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Application of the Extended *G'/G*-expansion Method to the Improved Eckhaus Equation

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Abstract

In this paper, the extended (G'/G)-expansion method is used to seek more general exact solutions of the improved Eckhaus equation and the (2+1)-dimensional improved Eckhaus equation. As a result, hyperbolic function solutions, trigonometric function solutions and rational function solutions with free parameters are obtained. When the parameters are taken as special values the solitary wave solutions are also derived from the traveling wave solutions. Moreover, it is shown that the proposed method is direct, effective and can be used for many other nonlinear evolution equations in mathematical physics.

Keywords: Extended (G'/G)-expansion method; Improved Eckhaus equation;
The (2+1)-dimensional Improved Eckhaus equation

MSC 2010 No.: 47F05, 35Q53, 35Q80, 35G25

1. Introduction

The investigation of the exact solutions to nonlinear partial differential equations (NLPDEs) plays an important role in the study of nonlinear physical phenomena. With the help of exact solutions, when they exist, the mechanism of complicated physical phenomena and dynamical processes modeled by these NLPDEs can be better understood. They can also help to analyze the stability of these solutions and to check numerical analysis for these NLPDEs. In recent years, reducing PDEs into ordinary differential equations (ODEs) has proved a successful idea to generate exact solutions of nonlinear wave equations. Many approaches to exact solutions in the literature follow such an idea, which contains the tanh and extended tanh methods [Abdou (2007), Wazwaz (2007), Taghizadeh et al. (2010, 2011)], (G'/G)-expansion method [Wang et al. (2008), Bekir (2008), Zhang et al. (2008, 2009), Zhu (2010), Zuo (2010), Jabbari et al. (2011), Kabir et al. (2011), Ebadi and Biswas (2010, 2011)], the homogeneous balance method [Wang (1995)], the Jacobi elliptic function method [Inc and Ergut (2005)], the exp-function method [Li et al. (2008)], the first-integral method [Feng (2002), Taghizadeh and Mirzazadeh (2011)], the sine-cosine method [Wazwaz (2004)] and so on.

The (G'/G)-expansion method, was proposed by Wang et al.(2008) for the first time, to look for traveling wave solutions of nonlinear evolution equations. This method is based on the assumptions that the traveling wave solutions can be expressed as a polynomial in (G'/G) and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (LODE). Next, Zhang et al. (2008, 2009) proposed a generalized (G'/G)-expansion method to improve and extend Wang et al.'s work for solving variable coefficient equations and high dimensional equations. Zhang et al. (2009) devised an algorithm for using the method to solve nonlinear differential-difference equations. Also, Zhang (2009) solved the equations with the balance numbers of which are not positive integers, by this method. More recently, Zhu (2010) proposed the extended (G'/G)-expansion method to seek the traveling wave solutions of nonlinear evolution equations.

In this paper, the extended (G'/G)-expansion method is investigated. For illustration, we consider the improved Eckhaus equation, and the (2+ 1)-dimensional improved Eckhaus equation.

2. The Extended (G'/G)-Expansion Method

We suppose that the given nonlinear partial differential equation for $u(x,t)$ to be in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{xxx}, \ldots) = 0, \tag{1}$$

where P is a polynomial in its arguments. The essence of the (G'/G)-expansion method can be presented in the following steps:

Step 1: Seek traveling wave solutions of equation (1) by taking $u(x,t) = U(\xi)$, $\xi = k(x - ct)$, and transform equation (1) to the ordinary differential equation

$$Q(U, U', U'', U''', \ldots) = 0, \tag{2}$$

where prime denotes the derivative with respect to ξ.

Step 2: If possible, integrate equation (2) term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

Step 3: Introduce the solution $U(\xi)$ of equation (2) in the finite series form

$$U(\xi)=\sum_{i=0}^{M} a_i \left(\frac{G'(\xi)}{G(\xi)}\right)^{i}+\sum_{i=1}^{M} b_i \left(\frac{G'(\xi)}{G(\xi)}\right)^{-i}, \tag{3}$$

where a_i, b_i are real constants to be determined, M is a positive integer to be determined, and the function $G(\xi)$ is the general solution of the auxiliary linear ordinary differential equation

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \tag{4}$$

where λ, μ are real constants to be determined.

Remark:

Expansion (3) reduces to the (G'/G)-expansion method for $b_i=0, (i=1,...,M)$.

