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Graphic Illustration of the Transmission Resonances for the DKP Particles

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Abstract

We consider the Duffin-Kemmer-Petiau (DKP) equation in the presence of a spatially one-dimensional Woods-Saxon (WS) potential and we show by graphics how the zero-reflection condition on the Klein interval depends on the shape of the potential.

Keywords: DKP algebra; KG operator; Dirac particle; WS potential; Transmission coefficient; reflection coefficient

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1. Introduction

The DKP equation Petiau (1936), Duffin (1938), Kemmer (1939) and Géhéniau (1938) is similar in structure to the Dirac equation. Over the succeeding years, a great number of papers dealing with this equation were published. However, after the early 1950's until approximately 1970, physical interest in it waned because it was ultimately believed that the Klein-Gordon (KG) and the DKP equations are equivalent, since for many classes of processes such as the quantum electrodynamics of spin-0 mesons, calculations based on the DKP and KG equations yield identical results including one-loop corrections. Afterwards, with the discovery of the parity violation and with the creation of the unified theory of electroweak interaction (the Weinberg-Salam theory of standard model), it was concluded that the DKP formalism in some cases yields different results from a second-order formalism, and this renewed the interest in the DKP equation and its corresponding algebra.

The KG equation in the WS potential well was solved in Rojas and Villalba (2005), Villalba and Rojas (2006) and it was shown that there is a critical value for the potential where the bound antiparticle mode appears. Also, transmission resonances for the KG particle in the WS potential barrier have been computed in Rojas and Villalba (2005), and it has been shown that the transmission coefficient as a function of the energy and the potential amplitude shows a behavior that resembles the one obtained for the Dirac particle.

Let us briefly recall that the DKP equation is a natural manner to extend the covariant Dirac formalism to the case of scalar (spin0) and vectorial (spin1) particles when interacting with an electromagnetic field, and will be written as ($\hbar = c = 1$):

$$i\beta^{\mu}\left[\left(\partial_{\mu} + ieA_{\mu}\right) - m\right]\Psi(\boldsymbol{r}, t) = 0, \tag{1}$$

the matrices β^{μ} verifying the DKP algebra:

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\nu\lambda}\beta^{\mu}. \tag{2}$$

The convention for the metric tensor is $g^{\mu\nu} = diag(1, -1, -1, -1)$. The algebra (2) has three irreductible representations whose degrees are $1, 5$ and 10. The first one is trivial, having no physical content, the second and the third ones correspond respectively to the scalar and vectorial representations. For the spin 0, the β^{μ} are given by

$$\beta^{0} = \begin{pmatrix} \boldsymbol{\theta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \beta^{i} = \begin{pmatrix} \mathbf{0} & \rho^{i} \\ -\rho_{T}^{i} & \mathbf{0} \end{pmatrix}, i = 1,2,3, \tag{3}$$

where

$$\rho^{1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \rho^{2} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\rho^{3} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\theta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4}$$

The ρ_{T} denoting the transposed matrix of ρ , and $\mathbf{0}$ denoting the zero matrix. For the spin 1, the β^{μ} are given by

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix},$$

$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix}, i = 1,2,3, \tag{5}$$

with

$$e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1), \bar{0} = (0,0,0). \tag{6}$$

The s_i being the standard nonrelativistic (3×3) spin 1 matrices, and $\mathbf{0}$ and $\mathbf{1}$ denoting respectively the zero matrix and the unity matrix.

We consider the case when the particle is interacting with the scalar and independent of time, potential of WS given by

$$V(z) = \frac{V_0}{1 + exp\left(\frac{|z| - a}{r}\right)}, \tag{7}$$

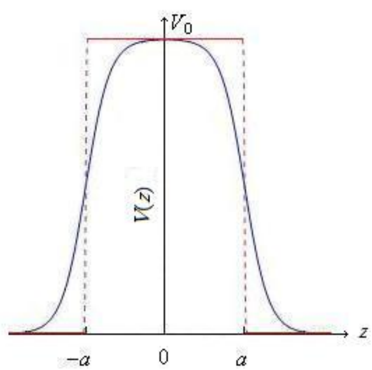

Figure 1. The WS scalar potential. $a = 2, r = (1/3\ )$ for solid lined and $a = 2, r = (1/100)$ for dotted lined.

