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Boundary Stabilization of Torsional Vibrations of a Solar Panel

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Abstract

In this paper, we study a boundary stabilization of the torsional vibrations of a solar panel. The panel is held by a rigid hub at one end and is totally free at the other. The dynamics of the overall system leads to hybrid system of equations. It is set to a certain initial vibrations with a control torque as a stabilizer at the hub end only. Taking a non-linear damping as boundary stabilizer, a uniform exponential energy decay rate is obtained directly. Thus an explicit form of uniform stabilization of the system is achieved by means of the exponential energy decay estimate.

Keywords: Solar panel; hybrid system; torsional vibrations; exponential energy decay estimate.

MSC 2010: 35L35; 37L15; 74H55; 93D15.

1. Introduction

Research in the area of stabilization problems for distributed parameter systems have been developing in a significant manner.The most common classes of vibration control mechanisms are of passive, active and of hybrid type. Passive vibration control uses resistance devices that absorbs

vibration energy. Active vibration control is also like that but involves the use of force actuators linked with external energy. Hybrid vibration control is a combination of passive approach with active control. The associated type of stability most commonly studied in the mathematical literature are strong stability and uniform stability. A system is called strongly stable, if the energy E(t) of each solution of the system converges to zero as time $t \to +\infty$. If the convergence is uniform for $t > 0$ with respect to all initial data in the energy space for which $E(0) < \infty$, the system is called uniformly stable. If the stability property can be achieved due to incorporation of a stabilizer or a damping device applied on the boundary, the system is then called boundary stabilization. The most important fact for studying the stability of such system is to suppress the vibrations to assure a good performance of the overall system.

During the last few decades, the use of flexible structures is on the rise. The vibrations of flexible structures are usually non-linear in practice. The analytical study of non-linear problems are cumbersome and the results so obtained are usually not in precise form. The linearized mathematical model are considered here just for simplicity and concise results. The vibrations of flexible structures are the problem of dynamical system mathematically governed by partial differential equations, in particular, the second order wave equation and the fourth-order Euler-Bernoulli beam equation. Stabilization for the wave equation in a bounded domain have been investigated by several authors (cf. Chen (1979), (1981), Lagnese (1983), (1988); Lions (1988), Komornik (1991)). Similarly, those governed by the fourth-order Euler-Bernoulli beam equation have been treated by Chen and Zhou (1990), Morgül (1992) and Krall (1989). The term 'hybrid systems' are those consists of coupled elastic and rigid parts. Hybrid systems in the category, when a lumped mass is present at one end, have been treated by Chen et. al. (1987), Littman and Markus (1988), Rao (1995) and Bose and Gorain (2003). In such systems, it is found that under very relaxed initials conditions no such uniform exponential decay of energy is possible. In many practical problems, it is very common to apply control force on the free end of the elastic part relative to rigid part to get a good result in the system.

The energy decay rate for the solutions of second order wave equations in a bounded domain has been established by several authors (Chen (1979), (1981); Lagnese (1983), (1988); Lions (1988), Komornik (1991)). Gorain (1997) treated the case of internally damped wave equations for the so called Voigt model of viscoelasticity together with undamped boundary conditions (without considering boundary feedback) to obtain a uniform exponential energy decay estimate. Such estimate is also found in Gorain (2009) for the case of n -dimensional vibrating equation modeling 'standard Linear model' of viscoelasticity. The approach adopted below is to formulate a distributed hybrid model of the dynamics of torsional vibrations of a flexible structure hoisted by a rigid hub at one end (cf. Fukuda et. al. (1985), (1986), (1988)). To establish the stability of the system by means of uniform exponential energy decay estimate for the solutions of such problem, a control torque is applied on the hub end. Finally we obtain explicitly the exponential energy decay estimate for the solution of this problem.