Step 4: Determine M .To determine the parameter M , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.

Step 5: Substituting (3) together with (4) into equation (2) yields an algebraic equation involving powers of (G'/G). Equating the coefficients of each power of (G'/G) to zero gives a system of algebraic equations for a_i, b_i, λ, μ and c. Then, we solve the system with the aid of a computer algebra system, such as Maple, to determine these constants. On the other hand, depending on the sign of the discriminant $\Delta=\lambda^2-4\mu,$ the solutions of equation (4) are well known to us. So, we can obtain exact solutions of equation (1).

3. Improved Eckhaus equation

We first consider the improved Eckhaus equation [Taghizadeh and Mirzazadeh (2010)]

$$iu_t+u_{xx}+2(|u|^2)_{xx}u+|u|^4u=0. \tag{5}$$

Using the wave variable

$$u(x,t)=e^{i\theta}U(\xi),\ \theta=\alpha x+\beta t,\ \xi=x-2\alpha t, \tag{6}$$

where $U(\xi)$ is real function, the constants α, β are to be determined. Substituting (6) into equation (5), we have

$$U'' - (\alpha^2 + \beta)U + 2(U^2)''U + U^5 = 0.$$

Hence,

$$U'' - (\alpha^2 + \beta)U + 4(U')^2 U + 4U''U^2 + U^5 = 0, \tag{7}$$

where prime denotes differentiation with respect to ξ.

"Balancing $U''U^2$ with U^5 in equation (7) gives $M + 2 + 2M = 5M$. Then, $M = 1$."

Consequently, using (6), the extended (*G′/G*)-expansion method (3) admits the use of the finite expansion

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + b_1\left(\frac{G'}{G}\right)^{-1}. \tag{8}$$

Substituting (8) into (7), setting coefficients of $\left(\frac{G'}{G}\right)^i$ to zero, we obtain the following underdetermined system of algebraic equations for a_i, b_i, λ, μ and β:

$$\left(\frac{G'}{G}\right)^5 : 12a_1^3 + a_1^5 = 0,$$

$$\left(\frac{G'}{G}\right)^4 : 20\lambda a_1^3 + 20a_0a_1^2 + 5a_0a_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^3 : 2a_1 + 8(\lambda^2 + 2\mu)a_1^3 + 32\lambda a_0a_1^2 + 8a_0^2a_1 + 10a_0^2a_1^3 + 12a_1^2b_1 + 5a_1^4b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^2 : 3\lambda a_1 + 12\mu\lambda a_1^3 + 12(\lambda^2 + 2\mu)a_0a_1^2 + 12\lambda a_0^2a_1 + 20\lambda a_1^2b_1 + 10a_0^3a_1^2 + 8a_0a_1b_1 + 20a_0a_1^3b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^1 : [(\lambda^2 + 2\mu) - (\alpha^2 + \beta)]a_1 - 4\mu a_1^2 + 16\mu\lambda a_0a_1^2 + 4(\lambda^2 + 2\mu)a_0^2a_1 + 5a_0^4a_1 + 4a_1b_1^2 + 8(\lambda^2 + 2\mu)a_1^2b_1 + 10a_1^3b_1^2 + 16\lambda a_0a_1b_1 + 30a_0^2a_1^2b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^0 : \mu\lambda a_1 + \lambda b_1 - (\alpha^2 + \beta)a_0 + a_0^5 + 4\mu^2a_0a_1^2 + 4a_0b_1^2 + 4\mu\lambda a_0^2a_1 + 4\lambda a_0^2b_1 + 12\lambda a_1b_1^2 + 12\mu\lambda a_1^2b_1 + 8(\lambda^2 + 2\mu)a_0a_1b_1 + 30a_0a_1^2b_1^2 + 20a_0^3a_1b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^{-1} : [(\lambda^2 + 2\mu) - (\alpha^2 + \beta)]b_1 - 4b_1^2 + 16\lambda a_0b_1^2 + 4(\lambda^2 + 2\mu)a_0^2b_1 + 5a_0^4b_1 + 4\mu^2a_1^2b_1 + 8(\lambda^2 + 2\mu)a_1b_1^2 + 10a_1^2b_1^3 + 16\mu\lambda a_0a_1b_1 + 30a_0^2a_1b_1^2 = 0,$$

