Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466
Vol. 11, Issue 1 (June 2016), pp. 408 – 423

Applications and Applied Mathematics:
An International Journal
(AAM)

Domination Integrity of Line Splitting Graph and Central Graph of Path, Cycle and Star Graphs

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Received: September 15, 2015; Accepted: May 17, 2016

Abstract

The domination integrity of a connected graph $G = (V(G), E(G))$ is denoted as $DI(G)$ and defined by $DI(G) = \min\{|S| + m(G - S)\}$, where S is a dominating set and $m(G - S)$ is the order of a maximum component of $G - S$. This paper discusses domination integrity of line splitting graph and central graph of some graphs.

Keywords: Integrity; Domination Integrity; Line Splitting graph; Central graph of graph

MSC 2010 No.: 05C38, 05C69, 05C76

1. Introduction

All graphs considered in this paper are finite and undirected, without loops and multiple edges. Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex v in a graph G is the number of edges of G incident with v and it is denoted by $deg(v)$. $\lceil x \rceil$ denotes the smallest integer number that greater than or equals to x with $\lfloor x \rfloor$ to the greatest integer number that smaller than or equals to x. For graph theoretic terminology, we refer to Harary (1969). For any undefined terminology and notation related to the concept of domination we refer to Haynes et al. (1998).

In the remaining portion of this section we will give brief summary of definitions and information related to the present work. The stability of a communication network composed of processing nodes and communication links is of prime importance to network designers. As the network begins losing links or nodes, eventually there is a loss in its effectiveness. Thus communication networks must be constructed to be as stable as possible, not only with respect to the initial disruption but also with respect to the possible reconstruction of the network. The communication network can be represented as an undirected graph. Tree, mesh, hypercube, and star graph are popular communication networks. If we think of the graph as modeling a network, there are many graph theoretical parameters used in the past to describe the stability of communication networks. Most notably, the vertex-connectivity and edge-connectivity have been frequently used. The best known measure of reliability of a graph is its vertex-connectivity $\kappa(G)$ defined to be the minimum number of vertices whose deletion results in a disconnected or trivial graph. The difficulty with these parameters is that they do not take into account what remains after the graph is disconnected. Consequently, a number of other parameters have recently been introduced in order to attempt to cope with this difficulty.

The concept of integrity was introduced as a measure of graph vulnerability in this sense. Formally, the vertex-integrity (frequently called just the integrity) is $I(G) = \min\{|S|+m(G-S) : S \subseteq V(G)\}$, where $m(G-S)$ denotes the order of a maximum component of $G-S$. This concept was introduced by Barefoot et al. (1987).

Definition.

A subset S of $V(G)$ is said to be an I -set, if $I(G) = |S| + m(G - S)$.

The connectedness of a graph is not essential to define integrity. The integrity of middle graphs is discussed by Mamut and Vumar (2007) while integrity of total graphs is discussed by Dundar and Aytac (2004).

A set $S \subseteq V(G)$ is called a dominating set of G if each vertex of $V - S$ is adjacent to at least one vertex of S. The domination number of a graph G, denoted as $\gamma(G)$, is the minimum cardinality of a dominating set in G. We refer to Haynes et al. (1998). If D is any minimal dominating set and if the order of the largest component of $G - D$ is small, then the removal of D will crash the communication network. The decision making process as well as communication between remaining members will also be highly affected. Considering this aspect, Sundareswaran and Swaminathan (2010) introduced the concept of domination integrity which is defined as follows.

Definition.

The domination integrity of a connected graph G denoted by $DI(G)$ is defined as $DI(G) = min\{|S| + m(G - S) : S$ is a dominating set$\}$, where $m(G - S)$ is the order of a maximum component of $G - S$.

Sundareswaran and Swaminathan (2010) have investigated domination integrity of middle graph of some graphs, while Vaidya and Kothari (2013) have investigated domination integrity of splitting graph of path and cycle. Kulli and Biradar (2002) introduced the concept of line splitting graph of a graph as follows.

Definition.

For each edge e_i of G, a new vertex e'_i is taken and the resulting set of vertices is denoted by $E_1(G)$. The line splitting graph $L_s(G)$ of a graph G is defined as the graph having vertex set $E(G) \cup E_1(G)$ with two vertices adjacent if they correspond to adjacent edges of G or one corresponds to an element e'_i of $E_1(G)$ and the other to an element e_j of $E(G)$ where e_j is in $N(e_i)$.

In the present work, we investigate domination integrity of line splitting graphs of path, cycle and star. We also investigate domination integrity of central graph of path, cycle and star.

2. Line Splitting Graphs

Theorem 2.1.

For $p \geq 2$,

$$\gamma(L_s(P_p)) = \begin{cases} 2, & \text{if } p = 2, 3, 4, \\ \frac{p}{2}, & \text{if } p \equiv 0, 2(mod\ 4), \text{ and } p > 4, \\ \frac{p-1}{2}, & \text{if } p \equiv 1(mod\ 4), \text{ and } p > 4, \\ \frac{p+1}{2}, & \text{if } p \equiv 3(mod\ 4), \text{ and } p > 4. \end{cases}$$

Proof:

By definition of line splitting graph, the graph $L_s(P_p)$ is having vertex set $\{e_1, e_2, ..., e_{p-1}, e'_1, e'_2, ..., e'_{p-1}\}$, where $e'_1, e'_2, ..., e'_{p-1}$ are the vertices corresponding to the edges $e_1, e_2, ..., e_{p-1}$ which are added to obtain $L_s(P_p)$. As $N(e'_1) = \{e_2\}$ and $N(e'_{p-1}) = \{e_{p-2}\}$, at least one vertex from each pair $\{e'_1, e_2\}$ and $\{e'_{p-1}, e_{p-2}\}$ must belong to any dominating set of $L_s(P_p)$. As well at least one vertex from e'_{i-1}, e'_{i+1} must belong to any dominating set as $N(e'_i) = \{e_{i-1}, e_{i+1}\}$, and $N(e_i) = \{e_{i-1}, e_{i+1}, e'_{i-1}, e'_{i+1}\}$ where $2 \leq i \leq p-2$. Accordingly,

$$|S| \geq \frac{p-1}{2} \text{ for any dominating set } S.$$

For $p = 2$, we consider $S = \{e_1, e'_1\}$, for $p = 3$, we consider $S = \{e_1, e_2\}$ and for $p = 4$, we consider $S = \{e_2, e'_2\}$.

