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3-Total Super Sum Cordial Labeling by Applying Operations on some Graphs

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Abstract

The sum cordial labeling is a variant of cordial labeling. In this paper, we investigate 3-Total Super Sum Cordial labeling. This labeling is discussed by applying union operation on some of the graphs. A vertex labeling is assigned as a whole number within the range. For each edge of the graph, assign the label, according to some definite rule, defined for the investigated labeling. Any graph which satisfies 3-Total Super Sum Cordial labeling is known as the 3-Total Super Sum Cordial graphs. Here, we prove that some of the graphs like the union of Cycle and Path graphs, the union of Cycle and Complete Bipartite graph and the union of Path and Complete Bipartite graph satisfy the investigated labeling and hence are called the 3-Total Super Sum Cordial graphs.

Keywords: 3-Total Sum Cordial labeling; 3-Total Sum Cordial graphs; 3-Total Super Sum Cordial labeling; 3-Total Super Sum Cordial graphs

MSC 2010 No.: 05C76, 05C78

1. Introduction

The graphs under consideration are finite, simple and undirected. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. Graphs G_1 and G_2 have disjoint point sets, V_1 and V_2 and edge sets, E_1 and E_2, respectively. The union of G_1 and G_2, is the graph $G_1 \cup G_2$, with

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2) \text{ and } E(G_1 \cup G_2) = E(G_1) \cup E(G_2).$$

The concept of Sum Cordial labeling of graph was introduced by [Shiama (2012, pp. 3271-3276)], and that of k-Sum Cordial labeling, by [Pethanachi and Lathamaheshwari (2013, pp. 253-259)]. The concept of Total Product Cordial labeling of a graph was introduced by [Sundaram et al. (2006, pp. 199-203). The concept of 3-Total Super Sum Cordial labeling of graph was introduced by [Tenguria and Verma (2014, pp. 117-121)]. The concept of 3-Total Super Product Cordial labeling of graph was introduced by [Tenguria and Verma (2015, pp. 557-559)]. The concept of 3-Total Super Sum Cordial labeling for union of some graphs was introduced by [Tenguria and Verma (2015, pp. 25-30)].

Definition 1.1.

Let G be a graph. Let f be a map from $V(G)$ to $\{0,1,2\}$. For each edge uv assign the label $\big(f(u)+f(v)\big)(mod\ 3)$. Then the map f is called 3-Total Sum Cordial labeling of G, if $|f(i)-f(j)| \leq 1$: $i, j\ \varepsilon\ \{0,1,2\}$, where $f(x)$, denotes the total number of vertices and edges labeled with $x = \{0,1,2\}$.

Definition 1.2.

A 3-Total Sum Cordial labeling of a graph G is called 3-Total Super Sum Cordial labeling, if for each edge uv $|f(u)-f(v)| \leq 1$. A graph G is 3-Total Super Sum Cordial if it admits 3-Total Super Sum Cordial labeling.

Definition 1.3.

The union of two graphs $C_m = (V_1, E_1)$, where $V_1 = \{u_1, u_2, \dots, u_m\}$ and $E_1 = \{u_1 u_2, u_2 u_3, \dots, u_m u_1\}$ and $P_n = (V_2, E_2)$, where $V_2 = \{v_1, v_2, \dots, v_n\}$ and $E_2 = \{v_1 v_2, v_2 v_3, \dots, v_{n-1} v_n\}$ is a graph denoted by $C_m \cup P_n$, and is defined by $C_m \cup P_n = (V_1 \cup V_2, E_1 \cup E_2)$.

Definition 1.4.

The union of two graphs $C_m = (V_1, E_1)$, where $V_1 = \{u_1, u_2, \dots, u_m\}$ and $E_1 = \{u_1 u_2, u_2 u_3, \dots, u_m u_1\}$ and $k_{1,n} = (V_2, E_2)$, where $V_2 = \{v, v_1, v_2, \dots, v_n\}$ and $E_2 = \{vv_1, vv_2, vv_3, \dots, vv_n\}$ is a graph denoted by $C_m \cup k_1, n$, and is defined by $C_m \cup k_1, n = (V_1 \cup V_2, E_1 \cup E_2)$.