$$\left(\frac{G'}{G}\right)^{-2} : 3\mu\lambda b_1 + 12\lambda b_1^3 + 12(\lambda^2 + 2\mu)a_0 b_1^2 + 12\mu\lambda a_0^2 b_1 + 20\mu\lambda a_1 b_1^2 + 10 a_0^3 b_1^2 + 8\mu^2 a_0 a_1 b_1 + 20 a_0 a_1 b_1^3 = 0,$$

$$\left(\frac{G'}{G}\right)^{-3} : 2\mu^2 b_1 + 8(\lambda^2 + 2\mu)b_1^3 + 32\mu\lambda a_0 b_1^2 + 8\mu^2 a_0^2 b_1 + 10 a_0^2 b_1^3 + 12\mu^2 a_1 b_1^2 + 5 a_1 b_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^{-4} : 20\mu\lambda b_1^3 + 20\mu^2 a_0 b_1^2 + 5 a_0 b_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^{-5} : 12\mu^2 b_1^3 + b_1^5 = 0.$$

Solving this system using Maple gives

Case 1:

$$a_0 = \pm i\sqrt{3}\lambda, \qquad a_1 = \pm 2i\sqrt{3}, \qquad b_1 = 0, \qquad \beta = \frac{1}{64} - \alpha^2. \tag{9}$$

Case 2:

$$a_0 = \pm i\sqrt{3}\lambda, \qquad a_1 = 0, \qquad b_1 = \pm 2i\sqrt{3}\mu, \qquad \beta = \frac{1}{64} - \alpha^2. \tag{10}$$

Case 3:

$$a_0 = \pm i\sqrt{3}\lambda, \qquad a_1 = \pm 2i\sqrt{3}, \qquad b_1 = \pm 2i\sqrt{3}\mu, \qquad \beta = \frac{1}{64} - \alpha^2, \tag{11}$$

where λ and μ are arbitrary constants. Substituting equation (9) into equation (8) yields

$$U(\xi) = \pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{G'}{G}\right). \tag{12}$$

Substituting general solutions of equation (4) into equation (12), we have three types of traveling wave solutions of the Improved Eckhaus equation as follow:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solutions

$$u_{11}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right) - \frac{\lambda}{2}\right)\right] \cdot \exp\left[i(\alpha x + \left(\frac{1-64\alpha^2}{64}\right)t)\right]. \tag{13}$$

When $\lambda^2 - 4\mu < 0,$ we obtain the trigonometric function traveling wave solutions

$$u_{12}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]} \right) - \frac{\lambda}{2} \right) \right] \cdot \exp\left[i(\alpha x + \left(\frac{1-64\alpha^2}{64} \right) t) \right]. \quad (14)$$

When $\lambda^2 - 4\mu = 0,$ we obtain the rational function traveling wave solutions

$$u_{13}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{B}{A+B\xi} - \frac{\lambda}{2} \right) \right] \times \exp\left[i(\alpha x + \left(\frac{1-64\alpha^2}{64} \right) t) \right], \quad (15)$$

where $\xi = x - 2\alpha t$, for (13)-(15).

In solutions (13) − (15), A and B are left as free parameters.

In particular, if $A \neq 0$, $B = 0$ and we set $\lambda = 0$, $\mu = \frac{1}{96}$ then u_{11} becomes

$$u_{11}(\xi) = \pm \frac{i}{2\sqrt{2}} \tanh[\frac{1}{4\sqrt{6}}(x - 2\alpha t)] \exp\left[i(\alpha x + \left(\frac{1-64\alpha^2}{64} \right) t) \right], \quad (16)$$

which is the soliton solution of the improved Eckhaus equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

Substituting equation (10) into equation (8) yields

$$U(\xi) = \pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\mu \left(\frac{G'}{G} \right)^{-1}. \quad (17)$$

Substituting general solutions of equation (4) into equation (17), we have three types of traveling wave solutions of the improved Eckhaus equation as follow:

When $\lambda^2 - 4\mu > 0,$ we obtain the hyperbolic function traveling wave solutions

$$u_{21}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\mu\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}\right]$$
$$\cdot\exp\left[i\left(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t\right)\right]. \qquad (18)$$

When $\lambda^2-4\mu<0,$ we obtain the trigonometric function traveling wave solutions

$$u_{22}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\mu\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}\right]$$
$$\cdot\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right]. \qquad (19)$$

When $\lambda^2-4\mu=0,$ we obtain the rational function traveling wave solutions

$$u_{23}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\mu\left(\frac{B}{A+B\xi}-\frac{\lambda}{2}\right)^{-1}\right]\times\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right], \qquad (20)$$

where $\xi=x-2\alpha t$, for (18)-(20).