Based upon the number of vertices in P_p, the following subsets are considered:
For $0 \leq j < \lfloor \frac{p}{4} \rfloor - 1$, and $p > 4$, $S = \{e_{2+4j}, e_{3+4j}, e_{p-3}, e_{p-2}\}$, $|S| = \frac{p}{2}$ for $p \equiv 0(mod\ 4)$.

For $p > 4, 0 \leq j < \lfloor \frac{p}{4} \rfloor$, $S = \{e_{2+4j}, e_{3+4j}\}, |S| = \frac{p-1}{2}$ for $p \equiv 1(mod\ 4)$,

$$S = \{e_{2+4j}, e_{3+4j}, e_{p-2}\}, |S| = \frac{p}{2} \text{ for } p \equiv 2(mod\ 4),$$

$$S = \{e_{2+4j}, e_{3+4j}, e_{p-3}, e_{p-2}\}, |S| = \frac{p+1}{2} \text{ for } p \equiv 3(mod\ 4).$$

Also each S is minimal since the vertex e'_{2+4j} will not be dominated by any of the vertices when the vertex e_{3+4j} is removed.

So S is minimal and minimum dominating set of $(L_s(P_p))$. Therefore

$$\gamma(L_s(P_p)) = \begin{cases} 2, & \text{if } p = 2, 3, 4, \\ \frac{p}{2}, & \text{if } p \equiv 0, 2(mod\ 4), \text{ and } p > 4, \\ \frac{p-1}{2}, & \text{if } p \equiv 1(mod\ 4), \text{ and } p > 4, \\ \frac{p+1}{2}, & \text{if } p \equiv 3(mod\ 4), \text{ and } p > 4. \end{cases}$$

□

Theorem 2.2.

$$DI(L_s(P_p)) = \begin{cases} p, & \text{if } p = 2, 3, \\ p-1, & \text{if } 4 \leq p \leq 10, \\ p-2, & \text{if } p = 11, 12, 13, \\ p-3, & \text{if } p = 14, 15, 16, \\ p-4, & \text{if } p = 17, 18, 19, \\ 15, & \text{if } p = 20. \end{cases}$$

Proof:

By definition, $L_s(P_p)$ has vertex set $\{e_1, e_2, ..., e_{p-1}, e'_1, e'_2, ..., e'_{p-1}\}$, where $\{e'_1, e'_2, ..., e'_{p-1}\}$ is the set of vertices corresponding to $e_1, e_2, ..., e_{p-1}$ which are added to obtain $L_s(P_p)$.

To prove this result we consider following four cases.

Case 1: $p = 2$. From Theorem 2.1, we have $DI(L_s(P_2)) = 2$, $p = 3$. From Theorem 2.1, $\gamma(L_s(P_3)) = 2$ and $D = \{e_1, e_2\}$ is a γ-set of $L_s(P_3)$. Then $m(L_s(P_3) - D) = 1$. This implies that $DI(L_s(P_3)) = \gamma(L_s(P_3)) + m(L_s(P_3) - D) = 2 + 1 = 3$. Since $\gamma(L_s(P_3)) \leq |S|$ and $m(L_s(P_3) - D) \leq m(L_s(P_3) - S)$ for any dominating set S of $L_s(P_3)$, it follows that $\gamma(L_s(P_3)) + m(L_s(P_3) - D) \leq |S| + m(L_s(P_3) - S)$ for any dominating set S of $L_s(P_3)$. For this reason $DI(L_s(P_3)) = 3$.

Case 2: $p = 4$. From Theorem 2.1, $\gamma(L_s(P_4)) = 2$ and $D = \{e_2, e'_2\}$ is a γ-set of $L_s(P_4)$. We have $DI(L_s(P_4)) = 3$. Similar to case 1.

Case 3: $5 \leq p \leq 7$. By Theorem 2.1, $\gamma(L_s(P_p)) = p - 3$ and $D = \{e_2, e_3, ..., e_{p-2}\}$ is a γ-set of $L_s(P_p)$. Then, $m(L_s(P_p) - D) = 2$.
Therefore,

$$\begin{aligned} DI(L_s(P_p)) &\leq \gamma(L_s(P_p)) + m(L_s(P_p) - D) \\ &\leq p - 3 + 2 = p - 1. \end{aligned} \tag{1}$$

If S is any dominating set of $L_s(P_p)$ other than D with $m(L_s(P_p) - S) = 1$, then $|S| \geq p - 1$. This implies that

$$|S| + m(L_s(P_p) - S) \geq p - 1 + 1 = p > p - 1. \tag{2}$$

If $m(L_s(P_p) - S) \geq 3$, then trivially $|S| + m(L_s(P_p) - S) > p - 1$. Thus for any dominating sets, $|S| + m(L_s(P_p) - S) > p - 1$. Hence, from (1) and (2), $DI(L_s(P_p)) = p - 1$.

Case 4: $p = 8$. From Theorem 2.1, $\gamma(L_s(P_8)) = 4$ and $D = \{e_2, e_3, e_5, e_6\}$ is a γ-set of $L_s(P_8)$. Then $m(L_s(P_8) - D) = 3$. Therefore,

$$DI(L_s(P_8)) \leq \gamma(L_s(P_8)) + m(L_s(P_8) - D) = 4 + 3 = 7. \tag{3}$$

Let's claim that if S_1 is any dominating set of $L_s(P_8)$ other than D with $m(L_s(P_8) - S_1) = 2$, then $|S_1| \geq 5$. So,

$$|S_1| + m(L_s(P_8) - S_1) \geq 5 + 2 = 7. \tag{4}$$

Consider S_2 is any dominating set of $L_s(P_8)$ other than D and S_1 with $m(L_s(P_8) - S_2) = 1$, then $|S_1| \geq 7$. Hence,

$$|S_2| + m(L_s(P_8) - S_2) \geq 7 + 1 = 8. \tag{5}$$

Therefore, from (3), (4) and (5), $DI(L_s(P_8)) = 7$.