Definition 1.5.

The union of two graphs $P_m = (V_1, E_1)$, where $V_1 = \{u_1, u_2, \dots, u_m\}$ and $E_1 = \{u_1 u_2, u_2 u_3, \dots, u_{m-1} u_m\}$ and $k_1, n = \big(V_2, E_2\big)$, where $V_2 = \{v, v_1, v_2, \dots, v_n\}$ and $E_2 = \{vv_1, vv_2, vv_3, \dots, vv_n\}$ is a graph denoted by $P_m \cup k_1, n$ and is defined by $P_m \cup k_1, n = (V_1 \cup V_2, E_1 \cup E_2)$.

2. Main Results

Theorem 2.1.

$C_m \cup P_n$, is 3-Total Super Sum Cordial.

***Proof*:**

Let C_m, be the cycle $u_1, u_2, \dots, u_m, u_1$ and P_n, be the path $v_1, v_2, \dots, v_n$.

Case I : $m \equiv 0\ (mod\ 3), n \equiv 0\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p,$$
$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is the 3-Total Super Sum Cordial.

Case II: $m \equiv 0\ (mod\ 3), n \equiv 1\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t + 1$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v_n) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case III: $m \equiv 0\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p, n = 3t + 2$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v_n) = 1,$$
$$f(v_{n-1}) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case IV: $m \equiv 1\ (mod\ 3), n \equiv 0\ (mod\ 3)$.

Let $m = 3p + 1, n = 3t$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \leq i < p,$$
$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case V: $m \equiv 1\ (mod\ 3), n \equiv 1\ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t + 1$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \leq i < p.$$

Assign:

$$f(v_n) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case VI: $m \equiv 1\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p + 1, n = 3t + 2$.

Assign:

$$f(u_m) = 1,.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v_n) = 1,$$
$$f(v_{n-1}) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case VII: $m \equiv 2\ (mod\ 3), n \equiv 0\ (mod\ 3)$.

Let $m = 3p + 2, n = 3t$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p,$$
$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case VIII: $m \equiv 2\ (mod\ 3), n \equiv 1\ (mod\ 3)$.

Let $m = 3p + 2, n = 3t + 1$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case IX: $m \equiv 2\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p + 2, n = 3t + 2$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \leq i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \leq i < p.$$

Assign:

$$f(v_n) = 1,$$
$$f(v_{n-1}) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Example 2.2.

A 3-Total Super Sum Cordial labeling, of $C_6 \cup P_7$.

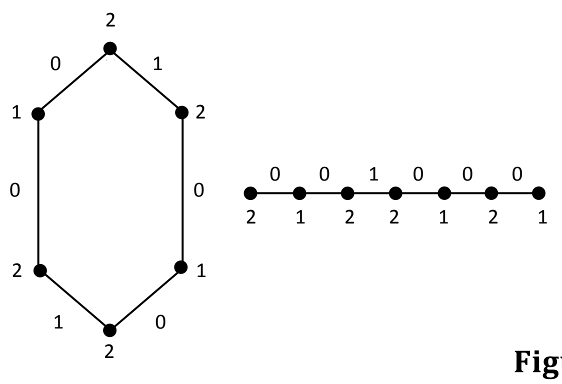

Figure 1. $C_6 \cup P_7$.