In solutions (18) − (20), A and B are left as free parameters.

In particular, if $A\neq 0,$ $B=0$ and we set $\lambda=0$, $\mu=\frac{1}{96}$ then u_{21} becomes

$$u_{21}(\xi)=\pm\frac{i}{2\sqrt{2}}\coth[\frac{1}{4\sqrt{6}}(x-2\alpha t)]\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right], \qquad (21)$$

which is the soliton solution of the improved Eckhaus equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

Substituting equation (11) into equation (8) yields

$$U(\xi)=\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\left(\frac{G'}{G}\right)\pm 2i\sqrt{3}\mu\left(\frac{G'}{G}\right)^{-1}. \qquad (22)$$

Substituting general solutions of equation (4) into equation (22), we have three types of traveling wave solutions of the improved Eckhaus equation as follow:

When $\lambda^2-4\mu>0$, we obtain the hyperbolic function traveling wave solutions

$$u_{31}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right)-\frac{\lambda}{2}\right)\right.$$
$$\left.\pm 2i\sqrt{3}\mu\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}\right]$$
$$\cdot\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right]. \qquad (23)$$

When $\lambda^2-4\mu<0$, we obtain the trigonometric function traveling wave solutions

$$u_{32}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}\right)-\frac{\lambda}{2}\right)\right.$$
$$\left.\pm 2i\sqrt{3}\mu\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}\right] \qquad (24)$$
$$\cdot\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right].$$

When $\lambda^2-4\mu=0$, we obtain the rational function traveling wave solutions

$$u_{33}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\left(\frac{B}{A+B\xi}-\frac{\lambda}{2}\right)\pm 2i\sqrt{3}\mu\left(\frac{B}{A+B\xi}-\frac{\lambda}{2}\right)^{-1}\right]$$
$$\cdot\exp\left[i(\alpha x+\left(\frac{1-64\alpha^2}{64}\right)t)\right], \qquad (25)$$

where $\xi=x-2\alpha t$, for (23)-(25).

In solutions (23) − (25), A and B are left as free parameters.

In particular, if $A\neq 0$, $B=0$ and we set $\lambda=0$, $\mu=\frac{1}{384}$ then u_{31} becomes

$$u_{31}(\xi) = \pm \frac{i}{4\sqrt{2}} \left(\tanh[\frac{1}{8\sqrt{6}}(x-2\alpha t)] - \coth[\frac{1}{8\sqrt{6}}(x-2\alpha t)] \right) \cdot \exp\left[i(\alpha x + \left(\frac{1-64\alpha^2}{64}\right) t) \right], \tag{26}$$

which is the soliton solution of the improved Eckhaus equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

These special results show that the extended (G'/G)-expansion method obtains general solutions and it can be seen that the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], are special cases of our solutions. Also, the rational function solutions (15), (20) and (25) have not been reported previously.

4. (2+1)-dimensional improved Eckhaus equation

In this section, we study the (2+1)-dimensional improved Eckhaus equation [Taghizadeh and Mirzazadeh (2010)]

$$iu_t + u_{xx} - u_{yy} + 2(|u|^2)_{xx} u + |u|^4 u = 0. \tag{27}$$

Using the wave variable

$$u(x,y,t) = e^{i\theta} U(\xi), \quad \theta = \alpha x + \beta y + \delta t, \quad \xi = x + cy + dt, \tag{28}$$

where $U(\xi)$ is real function, the constants α, β, δ, c, d are real constant. Substituting (28) into equation (27), we find the relation $d = -2(\alpha - c\beta)$, then (27) is following nonlinear ordinary differential equation

$$(1-c^2)U'' + (\beta^2 - \alpha^2 - \delta)U + 2(U^2)''U + U^5 = 0.$$

Hence,

$$(1-c^2)U'' + (\beta^2 - \alpha^2 - \delta)U + 4(U')^2 U + 4U''U^2 + U^5 = 0, \tag{29}$$

where prime denotes differentiation with respect to ξ.