Case 5: $9 \leq p \leq 20$. It is known

$$DI(L_s(P_p)) \leq \gamma(L_s(P_p)) + m(L_s(P_p) - D). \tag{6}$$

The domination number with $\gamma(L_s(P_p)) + m(L_s(P_p) - D)$ and set S with $|S| + m(L_s(P_p) - S)$ for different paths are shown in Table I.

Table I: The values of parameters: $\gamma(L_s(P_p)) + m(L_s(P_p) - D)$ and $|S| + m(L_s(P_p) - S)$

p	$\gamma(L_s(P_p))$	$m(L_s(P_p) - D)$	$\gamma(L_s(P_p)) + m(L_s(P_p) - D)$	S	$m(L_s(P_p) - S)$	$\|S\| + m(L_s(P_p) - S)$
9	4	6	10	$\{e_2, e_3, e_5, e_6, e_7\}$	3	8
10	5	6	11	$\{e_2, e_3, e_5, e_6, e_7, e_8\}$	3	9
11	6	6	12	$\{e_2, e_3, e_5, e_6, e_8, e_9\}$	3	9
12	6	6	12	$\{e_2, e_3, e_5, e_6, e_7, e_9, e_{10}\}$	3	10
13	6	6	12	$\{e_2, e_3, e_5, e_6, e_7, e_8, e_{10}, e_{11}\}$	3	11
14	7	6	13	$\{e_2, e_3, e_5, e_6, e_8, e_9, e_{11}, e_{12}\}$	3	11
15	8	6	14	$\{e_2, e_3, e_4, e_6, e_7, e_9, e_{10}, e_{12}, e_{13}\}$	3	12
16	8	6	14	$\{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{11}, e_{13}, e_{14}\}$	3	13
17	8	6	14	$\{e_2, e_3, e_5, e_6, e_8, e_9, e_{11}, e_{12}, e_{14}, e_{15}\}$	3	13
18	9	6	15	$\{e_2, e_3, e_5, e_6, e_8, e_9, e_{11}, e_{12}, e_{14}, e_{15}, e_{16}\}$	3	14
19	10	6	16	$\{e_2, e_3, e_5, e_6, e_8, e_9, e_{11}, e_{12}, e_{14}, e_{15}, e_{16}, e_{17}\}$	3	15
20	10	6	16	$\{e_2, e_3, e_5, e_6, e_8, e_9, e_{11}, e_{12}, e_{14}, e_{15}, e_{17}, e_{18}\}$	3	15

From Table I,

$$|S| + m(L_s(P_p) - S) \leq \gamma(L_s(P_p)) + m(L_s(P_p) - D). \tag{7}$$

Consider S_1 is any dominating set of $L_s(P_p)$ other than D and S with $m(L_s(P_p) - S_1) = 4$ *or* 5, then $|S_1| \geq |S|$. This implies that

$$|S| + m(L_s(P_p) - S) \leq |S_1| + m(L_s(P_p) - S_1). \tag{8}$$

If S_2 is any dominating set of $L_s(P_p)$ other than D, S_1 and S with $m(L_s(P_p) - S_2) = 2$, then $|S_2| \geq p - 3$. Therefore,

$$|S_2| + m(L_s(P_p) - S_2) \geq p - 3 + 2 = p - 1. \tag{9}$$

If S_3 is any dominating set of $L_s(P_p)$ other than D, S_1, S_2 and S with $m(L_s(P_p) - S_3) = 1$, then $|S_3| \geq p-1$. This implies that

$$|S_3| + m(L_s(P_p) - S_3) \geq p - 1 + 1 = p. \tag{10}$$

Thus from Table I and the results (6) to (10), conclude that

$$DI(L_s(P_p)) = |S| + m(L_s(P_p) - S) = \begin{cases} p, & \text{if } p = 2, 3, \\ p-1, & \text{if } 4 \leq p \leq 10, \\ p-2, & \text{if } p = 11, 12, 13, \\ p-3, & \text{if } p = 14, 15, 16, \\ p-4, & \text{if } p = 17, 18, 19, \\ 15, & \text{if } p = 20. \end{cases}$$

□

Theorem 2.3.

$$DI(L_s(P_p)) = \gamma(L_s(P_p)) + 6, for\ every\ p \geq 21. \tag{11}$$

Proof:

By definition of line splitting graph, the graph $L_s(P_p)$ has vertex set $\{e_1, e_2, ..., e_{p-1}, e'_1, e'_2, ..., e'_{p-1}\}$, where $\{e'_1, e'_2, ..., e'_{p-1}\}$ is the set of vertices corresponding to $e_1, e_2, ..., e_{p-1}$ which are added to obtain $L_s(P_p)$. Then from Theorem 2.1, for $p \geq 3$,

$$\gamma(L_s(P_p)) = \begin{cases} 2, & \text{if } p = 3, \\ \frac{p}{2}, & \text{if } p \equiv 0, 2(mod\ 4), \\ \frac{p-1}{2}, & \text{if } p \equiv 1(mod\ 4), \\ \frac{p+1}{2}, & \text{if } p \equiv 3(mod\ 4). \end{cases} \tag{12}$$

$D = \{e_{2+4j}, e_{3+4j}, e_{p-3}, e_{p-2}\}$ where $0 \leq j < \lfloor \frac{p}{4} \rfloor - 1$ for $p \equiv 0(mod\ 4)$, and $p > 4$;
$D = \{e_{2+4j}, e_{3+4j}\}$, where $0 \leq j < \lfloor \frac{p}{4} \rfloor\ for\ p \equiv 1(mod\ 4)$, and $p > 4$;
$D = \{e_{2+4j}, e_{3+4j}, e_{p-2}\}$, where $0 \leq j < \lfloor \frac{p}{4} \rfloor\ for\ p \equiv 2(mod\ 4)$, and $p > 4$;
$D = \{e_{2+4j}, e_{3+4j}, e_{p-3}, e_{p-2}\}$, where $0 \leq j < \lfloor \frac{p}{4} \rfloor\ for\ p \equiv 3(mod\ 4)$, and $p > 4$ are γ-sets of $L_s(P_p)$. Then, $m(L_s(P_p) - D) = 6$. Therefore,

$$DI(L_s(P_p)) \leq \gamma(L_s(P_p)) + 6. \tag{13}$$

Let's claim that if $m(L_s(P_p) - S) \neq 6$ for any dominating set S other than D, then

$$|S| + (L_s(P_p) - S) \geq \gamma(L_s(P_p)) + 6. \tag{14}$$

We have the following cases:

Case 1. If S_1 is any dominating set other than D and $m(L_s(P_p) - S_1) > 4$ or 5, then $|S_1| \geq \gamma(L_s(P_p)) + 2$. So, $|S_1| + m(L_s(P_p) - S_1) \geq \gamma(L_s(P_p)) + 6$.