V_0 is real and positive. The parameters of shape r, and of width a, are real, positive and adjustable. For this potential, the equation (1) will be written as

$$\left[i\beta^0 \left(\frac{\partial}{\partial t} + ieV\right) + i\beta^3 \frac{d}{dz} - m\right] \Psi(z,t) = 0. \tag{8}$$

The stationary states $\Psi(z,t)$ having the form

$$\Psi(z,t) = e^{-iEt}\widetilde{\phi(z)}, \tag{9}$$

so

$$\left[\beta^0(E-eV)+i\beta^3\frac{d}{dz}-m\right]\widetilde{\phi(z)}=0. \tag{10}$$

By taking the limit $r \to 0^+$, the potential (7) becomes of a square barrier form

$$V(z)=V_0\theta(a-|z|), \tag{11}$$

that leads to the following eigenvalue equation

$$\left[\beta^0(E-eV)+i\beta^3\frac{d}{dz}-m\right]\kappa(z)=0, \tag{12}$$

where $\kappa(z)^T=(\varphi,A,B,C\ )$, $A,B\ and\ C$ being respectively vectors of components $A_i,B_i\ and\ C_i;i=1{,}2{,}3$. The square barrier potential is one among the simple models of potentials realizing the concrete physical situation of the pair-creation phenomena Sugg et al. (1993), Calogeracos and Dombey (1999), Dombey and Calogeracos (1999), which is intimately related to the tunnel phenomena. This last is one of important and strange effects predicted by the quantum mechanics. Although it can be explained using the wave mechanics, it doesn't yet delivered its secret.

According to the equations they satisfy, one gathers the components of $\kappa(z)$ this way Boutabia-Chéraitia and Boudjedaa (2005)

$$\begin{aligned}&\Psi^T=(A_1,\ A_2,\ B_3),\Phi^T=(B_1,\ B_2,\ A_3),\\&\Theta^T=(C_2,-C_1,\varphi)\ and\ C_3=0,\end{aligned} \tag{13}$$

with

$$\boldsymbol{\Psi}=0,\begin{pmatrix}\Phi\\\Theta\end{pmatrix}=\begin{pmatrix}\dfrac{E-eV}{m}\\\dfrac{i}{m}\dfrac{d}{dz}\end{pmatrix}\otimes\Psi. \tag{14}$$

$O_{KG}=\frac{d^2}{dz^2}+[(E-eV)^2-m^2]$ being the KG operator. One will then designate by $\phi(z)^T=(\Psi,\phi,\Theta\ )$ the solution of (12).

2. The DKP Scattering States

When a wave representing a particle is incident on a potential, it is partially transmitted and partially reflected. The asymptotic forms of diverging waves can be determined at time very long after the interaction. The vectorial DKP particle we consider is subjected to the barrier potential (11). As $|z|\to\infty, V(z)\to 0$ sufficiently fast so that $\phi(z)$ solution of equation (8) becomes that of a free particle. By what follows, we will try to examine the transmission-reflection problem in which the particle is incident, say, from the left $(-\infty)$, or from the right $(+\infty)$. The asymptotic form for the wave function $\phi(z)$ is given in Boutabia-Chéraitia and Boudjedaa (2005) by

$$\phi(z) \underset{z\to-\infty}{\longrightarrow} Ae^{-ik(z+a)}\begin{pmatrix}1\\ \frac{E}{m}\\ \frac{-i\mu}{rm}\end{pmatrix}\otimes \boldsymbol{V} + Be^{ik(z+a)}\begin{pmatrix}1\\ \frac{E}{m}\\ \frac{i\mu}{rm}\end{pmatrix}\otimes \boldsymbol{V}, \tag{15}$$

and

$$\phi(z) \underset{z\to+\infty}{\longrightarrow} Ce^{ik(z-a)}\begin{pmatrix}1\\ \frac{E}{m}\\ \frac{i\mu}{rm}\end{pmatrix}\otimes \boldsymbol{V} + De^{-ik(z-a)}\begin{pmatrix}1\\ \frac{E}{m}\\ \frac{-i\mu}{rm}\end{pmatrix}\otimes \boldsymbol{V}, \tag{16}$$

where $\mu^2 = r^2(m^2 - E^2)$, $\mu = irk$ with k real $(|E| > m)$ and $\boldsymbol{V}$ is a constant vector of dimension (3×1) which components are related to the three directions of spin 1. B and D are, respectively, the coefficients of the incoming waves from $-\infty \to 0$ and from $+\infty \to 0$. A and C are, respectively, the coefficients of the reflected and transmitted wave. The reflection and transmission coefficients (respectively, $\boldsymbol{R}$ and $\boldsymbol{T}$) along each direction i of the spin are given in Boutabia-Chéraitia and Boudjedaa (2005) by

$$\boldsymbol{R} = \frac{\left(\frac{k^2-p^2}{2pk}\right)^2 sin^2 2pa}{1+\left(\frac{k^2-p^2}{2pk}\right)^2 sin^2 2pa}, \tag{17}$$

$$\boldsymbol{T} = \frac{1}{1+\left(\frac{k^2-p^2}{2pk}\right)^2 sin^2 2pa}, \tag{18}$$

which verify the equality

$$\boldsymbol{R} + \boldsymbol{T} = 1. \tag{19}$$

We propose to resolve numerically the equation (18) then we get the following graphics. When varying the energy E we obtain