"Balancing $U''U^2$ with U^5 in equation (29) gives $M + 2 + 2M = 5M$. Then, $M = 1$."

Consequently, the extended (G'/G)-expansion method (3) admits the use of the finite expansion

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + b_1\left(\frac{G'}{G}\right)^{-1}. \tag{30}$$

Substituting (30) into (29), setting coefficients of $\left(\frac{G'}{G}\right)^i$ to zero, we obtain the following underdetermined system of algebraic equations for a_i, b_i, λ, μ and β:

$$\left(\frac{G'}{G}\right)^5 : 12a_1^3 + a_1^5 = 0,$$

$$\left(\frac{G'}{G}\right)^4 : 20\lambda a_1^3 + 20a_0a_1^2 + 5a_0a_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^3 : 2(1-c^2)a_1 + 8(\lambda^2+2\mu)a_1^3 + 32\lambda a_0a_1^2 + 8a_0^2a_1 + 10a_0^2a_1^3 + 12a_1^2b_1 + 5a_1^4b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^2 : 3(1-c^2)\lambda a_1 + 12\mu\lambda a_1^3 + 12(\lambda^2+2\mu)a_0a_1^2 + 12\lambda a_0^2a_1 + 20\lambda a_1^2b_1 + 10a_0^3a_1^2 + 8a_0a_1b_1 + 20a_0a_1^3b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^1 : [(1-c^2)(\lambda^2+2\mu) + \beta^2 - \alpha^2 - \delta]a_1 + 4\mu^2a_1^3 + 16\mu\lambda a_0a_1^2 + 4(\lambda^2+2\mu)a_0^2a_1 + 5a_0^4a_1 + 4a_1b_1^2 + 8(\lambda^2+2\mu)a_1^2b_1 + 10a_1^3b_1^2 + 16\lambda a_0a_1b_1 + 30a_0^2a_1^2b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^0 : (1-c^2)\mu\lambda a_1 + (1-c^2)\lambda b_1 + (\beta^2-\alpha^2-\delta)a_0 + a_0^5 + 4\mu^2a_0a_1^2 + 4a_0b_1^2 + 4\mu\lambda a_0^2a_1 + 4\lambda a_0^2b_1 + 12\lambda a_1b_1^2 + 12\mu\lambda a_1^2b_1 + 8(\lambda^2+2\mu)a_0a_1b_1 + 30a_0a_1^2b_1^2 + 20a_0^3a_1b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^{-1} : [(1-c^2)(\lambda^2+2\mu) + \beta^2 - \alpha^2 - \delta]b_1 + 4b_1^3 + 16\lambda a_0b_1^2 + 4(\lambda^2+2\mu)a_0^2b_1 + 5a_0^4b_1 + 4\mu^2a_1^2b_1 + 8(\lambda^2+2\mu)a_1b_1^2 + 10a_1^2b_1^3 + 16\mu\lambda a_0a_1b_1 + 30a_0^2a_1b_1^2 = 0,$$

$$\left(\frac{G'}{G}\right)^{-2} : 3(1-c^2)\mu\lambda b_1 + 12\lambda b_1^3 + 12(\lambda^2+2\mu)a_0b_1^2 + 12\mu\lambda a_0^2b_1 + 20\mu\lambda a_1b_1^2 + 10a_0^3b_1^2 + 8\mu^2a_0a_1b_1 + 20a_0a_1b_1^3 = 0,$$

$$\left(\frac{G'}{G}\right)^{-3} : 2(1-c^2)\mu^2b_1 + 8(\lambda^2+2\mu)b_1^3 + 32\mu\lambda a_0b_1^2 + 8\mu^2a_0^2b_1 + 10a_0^2b_1^3 + 12\mu^2a_1b_1^2 + 5a_1b_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^{-4} : 20\mu\lambda b_1^3 + 20\mu^2a_0b_1^2 + 5a_0b_1^4 = 0,$$

$$\left(\frac{G'}{G}\right)^{-5} : 12\mu^2b_1^3 + b_1^5 = 0.$$

Solving this system using Maple gives

Case 1:

$$a_0 = \pm i\sqrt{3}\lambda, \quad a_1 = \pm 2i\sqrt{3}, \quad b_1 = 0, \quad \delta = \beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4. \tag{31}$$

Case 2:

$$a_0 = \pm i\sqrt{3}\lambda, \quad a_1 = 0, \quad b_1 = \pm 2i\sqrt{3}\mu, \quad \delta = \beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4. \tag{32}$$