Case 2. Consider S_2 is any dominating set of $L_s(P_p)$ other than D and S_1 with $m(L_s(P_p)-S_2)=3$, then $|S_2|\geq(p-1-\lceil\frac{p}{3}\rceil)$. Consequently, $|S_2|+m(L_s(P_p)-S_2)\geq(p-\lceil\frac{p}{3}\rceil)+2>\gamma(L_s(P_p))+6$.

Case 3. If S_3 is any dominating set of $L_s(P_p)$ other than D, S_1 and S_2 with $m(L_s(P_p)-S_3)=2$, then $|S_3|\geq p-3$. Then $|S_3|+m(L_s(P_p)-S_3)\geq(p-3)+2=p-1>\gamma(L_s(P_p))+6$.

Case 4. Consider S_4 is any dominating set of $L_s(P_p)$ other than D, S_1, S_2 and S_3 with $m(L_s(P_p)-S_4)=1$ then $|S_4|\geq p-1$. Consequently $|S_4|+m(L_s(P_p)-S_4)\geq p-1+1=p>\gamma(L_s(P_p))+6$.

Hence, from (13) and (14), $DI(L_s(P_p))=\gamma(L_s(P_p))+6$. □

Theorem 2.4.

For all $p\geq 3$,

$$\gamma(L_s(C_p))=\begin{cases}2, & \text{if } p=3,4,\\ \frac{p}{2}, & \text{if } p\equiv 0(mod\ 4)\ and\ p\geq 5,\\ \frac{p+1}{2}, & \text{if } p\equiv 1,3(mod\ 4)\ and\ p\geq 5,\\ \frac{p+2}{2}, & \text{if } p\equiv 2(mod\ 4)\ and\ p\geq 5.\end{cases}$$

Proof:

By definition of line splitting graph, $L_s(C_p)$ has vertex set $\{e_1,e_2,...,e_p,e'_1,e'_2,...,e'_p\}$, where $\{e'_1,e'_2,...,e'_p\}$ is the set of vertices corresponding to edges $e_1,e_2,...,e_p$ which are added to obtain $L_s(C_p)$. S is chosen as follows:

For $p=3,4$, consider $S=\{e_1,e_2\}$, Let's claim that S is a minimal dominating set of $L_s(C_3)$ and $L_s(C_4)$, because the vertex e'_2 will not be dominated when the vertex e_1 is removed. Thus S is minimal dominating set, hence $\gamma(L_s(C_3))=2$ and $\gamma(L_s(C_4))=2$.

For $1\leq j<\lfloor\frac{p}{4}\rfloor$, and $p\geq 5$, $S=\{e_1,e_{4j},e_{4j+1},e_p\}$, $|S|=\frac{p}{2}$ for $p\equiv 0(mod\ 4)$,
For $1\leq j\leq\lfloor\frac{p}{4}\rfloor$, and $p\geq 5$, $S=\{e_1,e_{4j},e_{4j+1}\}$, $|S|=\frac{p+1}{2}$ for $p\equiv 1(mod\ 4)$,
For $1\leq j\leq\lfloor\frac{p}{4}\rfloor$, and $p\geq 5$, $S=\{e_1,e_{4j},e_{4j+1},e_p\}$, $|S|=\frac{p+2}{2}$ for $p\equiv 2(mod\ 4)$,
For $1\leq j\leq\lfloor\frac{p}{4}\rfloor$, and $p\geq 5$, $S=\{e_1,e_{4j},e_{4j+1},e_p\}$, $|S|=\frac{p+1}{2}$ for $p\equiv 3(mod\ 4)$.

We claim that each S is a minimal dominating set of $L_s(C_p)$, as $N(e_1)=\{e_2,e_p,e'_2,e'_p\}$, $N(e_{4j})=\{e_{4j-1},e_{4j+1},e'_{4j-1},e'_{4j+1}\}$, $N(e_{4j+1})=\{e_{4j},e_{4j+2},e'_{4j},e'_{4j+2}\}$ and $N(e_p)=\{e_1,e_{p-1},e'_1,e'_{p-1}\}$, removal of e_{4j}, a vertex e'_{4j+1} will not be dominated by any vertex, hence the proof is completed. □

Theorem 2.5.

$$DI(L_s(C_p))=\begin{cases}p+1, & \text{if } 3\leq p\leq 6,\\ p-\lfloor\frac{p}{3}\rfloor+3, & \text{if } 7\leq p\leq 15.\end{cases}$$

Proof:

By definition of line splitting gresaph, $L_s(C_p)$ has vertex set $\{e_1,e_2,...,e_p,e'_1,e'_2,...,e'_p\}$, where $\{e'_1,e'_2,...,e'_p\}$ is the set of vertices corresponding to edges $e_1,e_2,...,e_p$ which are added to obtain $L_s(C_p)$.

We have the following cases:

Case 1: $p = 3$. From Theorem 2.4, we have $\gamma(L_s(C_3)) = 2$ and $D = \{e_1, e_2\}$ is a γ-set of $L_s(C_3)$, then $m(L_s(C_3) - D) = 3$. Therefore,

$$\begin{aligned} DI(L_s(C_3)) &\leq \gamma(L_s(C_3)) + m(L_s(C_3) - D) \\ &= 2 + 3 = 5. \end{aligned} \tag{15}$$

Consider S_1 is any dominating set of $L_s(C_3)$ other than D with $m(L_s(C_3) - S_1) = 2$, then $|S_1| \geq 3$. This implies that

$$|S_1| + m(L_s(C_3) - S_1) \geq 2 + 3 = 5. \tag{16}$$

Let $S_2 = \{e_1, e_2, e_3\}$ be dominating set of $L_s(C_3)$ and $m(L_s(C_3) - S_2) = 1$. Thus

$$|S_2| + m(L_s(C_3) - S_2) = 3 + 1 = 4. \tag{17}$$

Hence, from (15), (16), and (17), $DI(L_s(C_3)) = 4$.

