4	(0.3157,0.1944)	(0.1578,0.325)	(0.4210,0.1904)	

Overall preference of each alternative in intuitionistic fuzzy set and the score of these numbers to rank the alternatives are given in Table 6.

Table 6. Overall preference of alternatives

	Overall preference in IFS	Score of overall preference values
P_1	(0.1571,0.2328)	-0.0757
P_2	(0.3146,0.1012)	0.2134
P_3	(0.0544,0.2384)	-0.1839
P_4	(0.2236,0.1774)	0.0462

Rank all the alternatives P_i $(i = 1,\ 2,\ 3,\ 4)$:

$$P_2 \succ P_4 \succ P_1 \succ P_3.$$

Thus P_2 is most desirable alternative, so Adam won the best teacher award in the university.

5. Conclusion

Recently the modelling of real world decision-making problems with linguistic expression has been used by several researchers. These methods are less effective in conveying the imprecise nature of the linguistic assessment. Usually decision makers hesitate among more than one linguistic term to express their opinion. The combination of linguistic variables as HFLTS and intuitionistic fuzzy set provide the better way to cope with the uncertainty in decision-making problems. We developed a method for solving group decision-making problems in HIFLTS. This group decision-making method is capable of dealing with comparative linguistic expressions based on HIFLTS. It carries out the processes of computing with words by using linguistic computing procedure. Finally, an example has been solved by the proposed method to show its feasibility.

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