Case 3:

$$a_0 = \pm i\sqrt{3}\lambda, \quad a_1 = \pm 2i\sqrt{3}, \quad b_1 = \pm 2i\sqrt{3}\mu, \quad \delta = \beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4, \tag{33}$$

where λ and μ are arbitrary constants. Substituting equation (31) into equation (30) yields

$$U(\xi) = \pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{G'}{G}\right). \tag{34}$$

Substituting general solutions of equation (4) into equation (34), we have three types of traveling wave solutions of the (2+1)-dimensional improved Eckhaus equation as follow:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solutions

$$u_{11}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]} \right) - \frac{\lambda}{2} \right) \right] \cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right]. \tag{35}$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function traveling wave solutions

$$u_{12}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]} \right) - \frac{\lambda}{2} \right) \right] \cdot \exp\left[i(\alpha x + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right]. \tag{36}$$

When $\lambda^2 - 4\mu = 0$, we obtain the rational function traveling wave solutions

$$u_{13}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\left(\frac{B}{A+B\xi}-\frac{\lambda}{2}\right)\right]\cdot\exp\left[i(\alpha x+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right], \quad (37)$$

where $\xi=x+cy-2(\alpha-c\beta)t$, for (35)-(37).

In solutions (35) − (37), A and B are left as free parameters.

In particular, if $A\neq 0,\ B=0$ and we set $\lambda=0$, $\mu=\frac{1}{96}(1-c^2)$ then u_{11} becomes

$$u_{11}(\xi)=\pm\frac{i\sqrt{c^2-1}}{2\sqrt{2}}\tanh[\frac{\sqrt{c^2-1}}{4\sqrt{6}}(x-2\alpha t)]\cdot\exp\left[i(\alpha x+\beta y+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right], \quad (38)$$

and u_{12} becomes

$$u_{12}(\xi)=\pm\frac{i\sqrt{1-c^2}}{2\sqrt{2}}\tan[\frac{\sqrt{1-c^2}}{4\sqrt{6}}(x-2\alpha t)]\cdot\exp\left[i(\alpha x+\beta y+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right], \quad (39)$$

which are the solitary and periodic wave solutions of the (2+1)-dimensional improved Eckhaus equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

Substituting equation (32) into equation (30) yields

$$U(\xi)=\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\mu\left(\frac{G'}{G}\right)^{-1}. \quad (40)$$

Substituting general solutions of equation (4) into (40), we have three types of traveling wave solutions of the (2+1)-dimensional improved Eckhaus Equation as follow:

When $\lambda^2-4\mu>0$, we obtain the hyperbolic function traveling wave solutions

$$u_{21}(\xi)=\left[\pm i\sqrt{3}\lambda\pm 2i\sqrt{3}\mu\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]+B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}\right]\cdot\exp\left[i(\alpha x+\beta y+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right]. \quad (41)$$

When $\lambda^2 - 4\mu < 0,$ we obtain the trigonometric function traveling wave solutions

$$u_{22}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\mu \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]} \right) - \frac{\lambda}{2} \right)^{-1} \right] \cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right]. \tag{42}$$

When $\lambda^2 - 4\mu = 0,$ we obtain the rational function traveling wave solutions

$$u_{23}(\xi) = \left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\mu \left(\frac{B}{A+B\xi} - \frac{\lambda}{2} \right)^{-1} \right] \cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right], \tag{43}$$

where $\xi = x + cy - 2(\alpha - c\beta)t$, for (41)-(43).

In solutions (41) − (43), A and B are left as free parameters.

In particular, if $A \neq 0,$ $B = 0$ and we set $\lambda = 0$, $\mu = \frac{1}{96}(1-c^2)$ then u_{21} becomes

$$u_{21}(\xi) = \pm \frac{i\sqrt{c^2-1}}{2\sqrt{2}} \coth[\frac{\sqrt{c^2-1}}{4\sqrt{6}}(x - 2\alpha t)] \cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right], \tag{44}$$

and u_{22} becomes

$$u_{22}(\xi) = \pm \frac{i\sqrt{1-c^2}}{2\sqrt{2}} \cot[\frac{\sqrt{1-c^2}}{4\sqrt{6}}(x - 2\alpha t)] \cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right], \tag{45}$$

which are the solitary and periodic wave solutions of the (2+1)-dimensional improved Eckhaus equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