Case 2: $p = 4$. From Theorem 2.4, $\gamma(L_s(C_4)) = 2$ and $D = \{e_1, e_2\}$ is a γ-set of $L_s(C_4)$, then $m(L_s(C_4) - D) = 6$. Therefore,

$$\begin{aligned} DI(L_s(C_4)) &\leq \gamma(L_s(C_4)) + m(L_s(C_4) - D) \\ &= 2 + 6 = 8. \end{aligned} \tag{18}$$

If S_1 is any dominating set of $L_s(C_4)$ other than D with $m(L_s(C_4) - S_1) = 2$ or 1, then $|S_1| \geq 4$. This implies that

$$|S_1| + m(L_s(C_4) - S_1) \geq 4 + 2 = 6. \tag{19}$$

Let $S_2 = \{e_1, e_2, e_3, e_4\}$ be dominating set of $L_s(C_4)$ and $m(L_s(C_3) - S_2) = 1$. Thus,

$$|S_2| + m(L_s(C_4) - S_2) = 4 + 1 = 5. \tag{20}$$

Hence, from (18), (19), and (20), $DI(L_s(C_4)) = 5$.

Case 3: $p = 5$. From Theorem 2.4, we have $\gamma(L_s(C_5)) = 3$ and $D = \{e_2, e_3, e_4\}$ is a γ-set of $L_s(C_5)$, then $m(L_s(C_5) - D) = 6$. Therefore,

$$\begin{aligned} DI(L_s(C_5)) &\leq \gamma(L_s(C_5)) + m(L_s(C_5) - D) \\ &= 3 + 6 = 9. \end{aligned} \tag{21}$$

If S_1 is any dominating set of $L_s(C_5)$ other than D with $m(L_s(C_5) - S_1) = 4$ or 5, then $|S_1| \geq 4$. This implies that

$$|S_1| + m(L_s(C_5) - S_1) \geq 4 + 5 = 9. \tag{22}$$

Consider S_2 is any dominating set of $L_s(C_5)$ other than D and S_1 with $m(L_s(C_5) - S_2) = 2$ or 3, then $|S_2| \geq 5$. This implies that

$$|S_2| + m(L_s(C_5) - S_2) \geq 5 + 2 = 7. \tag{23}$$

Let $S_3 = \{e_1, e_2, e_3, e_4, e_5\}$ be dominating set of $L_s(C_5)$ and $m(L_s(C_5) - S_3) = 1$. Thus,

$$|S_3| + m(L_s(C_5) - S_3) = 5 + 1 = 6. \tag{24}$$

Hence, from (22), (23), and (24), $DI(L_s(C_5)) = 6$.

Case 4: $p = 6$. From Theorem 2.4, $\gamma(L_s(C_6)) = 4$ and $D = \{e_1, e_4, e_5, e_6\}$ is a γ-set of $L_s(C_6)$, then $m(L_s(C_6) - D) = 3$. Therefore,

$$\begin{aligned} DI(L_s(C_6)) &\leq \gamma(L_s(C_6)) + m(L_s(C_6) - D) \\ &= 4 + 3 = 7. \end{aligned} \tag{25}$$

If S_1 is any dominating set of $L_s(C_6)$ other than D with $m(L_s(C_6) - S_1) = 2$, then $|S_1| \geq 6$. This implies that

$$|S_1| + m(L_s(C_6) - S_1) \geq 6 + 2 = 8. \tag{26}$$

Let $S_2 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be dominating set of $L_s(C_6)$ and $m(L_s(C_6) - S_2) = 1$. Thus,

$$|S_2| + m(L_s(C_6) - S_2) = 6 + 1 = 7. \tag{27}$$

Hence, from (25), (26), and (27), $DI(L_s(C_6)) = 7$.

Case 5: $7 \leq p \leq 15$.

$$DI(L_s(C_p)) \leq \gamma(L_s(C_p)) + m(L_s(C_p) - D). \tag{28}$$

The set S with $|S| + m(L_s(C_p) - S)$ and domination number with $\gamma(L_s(C_p) + m(L_s(C_p) - D)$ are shown in Table II.

Table II: The values of parameters: $\gamma(L_s(C_p)) + m(L_s(C_p) - D)$ and $|S| + m(L_s(C_p) - S)$

p	$\gamma(L_s(C_p))$	$m(L_s(C_p) - D)$	$\gamma(L_s(C_p)) + m(L_s(C_p) - D)$	S	$m(L_s(C_p) - S)$	$\|S\| + m(L_s(C_p) - S)$
7	4	6	10	$\{e_1, e_2, e_4, e_5, e_7\}$	3	8
8	4	6	10	$\{e_1, e_2, e_3, e_5, e_6, e_8\}$	3	9
9	5	6	11	$\{e_1, e_2, e_4, e_5, e_7, e_8\}$	3	9
10	6	6	12	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}\}$	3	10
11	6	6	12	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}, e_{11}\}$	3	11
12	6	6	12	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}, e_{11}\}$	3	11
13	7	6	13	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}, e_{11}, e_{13}\}$	3	12
14	8	6	14	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}, e_{11}, e_{13}, e_{14}\}$	3	13
15	8	6	14	$\{e_1, e_2, e_4, e_5, e_7, e_8, e_{10}, e_{11}, e_{13}, e_{14}\}$	3	13

From Table II, for any domination set S of $L_s(C_p)$ other than D, we have

$$|S| + m(L_s(C_p) - S) \leq \gamma(L_s(C_p) + m(L_s(C_p) - D). \tag{29}$$

If S_1 is any dominating set of $L_s(C_p)$ other than D and S with $m(L_s(C_p) - S_1) = 4$ or 5, then $|S| \leq |S_1|$. This implies that

$$|S| + m(L_s(C_P) - S) < |S_1| + m(L_s(C_P) - S_1). \tag{30}$$

If S_2 is any dominating set of $L_s(C_p)$ other than D , S and S_1 with $m(L_s(C_p) - S_2) = 2$ or 1, then $|S_2| \geq p$. This implies that

$$|S_2| + m(L_s(C_P) - S_2) \geq p + 1. \tag{31}$$

Thus from Table II and the results (28) to (31), we have that

$$\begin{aligned} DI(L_s(C_p)) &= |S| + m(L_s(C_p) - S) \\ &= p - \lfloor\frac{p}{3}\rfloor + 3. \end{aligned} \quad (32)$$

□

Theorem 2.6.