Table 1. Vertex and edge conditions for 3-Total Super Sum Cordial labeling of $C_m \cup P_n$

Case	**Vertex Condition**	**Edge Condition**	$\boldsymbol{f(i) = v_f(i) + e_f(i)}$
$m = 3p$ & $n = 3t$.	$v_f(0) = 0,$ $v_f(1) = p + t,$ $v_f(2) = 2p + 2t,$	$e_f(0) = 2p + 2t - 1,$ $e_f(1) = p + t,$ $e_f(2) = 0,$	$f(0) = 2p + 2t - 1.$ $f(1) = 2p + 2t.$ $f(2) = 2p + 2t.$
$m = 3p$ & $n = 3t$+1.	$v_f(0) = 0,$ $v_f(1) = p + t + 1,$ $v_f(2) = 2p + 2t,$	$e_f(0) = 2p + 2t + 1,$ $e_f(1) = p + t - 1,$ $e_f(2) = 0,$	$f(0) = 2p + 2t + 1.$ $f(1) = 2p + 2t.$ $f(2) = 2p + 2t.$
$m = 3p$ & $n = 3t + 2$.	$v_f(0) = 0,$ $v_f(1) = p + t + 1,$ $(2) = 2p + 2t + 1,$	$e_f(0) = 2p + 2t + 1,$ $e_f(1) = p + t,$ $e_f(2) = 0,$	$f(0) = 2p + 2t + 1.$ $f(1) = 2p + 2t + 1.$ $f(2) = 2p + 2t + 1.$
$m = 3p + 1$ & $n = 3t$.	$v_f(0) = 0,$ $v_f(1) = p + t + 1,$ $v_f(2) = 2p + 2t,$	$e_f(0) = 2p + 2t,$ $e_f(1) = p + t - 1,$ $e_f(2) = 1,$	$f(0) = 2p + 2t.$ $f(1) = 2p + 2t.$ $f(2) = 2p + 2t + 1.$
$m = 3p + 1$ & $n = 3t + 1$.	$v_f(0) = 0,$ $v_f(1) = p + t + 2,$ $(2) = 2p + 2t,$	$e_f(0) = 2p + 2t + 1,$ $e_f(1) = p + t - 1,$ $e_f(2) = 1,$	$f(0) = 2p + 2t + 1.$ $f(1) = 2p + 2t + 1.$ $f(2) = 2p + 2t + 1.$
$m = 3p + 1$ & $n = 3t + 2$.	$v_f(0) = 0,$ $v_f(1) = p + t + 2,$ $(2) = 2p + 2t + 1,$	$e_f(0) = 2p + 2t + 1,$ $e_f(1) = p + t,$ $e_f(2) = 1,$	$f(0) = 2p + 2t + 1.$ $f(1) = 2p + 2t + 2.$ $f(2) = 2p + 2t + 2.$
$m = 3p + 2$ & $n = 3t$.	$v_f(0) = 0,$ $v_f(1) = p + t + 1,$ $(2) = 2p + 2t + 1,$	$e_f(0) = 2p + 2t + 1,$ $e_f(1) = p + t,$ $e_f(2) = 0,$	$f(0) = 2p + 2t + 1.$ $f(1) = 2p + 2t + 1.$ $f(2) = 2p + 2t + 1.$
$m = 3p + 2$ & $n = 3t + 1$.	$v_f(0) = 0,$ $v_f(1) = p + t + 1,$ $(2) = 2p + 2t + 2,$	$e_f(0) = 2p + 2t + 2,$ $e_f(1) = p + t,$ $e_f(2) = 0,$	$f(0) = 2p + 2t + 2.$ $f(1) = 2p + 2t + 1.$ $f(2) = 2p + 2t + 2.$
$m = 3p + 2$ & $n = 3t + 2$.	$v_f(0) = 0,$ $v_f(1) = p + t + 2,$ $(2) = 2p + 2t + 2,$	$e_f(0) = 2p + 2t + 3,$ $e_f(1) = p + t,$ $e_f(2) = 0,$	$f(0) = 2p + 2t + 3.$ $f(1) = 2p + 2t + 2.$ $f(2) = 2p + 2t + 2.$

Theorem 2.3.

$C_m \cup k_1, n,$ is 3-Total Super Sum Cordial.

Proof:

Let, C_m, be the cycle $u_1, u_2, \dots, u_m, u_1$ and let,

$$V(k_1, n) = \{v, v_i : 1 \leq i \leq n\} \text{ and } E(k_1, n) = \{vv_i : 1 \leq i \leq n\}.$$

Case I: $m \equiv 0 \ (mod\ 3), n \equiv 0 \ (mod\ 3)$.