Substituting equation (33) into equation (30) yields

$$U(\xi) = \pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{G'}{G}\right) \pm 2i\sqrt{3}\mu\left(\frac{G'}{G}\right)^{-1}. \tag{46}$$

Substituting general solutions of equation (4) into equation (46), we have three types of traveling wave solutions of the (2+1)-dimensional improved Eckhaus Equation as follow:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solutions

$$\begin{aligned} u_{31}(\xi) = &\left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]} \right) - \frac{\lambda}{2} \right) \right. \\ &\left. \pm 2i\sqrt{3}\mu\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A\cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + B\sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]} \right) - \frac{\lambda}{2} \right)^{-1} \right] \\ &\cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right]. \end{aligned} \tag{47}$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function traveling wave solutions

$$\begin{aligned} u_{32}(\xi) = &\left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]} \right) - \frac{\lambda}{2} \right) \right. \\ &\left. \pm 2i\sqrt{3}\mu\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A\cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + B\sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]} \right) - \frac{\lambda}{2} \right)^{-1} \right] \\ &\cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right]. \end{aligned} \tag{48}$$

When $\lambda^2 - 4\mu = 0$, we obtain the rational function traveling wave solutions

$$\begin{aligned} u_{33}(\xi) = &\left[\pm i\sqrt{3}\lambda \pm 2i\sqrt{3}\left(\frac{B}{A+B\xi} - \frac{\lambda}{2} \right) \pm 2i\sqrt{3}\mu\left(\frac{B}{A+B\xi} - \frac{\lambda}{2} \right)^{-1} \right] \\ &\cdot \exp\left[i(\alpha x + \beta y + \left(\beta^2 - \alpha^2 + \frac{1}{64} - \frac{1}{32}c^2 + \frac{1}{64}c^4 \right) t) \right], \end{aligned} \tag{49}$$

where $\xi = x + cy - 2(\alpha - c\beta)t$, for (47)-(49).

In solutions (47) − (49), A and B are left as free parameters.

In particular, if $A \neq 0$, $B = 0$ and we set $\lambda = 0$, $\mu = \frac{1}{384}(1-c^2)$, then u_{31} becomes

$$u_{31}(\xi) = \pm\frac{i\sqrt{c^2-1}}{4\sqrt{2}}\left(\tanh[\frac{\sqrt{c^2-1}}{8\sqrt{6}}(x-2\alpha t)] - \coth[\frac{\sqrt{c^2-1}}{8\sqrt{6}}(x-2\alpha t)]\right) \cdot \exp\left[i(\alpha x+\beta y+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right], \tag{50}$$

and u_{32} becomes

$$u_{32}(\xi) = \pm\frac{i\sqrt{1-c^2}}{4\sqrt{2}}\left(\tan[\frac{\sqrt{1-c^2}}{8\sqrt{6}}(x-2\alpha t)] - \cot[\frac{\sqrt{1-c^2}}{8\sqrt{6}}(x-2\alpha t)]\right) \cdot \exp\left[i(\alpha x+\beta y+\left(\beta^2-\alpha^2+\frac{1}{64}-\frac{1}{32}c^2+\frac{1}{64}c^4\right)t)\right], \tag{51}$$

which are the solitary and periodic wave solutions of the (2+1)-dimensional improved Eckhaus Equation.

Comparing our result with the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], we can see that the results are the same.

These special results show that the extended (G'/G)-expansion method obtains general solutions and it can be seen that the solutions obtained in [Taghizadeh and Mirzazadeh (2010)], are special cases of our solutions. Also, the rational function solutions (37), (43) and (49) have not been reported previously.

5. Conclusion

In this paper, the extended (G'/G)-expansion methods have been successfully applied to find the more travelling wave solutions for the improved Eckhaus equation and (2+1)-dimensional improved Eckhaus equation. The performance of this method is reliable and effective and gives more solutions. This method has more advantages: It is direct and concise. It is elementary that the general solutions of the second order LODE have been well known for the researchers and effective that it can be used in many other nonlinear evolution equations. The availability of computer systems like *Mathematica* or *Maple* facilitates the tedious algebraic calculations. The method which we have proposed in this Letter is also a standard, direct and computerizable method, which allows us to solve complicated and tedious algebraic calculation.

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