For all $p \geq 16$, $DI(L_s(C_p)) = \gamma(L_s(C_p)) + 6$.

Proof:

By definition of line splitting graph, $L_s(C_p)$ has vertex set $\{e_1, e_2, ..., e_p, e'_1, e'_2, ..., e'_p\}$, where $\{e'_1, e'_2, ..., e'_p\}$ is the set of vertices corresponding to edges $e_1, e_2, ..., e_p$ which are added to obtain $L_s(C_p)$. Then from Theorem 2.4,

For $1 \leq j < \lfloor\frac{p}{4}\rfloor$, and $p \geq 5$, $S = \{e_1, e_{4j}, e_{4j+1}, e_p\}$, for $p \equiv 0(mod\, 4)$, For $1 \leq j \leq \lfloor\frac{p}{4}\rfloor$, and $p \geq 5$, $S = \{e_1, e_{4j}, e_{4j+1}\}$, for $p \equiv 1(mod\ 4)$, For $1 \leq j \leq \lfloor\frac{p}{4}\rfloor$, and $p \geq 5$, $S = \{e_1, e_{4j}, e_{4j+1}, e_p\}$, for $p \equiv 2(mod\ 4)$, For $1 \leq j \leq \lfloor\frac{p}{4}\rfloor$, and $p \geq 5$, $S = \{e_1, e_{4j}, e_{4j+1}, e_p\}$, for $p \equiv 3(mod\ 4)$ are γ-sets of $L_s(C_p)$. Then $m(L_s(C_p) - D) = 6$. Thus,

$$DI(L_s(C_p)) \leq \gamma(L_s(C_p)) + 6. \quad (33)$$

If $m(L_s(C_p) - S) \neq 6$ for any dominating set of $L_s(C_p)$ other than D, then

$$|S| + m(L_s(C_p) - S) \geq \gamma(L_s(C_p)) + 6. \quad (34)$$

Case 1. Consider S_1 is any dominating set of $L_s(C_p)$ other than D and $m(L_s(C_p) - S_1) = 4$ or 5, then $|S_1| \geq \gamma(L_s(C_p)) + 3$. So $|S_1| + m(L_s(C_p) - S_1) \geq \gamma(L_s(C_p)) + 6$.

Case 2. If S_2 is any dominating set of $L_s(C_p)$ other than D and S_1 with $m(L_s(C_p) - S_2) = 3$, then $|S_2| \geq (p - \lfloor\frac{p}{3}\rfloor)$. Thus $|S_2| + m(L_s(C_p) - S_2) \geq (p - \lfloor\frac{p}{3}\rfloor) + 3 \geq \gamma(L_s(C_p)) + 6$.

Case 3. If S_3 is any dominating set of $L_s(C_p)$ other than D, S_1 and S_2 with $m(L_s(C_p) - S_3) = 2$ or 1, then $|S_3| \geq p$. Consequently $|S_3| + m(L_s(C_p) - S_3) \geq p + 1 > \gamma(L_s(C_p)) + 6$. Thus, from (33) and (34), we get the result. □

Theorem 2.7.

$$\gamma(L_s(K_{1,p-1})) = 2.$$

Proof:

By definition of line splitting graph, $L_s(K_{1,p-1})$ has vertex set $\{e_1, e_2, ..., e_{p-1}, e'_1, e'_2, ..., e'_{p-1}\}$, where $\{e'_1, e'_2, ..., e'_{p-1}\}$ is the set of vertices corresponding to edges $e_1, e_2, ..., e_{p-1}$ which are added to obtain $L_s(K_{1,p-1})$.

Consider $S = \{e_1, e_2\}$, a dominating set of $L_s(K_{1,p-1})$ and $|S| = 2$. We claim that S is a minimal dominating set of $L_s(K_{1,p-1})$. Because e_1 is adjacent to all vertices except the vertex e'_1, and e_2

is adjacent to all vertices except the vertex e'_2, if e_1 or e_2 is removed from set S, then e'_1 or e'_2 will not be dominated. Thus, S is minimal dominating set, hence $\gamma(L_s(K_{1,p-1})) = 2$. □

Theorem 2.8.

$$DI(L_s(K_{1,p-1})) = p.$$

Proof:

From Theorem 2.7, $\gamma(L_s(K_{1,p-1})) = 2$. Let $D = \{e_1, e_2\}$ be a γ-set of graph $L_s(K_{1,p-1})$. Then, $m(L_s(K_{1,p-1}) - D) = 2p - 4$. Therefore,

$$\begin{aligned} DI(L_s(K_{1,p-1})) &\leq \gamma(L_s(K_{1,p-1})) + m(L_s(K_{1,p-1}) - D) \\ &= 2 + 2p - 4 = 2p - 2. \end{aligned} \tag{35}$$

Consider $S = \{e_1, e_2, ..., e_{p-1}\}$, a dominating set of $L_s(K_{1,p-1})$, where e_i is correspond to the edges of $K_{1,p-1}$. Then, the remaining graph $L_s(K_{1,p-1}) - S$ is totally disconnected. That is, $m(L_s(K_{1,p-1}) - S) = 1$, so

$$\begin{aligned} DI(L_s(K_{1,p-1})) &\leq |S| + m(L_s(K_{1,p-1}) - S) \\ &\leq p. \end{aligned} \tag{36}$$

To show that the number $|S|+m(L_s(K_{1,p-1})-S)$ is minimum. It is assumed that $m(L_s(K_{1,p-1})-S) \geq 1$, then trivially $|S| + m(L_s(K_{1,p-1}) - S) \geq p$. Thus, from (35) and (36), $DI(L_s(K_{1,p-1})) = p$. □

3. Central Graphs

For a given graph $G = (V, E)$ of order p, the central graph $C(G)$ is obtained by subdividing each edge in G exactly once and joining all the nonadjacent vertices of G (Thilagavathi and Roopesh (2007)).

Theorem 3.1.