Let $m = 3p$ and $n = 3t$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case II: $m \equiv 0\ (mod\ 3), n \equiv 1\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t + 1$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case III: $m \equiv 0\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t + 2$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 0.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case IV: $m \equiv 1 \ (mod\ 3),\ n \equiv 0 \ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p,$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 2,$$
$$f(v_{n-2}) = 0.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t-1,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t-1,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t-1.$$

Hence, f is 3-Total Super Sum Cordial.

Case V: $m \equiv 1 \ (mod\ 3), n \equiv 1 \ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t + 1$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case VI: $\equiv 1\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t + 2$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case VII: $m \equiv 2\ (mod\ 3), n \equiv 0\ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 2.$$
$$f(v_{n-2}) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t-1,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t-1,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t-1.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case VIII: $m \equiv 2 \ (mod\ 3), n \equiv 1 \ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t + 1$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Case IX: $\equiv 2 \ (mod\ 3), n \equiv 2 \ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t + 2$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 1,$$
$$f(v_{n-1}) = 2.$$

Define:

$$f(v_{3i+1}) = \ 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = \ 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = \ 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial labeling.

Table 2. Vertex and edge conditions for 3-Total Super Sum Cordial labeling of $C_m \cup k_1, n$

Case	Vertex Condition	Edge Condition	$f(i) = v_f(i) + e_f(i)$
m=3p & n=3t.	$v_f(0) = t$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t$,	$e_f(0) = 2p + t$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t$.
m=3p & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t + 1$.
m=3p & n=3t+2.	$v_f(0) = t + 1$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t + 1$, $e_f(2) = t$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 1$.
m=3p+1 & n=3t.	$v_f(0) = t$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t + 1$.
m=3p+1 & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 2$.
m=3p+1 & n=3t+2.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 2$, $e_f(1) = p + t$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 3$.
m=3p+2 & n=3t.	$v_f(0) = t - 1$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 3$, $e_f(1) = p + t - 1$, $e_f(2) = t$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t + 2$.
m=3p+2 & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 3$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t + 3$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 2$.
m=3p+2 & n=3t+2.	$v_f(0) = t$, $v_f(1) = p + t + 3$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 3$, $e_f(1) = p + t$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 3$. $f(1) = 2p + 2t + 3$. $f(2) = 2p + 2t + 3$.

Example 2.4. A 3-Total Super Sum Cordial labeling, of $C_5 \cup k_1, 5$.

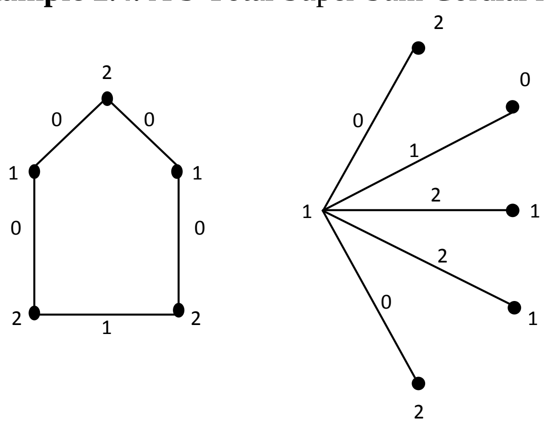

Figure 2. $C_5 \cup k_1, 5$.

Theorem 2.5.

$P_m \cup k_1, n$ is 3-Total Super Sum Cordial.

Proof:

Let P_m be the path $u_1, u_2, \dots, u_m$ and let $V(k_1, n) = \{v, v_i : 1 \leq i \leq n\}$ and $E(k_1, n) = \{vv_i : 1 \leq i \leq n\}$.

Case I: $m \equiv 0\ (mod\ 3)$, $n \equiv 0\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \leq i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \leq i <.,$$

Assign:

$$f(v) = 1,$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case II: $m \equiv 0\ (mod\ 3)$, $n \equiv 1\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t + 1$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \leq i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \leq i < p,$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 0; \quad 0 \leq i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case III: $m \equiv 0\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p$ and $n = 3t + 2$.