2. Mathematical Formulation of the problem

Here we study the exponential stabilization of a hybrid solar panel consisting of a long uniform

rectangular panel and a rigid hub fixed at one end. The panel is of length L, which is held at one end by the rigid hub and it is totally free at the other end. Our objective is to study the uniform exponential stability of the total system by applying a suitable boundary control torque $Q(t)$ at the hub end only. Referring to the fig. 1, if $\phi_h(t)$ is the rotation of the hub and $\phi_p(x,t)$ that of the panel at the position x along the span of the panel relative to the hub at time t, then the total rotational angle $\phi(x,t)$ of the panel obviously satisfies the relation

$$\phi(x,t) = \phi_h(t) + \phi_p(x,t), \qquad 0 \le x \le L,\ t > 0. \tag{1}$$

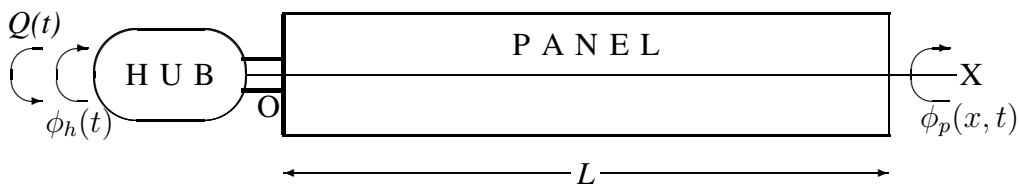

Figure 1. *Schematic of the rigid hub and the panel for torsional vibrations.*

This total rotation $\phi(x,t)$ then satisfies the governing differential equation (cf. Gorain and Bose (1998))

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \frac{\partial^2 \phi}{\partial x^2} \tag{2}$$

under the assumption $\left|\frac{\partial \phi}{\partial x}(x,t)\right| << 1$, where $c^2 = \frac{D_p}{\rho_p J_p}$, D_p, ρ_p and J_p being the torsional rigidity, density and moment of inertia of the area of cross section about the central axis of the panel. Initially at time t=0, the panel is set to vibrations with initial values

$$\phi(x,0) = \phi_0(x), \quad \frac{\partial \phi}{\partial t}(x,0) = \phi_1(x), \qquad 0 \le x \le L. \tag{3}$$

When control torque $Q(t)$ is applied at the hub end, its equation of motion is (cf. Gorain and Bose (1998))

$$I_h \frac{\partial^2 \phi_h}{\partial t^2}(0,t) = D_p \frac{\partial \phi_p}{\partial x}(0,t) - Q(t), \tag{4}$$

where I_h is the total moment of inertia of the hub about its axis of rotation. At the hub end $x = 0$, we have $\phi_p(0,t) = 0$, that yields $\phi_h(t) = \phi(0,t)$, then equation (4) becomes

$$I_h \frac{\partial^2 \phi}{\partial t^2}(0,t) = D_p \frac{\partial \phi_p}{\partial x}(0,t) - Q(t) \tag{5}$$

Again,

$$\frac{\partial \phi}{\partial x}(x,t) = \frac{\partial \phi_p}{\partial x}(x,t) \quad \text{so that,} \quad \frac{\partial \phi}{\partial x}(0,t) = \frac{\partial \phi_p}{\partial x}(0,t). \tag{6}$$

Hence, equation (5) reduces to

$$I_h \frac{\partial^2 \phi}{\partial t^2}(0,t) = D_p \frac{\partial \phi}{\partial x}(0,t) - Q(t). \tag{7}$$

Writing $\alpha = \dfrac{I_h}{D_p}$ and $\lambda = \dfrac{1}{D_p}$, we have from (7)

$$\frac{\partial \phi}{\partial x}(0,t) = \alpha \frac{\partial^2 \phi}{\partial t^2}(0,t) + \lambda Q(t). \tag{8}$$

The free end of the panel satisfies the equation

$$\frac{\partial \phi}{\partial x}(L,t) = 0. \tag{9}$$

Thus, we are concerned about uniform stabilization for the vibrations of a solar panel governed by the following initial-boundary value problem.

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \frac{\partial^2 \phi}{\partial x^2}, \qquad 0 \le x \le L,\ t > 0, \tag{10}$$

$$\phi(x,0) = \phi_0(x), \quad \frac{\partial \phi}{\partial t}(x,0) = \phi_1(x), \qquad 0 \le x \le L, \tag{11}$$

$$\frac{\partial \phi}{\partial x}(0,t) = \alpha \frac{\partial^2 \phi}{\partial t^2}(0,t) + \lambda Q(t), \quad \frac{\partial \phi}{\partial x}(L,t) = 0, \qquad t > 0. \tag{12}$$