For any path $P_p, p \geq 2$

$$\gamma(C(P_p)) = \begin{cases} \frac{p}{2}, & \text{if } p \equiv 0(mod\ 2), \\ \lceil \frac{p}{2} \rceil, & \text{if } p \equiv 1(mod\ 2). \end{cases}$$

Proof:

Let P_p be path of length $p - 1$ with vertices $\{v_1, v_2, ..., v_p\}$. Let u_i be the vertex of subdivision of edges $v_i v_{i+1} (1 \leq i \leq p)$. Also let $v_i u_i = e_i$ and $u_i v_{i+1} = e'_i (1 \leq i \leq p - 1)$.

By the definition of central graph, the non-adjacent vertices v_i and v_j of P_p are adjacent in $C(P_p)$ by the edge e_{ij}. Therefore, $V(C(P_p)) = \{v_i/1 \leq i \leq p\} \cup \{u_i/1 \leq i \leq p - 1\}$ and $E(C(P_p)) = \{e_i/1 \leq i \leq p - 1\} \cup \{e'_i/1 \leq i \leq p - 1\} \cup \{e_{ij}/1 \leq i \leq p - 2, i + 2 \leq j \leq p\}$.

Based upon the number of vertices in P_p the following subsets are considered:

- if $p = 2, S_1 = \{u_1\}$.
- if $p = 3, S_2 = \{v_1, u_2\}$.
- if $p = 4, S_3 = \{v_2, v_3\}$.
- for $0 \leq j < \frac{p}{2} : S_4 = \{v_{1+2j}\}, |S_4| = \frac{p}{2}$, for $p \equiv 0(mod\ 2), p \geq 5$.
- for $0 \leq j < \lceil \frac{p}{2} \rceil : S_5 = \{v_{1+2j}\}, |S_5| = \lceil \frac{p}{2} \rceil$, for $p \equiv 1(mod\ 2), p \geq 5$.

It is clear that S_1, S_2 and S_3 is a minimal dominating set. We claim that S_4 and S_5 is a minimal dominating set. None of the vertices does dominated the vertices $u_{2j}, j \geq 1$ when the vertex v_{1+2j} is removed. Hence,

$$\gamma(C(P_p)) = \begin{cases} \frac{p}{2}, & \text{if } p \equiv 0(mod\ 2), \\ \lceil \frac{p}{2} \rceil, & \text{if } p \equiv 1(mod\ 2). \end{cases}$$

□

Corollary 3.2.

For any path $P_p, p \geq 3$

$$\gamma(C(L(P_p))) = \gamma(C(P_{p-1})).$$

Theorem 3.3.

For any path $P_p, p \geq 2$

$$DI(C(P_p)) = \begin{cases} 2, & \text{if } p = 2, \\ p+1, & \text{if } p \geq 3. \end{cases}$$

Proof:

Let P_p be any path of length $p-1$, and let $VL(P_p) = \{v_1, v_2, ..., v_p\}$ and $E(P_p) = \{e_1, e_2, ..., e_{p-1}\}$. By the definition of central graph, $C(P_p)$ has the vertex set $V(P_p) \cup \{u_i : 1 \leq i \leq p-1\}$, where u_i is a vertex of subdivision of the edge $v_i v_{i+1} (1 \leq i \leq p-1)$.

Case 1: $p = 2$. From Theorem 3.1, $\gamma(C(P_2)) = 1$ and $D = \{u_1\}$, is a γ-set of $C(P_2)$. Then $m(C(P_2)) - D) = 1$. This implies that $DI(C(P_2)) = \gamma(C(P_2)) + m(C(P_2) - D) = 1 + 1 = 2$.

Case 2: $p = 3$. From Theorem 3.1, $\gamma(C(P_3)) = 2$ and $D = \{v_1, u_2\}$, is a γ-set of $C(P_3)$. Then $m(C(P_3) - D) = 2$. This implies that $DI(P_3) = \gamma(C(P_3)) + m(C(P_3) - D) = 2 + 2 = 4$.

Case 3: $p \geq 4$. Consider $S = \{v_1, v_2, ..., v_p\}$ is dominating set of $C(P_p)$. This implies that

$$DI(C(P_p)) \leq |S| + m(C(P_p) - S) = p + 1. \tag{37}$$

For showing that the number $|S| + m(C(P_p) - S)$ is minimum, $m(C(P_p) - S_1) \geq 2$ be considered, then trivially $|S_1| + m(C(P_p) - S_1) \geq p + 1$. Hence for any dominating set S_1,

$$|S_1| + m(C(P_p) - S_1) \geq p + 1. \tag{38}$$

From (37) and (38), $DI(C(P_p)) = p + 1$. □

Corollary 3.4.

For any path $P_p, p \geq 3$

$$DI(C(L(P_p))) = \begin{cases} 2, & \text{if } p = 3, \\ p, & \text{if } p \geq 4. \end{cases}$$

Theorem 3.5.

For any cycle C_p, $\gamma(C(C_p)) = \begin{cases} 2, & \text{if } p = 3, \\ 3, & \text{if } p = 4, \\ \frac{p}{2}, & \text{if } p \equiv 0(mod\ 2), \\ \lceil \frac{p}{2} \rceil, & \text{if } p \equiv 1(mod\ 2). \end{cases}$

Proof:

Let C_p be any cycle of length p and let $V(C_p) = \{v_1, v_2, ..., v_p\}$ and $E(C_p) = \{e_1, e_2, ..., e_p\}$. By the definition of central graph, $C(C_p)$ has the vertex set $V(C_p) \cup \{u_i : 1 \leq i \leq p\}$, where u_i is a vertex of subdivision of the edge $v_i v_{i+1} (1 \leq i \leq p)$ and u_p is a vertex of subdivision of the edge $v_p v_1$. We note that in $C(C_p)$,

(1) $deg(v_i) = p - 1$ for every i,
(2) $deg(u_i) = 2$ for every i, also $\{u_i : 1 \leq i \leq p\}$ is an independent set.

Depending upon the number of vertices in C_p, the following subsets are available:

- if $p = 3, S_1 = \{v_1, u_2\}$.
- if $p = 4, S_2 = \{v_1, u_2, v_4\}$.
- for $0 \leq j < \frac{p}{2}, p > 4,$ and for $p \equiv 0(mod\ 2), S_3 = \{v_{1+2j}\}, |S_3| = \frac{p}{2}$,
- for $0 \leq j \leq \lfloor \frac{p}{2} \rfloor, p > 4,$ and for $p \equiv 1(mod\ 2), S_4 = \{v_{1+2j}\}, |S_4| = \lceil \frac{p}{2} \rceil$.