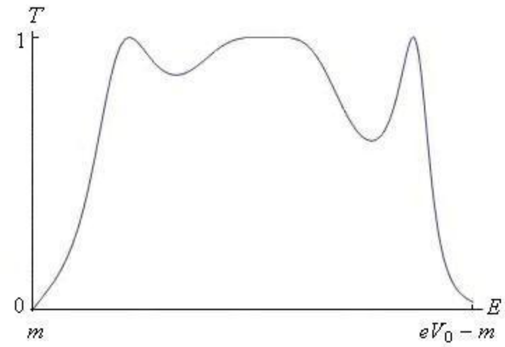

Figure 2. The plot illustrates T for varying energy E with $a = 2, eV_0 = 4, m = 1$

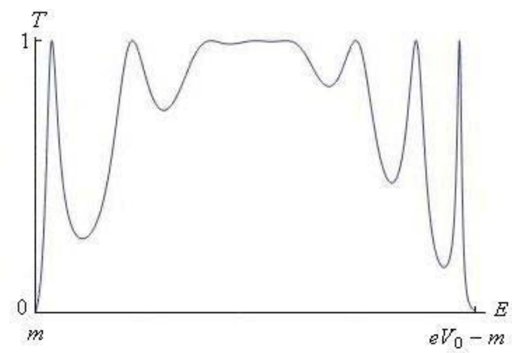

Figure 3. The plot illustrates T for varying energy E with $a = 4, eV_0 = 4, m = 1$

and when varying the barrier height eV_0 we obtain

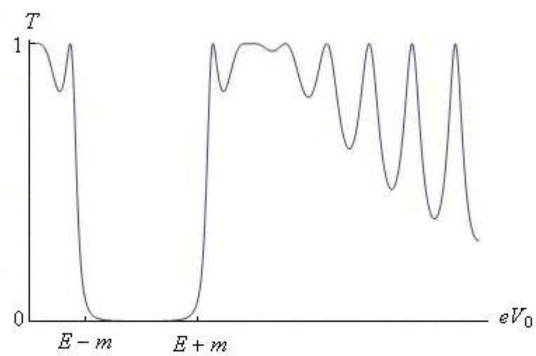

Figure 4. The plot illustrates T for varying barrier height eV_0 with $a = 2, E = 2m, m = 1$

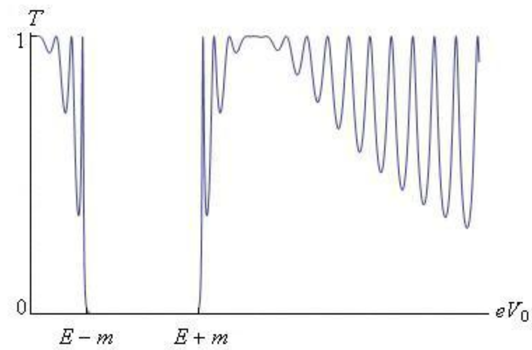

Figure 5. The plot illustrates T for varying barrier height eV_0 with $a = 4, E = 2m, m = 1$

From figures 2 and 3, one can see that analogous to the Dirac particle Sugg et al. (1993), Calogeracos and Dombey (1999), Dombey and Calogeracos (1999) and to the KG particle Rojas and Villalba (2005), the DKP particle exhibits transmission resonances in the Klein interval $(m < E < eV_0 - m)$.

Also, figures 4 and 5 show that analogous to the Dirac and KG cases, transmission resonances appear for $eV_0 > E + m$ (thus $E < eV_0 - m$). The zero-transmission happening for values of potential strength $E - m < eV_0 < E + m$ (thus$eV_0 - m < E < eV_0 + m$).

One can also mention for the four figures, that the occurrence of the transmission resonances increases with the width a of the square barrier.

3. Conclusions

We have showed a similarity in behavior between DKP, KG and Dirac particles when interacting with a one-dimensional scalar potential. For the DKP and KG particles this can be interpreted as a demonstration of the equivalence between DKP and KG theories. For DKP and Dirac particles, it gives us the hope of establishing in the future, some similarities in their behavior in the case of the potential well. This work is in pending.

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