We now introduce a suitable stabilizer that is observable to the velocity the panel at the free end. In other words, we assume here that the control torque $Q(t)$ is comparable with $\dfrac{\partial \phi}{\partial t}(L,t)$ in the sense

$$|Q(t)| \ge k \Big| \frac{\partial \phi}{\partial t}(L,t) \Big|, \tag{13}$$

where $k > 0$ is a dimensionality constant. Again due to the damping character of the control torque $Q(t)$, it must be an odd function of velocity at the hub end, that means,

$$Q(t) = f\Big(\frac{\partial \phi}{\partial t}(0,t)\Big), \tag{14}$$

where f is an odd function of its argument such that $f(0) = 0$ and $u.f(u) > 0$ for every $u \in \mathbb{R} - \{0\}$. For example, if $f(u) = u$, we have a simple viscous damper. The others types of dampers can be found in the literature (cf. Chen (1979), (1981), Lagnese (1983), (1988); Lions (1988), Komornik (1991), Gorain (1997), Bose and Gorain (2003), Gorain (2009)). Under the assumption (13), a restriction on the controller $f\Big(\dfrac{\partial \phi}{\partial t}(0,t)\Big)$ is given by

$$\Big| f\Big(\frac{\partial \phi}{\partial t}(0,t)\Big) \Big| \ge k \Big| \frac{\partial \phi}{\partial t}(L,t) \Big|. \tag{15}$$

3. Energy of the system

As defined by Lagnese (1988), the total energy of the system is given by the functional

$$E(t) = \frac{1}{2} \int_0^L \Big[\Big(\frac{\partial \phi}{\partial t}\Big)^2 + c^2 \Big(\frac{\partial \phi}{\partial x}\Big)^2 \Big] dx + \frac{1}{2} c^2 \alpha \Big[\frac{\partial \phi}{\partial t}(0,t) \Big]^2 \quad \text{for } t \ge 0. \tag{16}$$

Differentiating this with respect to t and using the governing equation (10), we obtain

$$\begin{aligned}\frac{dE}{dt} &= c^2\int_0^L\Big[\frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial x^2}+\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\partial t}\Big]dx + c^2\alpha\frac{\partial\phi}{\partial t}(0,t)\frac{\partial^2\phi}{\partial t^2}(0,t)\\ &= c^2\Big[\frac{\partial\phi}{\partial t}(L,t)\frac{\partial\phi}{\partial x}(L,t)-\frac{\partial\phi}{\partial t}(0,t)\frac{\partial\phi}{\partial x}(0,t)\Big]+c^2\alpha\frac{\partial\phi}{\partial t}(0,t)\frac{\partial^2\phi}{\partial t^2}(0,t)\\ &= -c^2\lambda\frac{\partial\phi}{\partial t}(0,t)Q(t), \qquad (17)\end{aligned}$$

where the integration is performed by parts and the boundary conditions (12) are used. By the help of the equation (14), the above expression can be reduced to

$$\frac{dE}{dt} = -c^2\lambda\frac{\partial\phi}{\partial t}(0,t)f\Big(\frac{\partial\phi}{\partial t}(0,t)\Big) = -c^2\lambda u f(u) \le 0 \qquad (18)$$

where $u = \dfrac{\partial\phi}{\partial t}(0,t)$ is the velocity at the hub end. The negativity of (18) shows that some amount of energy of the system (10)-(14) is dissipating due to incorporation of the feedback controller $f\Big(\frac{\partial\phi}{\partial t}(0,t)\Big)$ at the hub end $x=0$. Thus the energy functional E is a non-creasing function of time t. Hence, the solution of the system satisfies the energy estimate $E(t)\le E(0)$, where

$$E(0) = \frac{1}{2}\int_0^L\Big[\phi_1^2 + c^2\phi_0'^2\Big]dx + \frac{1}{2}c^2\alpha\Big[\phi_1(0)\Big]^2. \qquad (19)$$

The above estimate (16) suggests that, if $\phi_0\in H^1(0,L)$ and $\phi_1\in L_0^2(0,L)$, where

$$H^1(0,L) = \Big\{F \;\Big|\; F\in L^2(0,L), F'\in L^2(0,L)\Big\} \qquad (20)$$

is the classical Sobolev space of real valued functions of order one and

$$L_0^2(0,L) := \Big\{F \;\Big|\; F\in L^2(0,L) \text{ and } F(0)=0\Big\}, \qquad (21)$$

then $E(0)<\infty$. Hence, it follows that $E(t)<\infty$ for every $t\ge 0$ and the system (10)-(14) has a unique solution for $(\phi_0,\phi_1)\in H^1(0,L)\times L_0^2(0,L)$.