It is clear that S_1 and S_2 are a minimal dominating sets. We claim that S_3 and S_4 are also minimal dominating sets, since the vertices $u_{2j}, j \geq 1$ will not be dominated by any of the vertices when the vertex v_{1+2j} is removed. Hence,

$$\gamma(C(C_p)) = \begin{cases} 2, & \text{if } p = 3, \\ 3, & \text{if } p = 4, \\ \frac{p}{2}, & \text{if } p \equiv 0(mod\ 2), \\ \lceil \frac{p}{2} \rceil, & \text{if } p \equiv 1(mod\ 2). \end{cases}$$

□

Corollary 3.6.

For any cycle C_p, $\gamma(C(L(C_p))) = \begin{cases} 2, & \text{if } p = 3, \\ 3, & \text{if } p = 4, \\ \frac{p}{2}, & \text{if } p \equiv 0(mod\ 2), \\ \lceil \frac{p}{2} \rceil, & \text{if } p \equiv 1(mod\ 2). \end{cases}$

Proof:

Since $L(C_p) = C_p$, hence the result follows. □

Theorem 3.7.

For any cycle C_p, $DI(C(C_p)) = p + 1$.

Proof:

Let C_p be any cycle of length p, let $V(C_p) = \{v_1, v_2, ..., v_p\}$ and $E(C_p) = \{e_1, e_2, ..., e_p\}$.
By the definition of central graph, $C(C_p)$ has the vertex set $V(C_p) \cup \{u_i : 1 \leq i \leq p\}$, where u_i is a vertex of subdivision of the edge $v_i v_{i+1} (1 \leq i \leq p)$ and u_p is a vertex of subdivision of the edge $v_p v_1$. We consider the following cases:

Case 1: $p = 3$. From Theorem 3.5, $\gamma(C(C_3))) = 2$ and $D = \{v_1, u_2\}$ is a γ-set of $C(C_3))$. Then, $m(C(C_3) - D) = 2$. This implies that $DI(C(C_3)) = \gamma(C(C_3)) + m(C(C_3) - D) = 2 + 2 = 4$.

Case 2: $p = 4$. From Theorem 3.5, $\gamma(C(C_4)) = 3$ and $D = \{v_1, u_2, v_4\}$. is a γ-set of $C(C_4))$. Then, $m(C(C_4) - D) = 2$. This implies that $DI(C(C_4)) = \gamma(C(C_4)) + m(C(C_4) - D) = 3+2 = 5$.

Case 3: $p = 5$. Consider $S = \{v_1, u_2, v_4, v_5\}$, a dominating set of $C(C_5)$. Then, $m(C(C_5) - S) = 2$. This implies that

$$DI(C(C_5)) = |S| + m(C(C_5) - S) = 4 + 2 = 6. \tag{39}$$

From Theorem .13, $\gamma(C(C_5)) = 3$ and $D = \{v_1, v_3, v_5\}$ is a γ-set of $C(C_5)$. Then, $m(C(C_5) - D) = 6$. This implies that

$$DI(C(C_5)) = \gamma(C(C_5)) + m(C(C_5) - D) = 3 + 6 = 9. \tag{40}$$

If S_1 is any dominating set other than D and S with $m(C(C_5)) - S_1) \geq 3$, then $|S_1| \geq |S|$. This implies that

$$|S| + m(C(C_5) - S) \leq |S_1| + m(C(C_5) - S_1). \tag{41}$$

Hence from (39), (40) and (41), $DI(C(C_5)) = 6$.

Case 4: $p \geq 6$. Consider $S = \{v_1, v_2, ..., v_p\}$ a dominating set of $C(C_p)$. This implies that

$$DI(C(C_p)) \leq |S| + m(C(C_p) - S) = p + 1. \tag{42}$$

We will show that the number $|S| + m(C(C_p) - S)$ is minimum. If $m(C(C_p) - S_1) \geq 2$, then trivially $|S_1| + m(C(C_p) - S_1) \geq p + 1$. Hence for any dominating set S_1,

$$|S_1| + m(C(C_p) - S_1) \geq p + 1. \tag{43}$$

From (42) and (43), $DI(C(C_p)) = p + 1$. □

Corollary 3.8.

For any cycle C_p, $DI(C(L(C_p))) = p + 1$.

Proof:

Since $L(C_p) = C_p$, hence the result follows. □

Theorem 3.9.

For any star $K_{1,p-1}, p \geq 4$

$$DI(C(K_{1,p-1})) = p + 1.$$

Proof:

Let $V(K_{1,p-1}) = \{v, v_1, v_2, ..., v_{p-1}\}$. By the definition of central graph, $C(K_{1,p-1})$ has the vertex set $\{v_i/1 \leq i \leq p-1\} \cup \{u_i/1 \leq i \leq p-1\} \cup \{v\}$, where u_i is a vertex of subdivision of the edge $vv_i(1 \leq i \leq p-1)$.

Consider $S = \{v, v_1, v_2, ..., v_{p-1}\}$, a dominating set of $C(K_{1,p-1})$, and $m(C(K_{1,p-1}) - S) = 1$. Therefore

$$\begin{aligned} DI(C(K_{1,p-1})) &\leq |S| + m(C(K_{1,p-1}) - S) \\ &\leq p + 1. \end{aligned}$$

To show that the number $|S| + m(C(K_{1,p-1}) - S)$ is minimum. Consider $m(C(K_{1,p-1}) - S) \geq 1$, then $|S| + m(C(K_{1,p-1}) - S) \geq p + 1$. Thus $DI(C(K_{1,p-1})) = p + 1$. □

4. Conclusion

The domination and vulnerability of a network are two important concepts for the network security. We have studied an important measure of vulnerability known as domination integrity and investigate the domination integrity of line splitting graph and central graph of path, cycle and star graphs. We propose the following open problems to the readers for further research work.

Open Problem 1.

Find domination integrity of line splitting graph and central graph of other family of graphs.

Open Problem 2.

Investigate the edge-integrity for line splitting graph and central graph of graphs.

Acknowledgment:

The authors would like to express their gratitude to the referees and the Editor-in-Chief Professor Aliakbar Montazer Haghighi for their useful comments and suggestions towards the improvement of this paper.

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