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence f is 3-Total Super Sum Cordial.

Case IV: $m \equiv 1\ (mod\ 3),\ n \equiv 0\ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case V: $m \equiv 1\ (mod\ 3), n \equiv 1\ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t + 1$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case VI: $m \equiv 1\ (mod\ 3), n \equiv 2\ (mod\ 3)$.

Let $m = 3p + 1$ and $n = 3t + 2$.

Assign:

$$f(u_m) = 1.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case VII: $m \equiv 2\ (mod\ 3)$ and $n \equiv 0\ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1.$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case VIII: $m \equiv 2\ (mod\ 3)$, $n \equiv 1\ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t + 1$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2.$$

Define:

$$f(v_{3i+1}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Case IX: $m \equiv 2\ (mod\ 3)$, $n \equiv 2\ (mod\ 3)$.

Let $m = 3p + 2$ and $n = 3t + 2$.

Assign:

$$f(u_m) = 1,$$
$$f(u_{m-1}) = 2.$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p,$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p,$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p.$$

Assign:

$$f(v) = 1,$$
$$f(v_n) = 2,$$
$$f(v_{n-1}) = 1.$$

Define:

$$f(v_{3i+1}) = 0; \quad 0 \le i < t,$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t,$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t.$$

Hence, f is 3-Total Super Sum Cordial.

Table 3. Vertex and edge conditions for 3-Total Super Sum Cordial labeling of $P_m \cup k_1, n$

Case	**Vertex Condition**	**Edge Condition**	$f(i) = v_f(i) + e_f(i)$
m=3p & n=3t.	$v_f(0) = t$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t$,	$e_f(0) = 2p + t$, $e_f(1) = p + t - 1$, $e_f(2) = t$,	$f(0) = 2p + 2t$. $f(1) = 2p + 2t$. $f(2) = 2p + 2t$.
m=3p & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 1$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t - 1$, $e_f(2) = t$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t$. $f(2) = 2p + 2t + 1$.
m=3p & n=3t+2.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t - 1$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t + 2$.
m=3p+1 & n=3t.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t - 1$, $e_f(2) = t$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t$.
m=3p+1 & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 2$, $e_f(1) = p + t - 1$, $e_f(2) = t$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 1$. $f(2) = 2p + 2t + 1$.
m=3p+1 & n=3t+2.	$v_f(0) = t$, $v_f(1) = p + t + 3$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 2$, $e_f(1) = p + t - 1$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 2$.
m=3p+2 & n=3t.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 1$,	$e_f(0) = 2p + t + 1$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t + 1$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 1$.
m=3p+2 & n=3t+1.	$v_f(0) = t$, $v_f(1) = p + t + 2$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 2$, $e_f(1) = p + t$, $e_f(2) = t$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 2$. $f(2) = 2p + 2t + 2$.
m=3p+2 & n=3t+2.	$v_f(0) = t$, $v_f(1) = p + t + 3$, $v_f(2) = 2p + t + 2$,	$e_f(0) = 2p + t + 2$, $e_f(1) = p + t$, $e_f(2) = t + 1$,	$f(0) = 2p + 2t + 2$. $f(1) = 2p + 2t + 3$. $f(2) = 2p + 2t + 3$.

Example 2.6.

A 3-Total Super Sum Cordial labeling, of $P_5 \cup k_{1}, 8$.

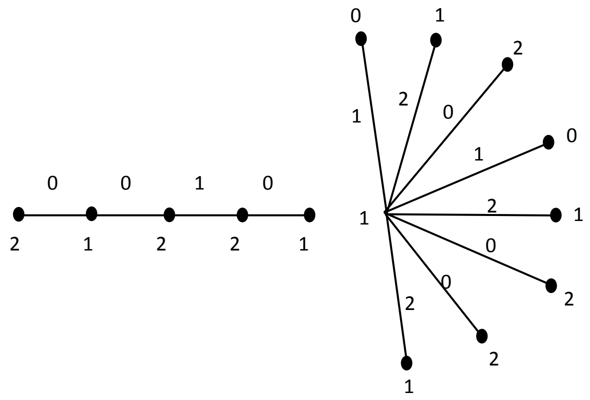

Figure 3. $P_5 \cup k_{1}, 8$.