As the energy decays, our main interest is to establish whether this decay is uniformly exponential or not. An affirmative answer can be found in the next section.

4. Uniform Stability Result

The main result of this paper can be stated in the following theorem.

Theorem 1.: Let $\phi(x,t)$ be a solution of the system (10)-(14) with the initial values $\{\phi_0,\phi_1\}\in H^1(0,L)\times L_0^2(0,L)$, then the energy E of the system decays uniformly exponentially with time, that means

$$E(t) < Me^{-\mu t}E(0) \quad \text{for } t>0,$$

and for some finite reals $M>1$ and $\mu>0$.

The theorem will be proved after some preliminary steps. First, we require a trivial inequality

$$(u.v) \leq \frac{1}{2s}(u^2 + s^2v^2), \tag{22}$$

for any two real functions u and v with a real number $s > 0$.

Next we consider the following lemma:

Lemma 1.: For every solution $\phi(x,t)$ be a solution of the system (10)-(14), the time derivative of the functional G (cf. Bose and Gorain (1998), Gorain (2006) and Nandi et. al. (2011)) defined by

$$G(t) = \int_0^L x\frac{\partial\phi}{\partial t}\frac{\partial\phi}{\partial x}dx \quad \text{for } t \geq 0 \tag{23}$$

satisfies

$$\frac{dG}{dt} = \frac{1}{2}L\Big[\frac{\partial\phi}{\partial t}(L,t)\Big]^2 + \frac{1}{2}c^2\alpha\Big[\frac{\partial\phi}{\partial t}(0,t)\Big]^2 - E(t). \tag{24}$$

Proof:

If we differentiate (23) with respect to t and using the governing equation (10), we obtain

$$\begin{aligned}\frac{dG}{dt} &= \int_0^L x\Big[c^2\frac{\partial^2\phi}{\partial x^2}\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial x\partial t}\Big]dx \\ &= \frac{1}{2}L\Big[\frac{\partial\phi}{\partial t}(L,t)\Big]^2 - E(t) + \frac{1}{2}c^2\alpha\Big[\frac{\partial\phi}{\partial t}(0,t)\Big]^2,\end{aligned} \tag{25}$$

where the integration is done by parts and the boundary conditions (12) are used.

Proof of Theorem 1:

Proceeding as in Gorain (2006) and Bose and Gorain (1998), we define energy like Lyapunov functional V by

$$V(t) = E(t) + \epsilon G(t) \quad \text{for } t \geq 0 \tag{26}$$

Differentiating (26) with respect to t, and using (18) and (24) we obtain

$$\begin{aligned}\frac{dV}{dt} &\leq \frac{1}{2}\epsilon\Big[c^2\alpha\Big(\frac{\partial\phi}{\partial t}(0,t)\Big)^2 + L\Big(\frac{\partial\phi}{\partial t}(L,t)\Big)^2\Big] \\ &\quad -c^2\lambda\frac{\partial\phi}{\partial t}(0,t)f\Big(\frac{\partial\phi}{\partial t}(0,t)\Big) - \epsilon E(t).\end{aligned} \tag{27}$$

In view of (15), the relation (27) becomes

$$\begin{aligned}\frac{dV}{dt} &\leq \frac{\epsilon c^2\alpha}{2}\Big[\Big(\frac{\partial\phi}{\partial t}(0,t)\Big)^2 + \frac{L}{c^2k^2\alpha}f^2\Big(\frac{\partial\phi}{\partial t}(0,t)\Big)\Big] \\ &\quad -c^2\lambda\frac{\partial\phi}{\partial t}(0,t)f\Big(\frac{\partial\phi}{\partial t}(0,t)\Big) - \epsilon E(t).\end{aligned} \tag{28}$$

Since $\epsilon > 0$ is small and $\frac{\partial\phi}{\partial t}(0,t)f\Big(\frac{\partial\phi}{\partial t}(0,t)\Big) > 0$ for every $t > 0$, we assume that

$$0 < \epsilon < \epsilon_0, \tag{29}$$

where

$$\epsilon_0 = \frac{\lambda \frac{\partial \phi}{\partial t}(0,t) f\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)}{\frac{\alpha}{2}\Big[\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)^2 + \frac{L}{c^2k^2\alpha} f^2\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)\Big]}. \tag{30}$$

By the inequality (22), an upper bound of ϵ_0 can be obtained as

$$\epsilon_0 \leq \frac{\frac{ck\lambda}{2}\sqrt{\frac{\alpha}{L}}\Big[\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)^2 + \frac{L}{c^2k^2\alpha} f^2\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)\Big]}{\frac{\alpha}{2}\Big[\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)^2 + \frac{L}{c^2k^2\alpha} f^2\Big(\frac{\partial \phi}{\partial t}(0,t)\Big)\Big]} = \frac{ck\lambda}{\sqrt{\alpha L}}, \tag{31}$$

a positive constant independent of time t. In particular, if we take $k = \dfrac{\sqrt{\alpha L}}{c\lambda}$, then $\epsilon_0 \leq 1$. According to our assumption (29), the lower bound of ϵ_0 is a real number > 0 that may be sufficiently small.

For the choice of ϵ, as defined in (29), where ϵ_0 is given by (30), the differential relation (28) reduces to

$$\frac{dV}{dt} + \epsilon E(t) \leq 0 \quad \text{for } t > 0. \tag{32}$$

Now applying the inequality (22), we have from (23),

$$\big|G(t)\big| \leq \frac{L}{c}\int_0^L \Big|\frac{\partial \phi}{\partial t}\Big|\Big|c\frac{\partial \phi}{\partial x}dx\Big| \leq \frac{L}{c}E(t), \tag{33}$$

that means,

$$-\frac{L}{c}E(t) \leq G(t) \leq \frac{L}{c}E(t) \quad \text{for } t > 0. \tag{34}$$

So the functional V defined by (26) can be estimated as

$$\Big(1 - \epsilon\frac{L}{c}\Big)E(t) \leq V(t) \leq \Big(1 + \epsilon\frac{L}{c}\Big)E(t) \quad \text{for } t \geq 0. \tag{35}$$

Since $\epsilon > 0$ is small, we may further assume that

$$0 < \epsilon < \frac{c}{L}. \tag{36}$$

Then it follows from (35) that $V(t) > 0$ for every $t \geq 0$. Invoking the inequality (35), the relation (32) leads to the differential inequality

$$\frac{dV}{dt} + \mu V(t) < 0, \tag{37}$$

where

$$\mu = \frac{\epsilon}{1 + \epsilon\frac{L}{c}} > 0. \tag{38}$$

Multiplying (37) by $e^{\mu t}$ and integrating from 0 to t, we obtain

$$V(t) < e^{-\mu t}V(0) \quad \text{for } t > 0. \tag{39}$$

Applying again the inequality (35) in (39), we get

$$E(t) < Me^{-\mu t}E(0) \quad \text{for } t > 0, \tag{40}$$

where

$$M = \frac{1 + \epsilon\frac{L}{c}}{1 - \epsilon\frac{L}{c}} > 1. \tag{41}$$

Hence the theorem is complete.

5. Conclusion

This study deals with uniform stability of the torsional vibrations of a prototype hybrid flexible structure – solar cell array. The significant result in this paper is that the solution of the system governed by (10)-(14) converges uniformly to zero as time $t \to \infty$. At the same time, we have estimated the exponential energy decay rate μ explicitly by a direct method. The result shows that the vibration energy of the system decays rapidly for larger values of μ. Again,

$$\frac{d\mu}{d\epsilon} = \left(1 + \frac{\epsilon L}{c}\right)^{-2} > \frac{1}{2}, \tag{42}$$

as $\epsilon L < c$ followed from (36). Hence the exponential decay rate as a function of ϵ will be maximum for largest admissible value ϵ, the least upper bound of which can be determined in view of the restrictions (29) and (36) simultaneously. Study of this type of vibrations assumes significance in treating the similar vibrations of flexible beams, plates, etc. capable of withstanding finite deformation.

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