Corollary.

If $G_1 \cup G_2$ is 2-Total Sum Cordial graph then, it is 2-Total Super Sum Cordial graph, and for each edge $uv, |f(u) - f(v)| \leq 1$.

3. Conclusion

Labeling of discrete structure is a potential area of research. Labeled graphs play an important role in the study of X-ray, Crystallography, Circuit design, Astronomy, Communication network and the design of optimal circuit layout. We have investigated 3-Total Super Sum Cordial labeling, by applying union operation on some graphs. The investigation of analogous results for different graphs as well in the context of various graph labeling problems is an open area of research.

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REFERENCES

Bloom G. S. and Golomb S. W. (1977). Application of numbered undirected graphs, Proceedings of IEEE, 165 (4): 562-570.

Cahit, I. Cordial graphs (1987). A weaker version of graceful and harmonious graphs, Ars combinatorial, 23 : 201-207.

Gallian, J. A. (2010). A dynamic survey of graph labeling, the Electronic Journal of Combinatorics, 17 (#DS6).

Ghodasara G. V. and Rokad A. H. (2013). Cordial labeling of $k_{n,n}$ related graphs, International Journal of Science and Research (IJSR), India Online, 2 (5): 74-77.

Pethanachi Selvam S. and Lathamaheshwari G. (2013). *k* sum cordial labeling for some graphs, IJMA, 4 (3): 253-259.

Ponraj R., Sivakumar M. and Sundaram M. (2012). 3-Total product cordial labeling of union of some graphs, Journal of Indian Acad. Math., 34 (2): 511-530.

Ponraj R., Siva Kumar M., Sundaram M. (2012). *k*-product cordial labeling of graphs, Int. J. Contemp. Math. Sciences, 7 (15): 733-742.

Shee S. C., Ho Y. S. (1916). The cordiality of path union of n copies of a graph, Discrete Math, 151: 221-229.

Shiama J. (2012). Sum cordial labeling for some graphs, IJMA, 3 (a): 3271-3276.

Sundaram M., Ponraj R. and Somasundaram S. (2004). Product cordial labeling of graphs, Bull. Pure and Applied Sciences (Mathematics and Statistics), 23 (E): 155-163.

Sundaram M., Ponraj R. and Somasundaram S. (2006). Total product cordial labeling of graphs, Bulletin of pure and applied sciences, 25 (1): 199-203.

Tenguria Abha and Verma Rinku (2014). 3-Total super sum cordial labeling for some graphs IJMA, 5 (12): 117-121.

Tenguria Abha and Verma Rinku (2015). 3-Total super sum cordial labeling for union of some graphs, IJAIS, Foundation of computer science, New York, USA, 8 (4): 25-30.

Tenguria Abha and Verma Rinku (2015). 3-Total super product cordial labeling for some graphs, International journal of science and Research , 4 (2): 557-559.

Vaidya S. K., Ghodasara G. V., Srivastav S. and Kaneria V. J. (2007). Cordial labeling for two cycle related graphs, The mathematics student, 76: 237-246.

Vaidya S. K., Ghodasara G. V., Srivastav Sweta and Kaneria V. J. (2008). Some new cordial graphs, Int. J. of Math & Math. Sci., 4 (2): 81-92.

Vaidya S. K., Ghodasara G. V., Srivastav Sweta and Kaneria V. J. (2008). Cordial and 3-equitable labeling of star of a cycle, Math. today, 24: 54-64.

Vaidya S. K., Ghodasara G. V., Srivastav Sweta and Kaneria V. J. (2008). Cordial labeling for cycle with one chord & its related graphs, Indian J. Math Science, 49: 146.

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