Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 9, Issue 1 (June 2014), pp. 330-341

Applications and Applied Mathematics:
An International Journal
(AAM)

Unstable Solutions to Nonlinear Vector Differential Equations of Sixth Order with Delay

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Received: October 22, 2013; Accepted: May 5, 2014

Abstract

This paper investigates the instability of the zero solution of a certain vector differential equation of the sixth order with delay. Using the Lyapunov- Krasovskiĭ functional approach, we obtain a new result on the topic and give an example for the related illustrations

Keywords: Vector, nonlinear differential equation; sixth order; Lyapunov- Krasovskiĭ Functional; instability; delay

MSC (2010) No.: 34K20

1. Introduction

To the best of our knowledge, the instability of the solutions to the sixth order nonlinear scalar differential equations was investigated by Ezeilo (1982). Ezeilo (1982) proved a theorem on the instability of the zero solution of the sixth order nonlinear scalar differential equation without delay:

$$x^{(6)}(t)+a_1x^{(5)}(t)+a_2x^{(4)}(t)+e(x(t),\dot{x}(t),...,x^{(5)}(t))\dddot{x}(t)+f(\dot{x}(t))\ddot{x}(t)$$
$$+g(x(t),\dot{x}(t),...,x^{(5)}(t))\dot{x}(t)+h(x(t))=0.$$

Later, Tiryaki (1990) discussed the instability of the zero solution of the following sixth order nonlinear scalar differential equation without delay:

$$x^{(6)}(t)+a_1x^{(5)}(t)+f_1(x(t),\dot{x}(t),...,x^{(5)}(t))x^{(4)}(t)+f_2(\ddot{x}(t))\dddot{x}(t)$$
$$+f_3(x(t),\dot{x}(t),...,x^{(5)}(t))\ddot{x}(t)+f_4(\dot{x}(t))+f_5(x(t))=0.$$

After that Tejumola (2000) studied the same topic for the following sixth order nonlinear scalar differential equations without delay:

$$x^{(6)}(t)+a_1x^{(5)}(t)+a_2x^{(4)}(t)+g_3(\ddot{x}(t))\dddot{x}(t)+g_4(\dot{x}(t))\ddot{x}(t)$$
$$+g_5(\dot{x}(t),\ddot{x}(t))\dot{x}(t)+g_6(x(t))=0$$

and

$$x^{(6)}(t)+a_1x^{(5)}(t)+a_2x^{(4)}(t)+a_3\dddot{x}(t)+\varphi_4(\dot{x}(t),...,x^{(5)}(t))\ddot{x}(t)$$
$$+\varphi_5(x(t))\dot{x}(t)+\varphi_6(x(t),\dot{x}(t),...,x^{(5)}(t))=0.$$

It should be noted that using the Lyapunov's direct method and based on the Krasovskiĭ's (1963) properties, Ezeilo (1982), Tiryaki (1990) and Tejumola (2000) proved their results. Later, Tunc (2011, 2012b) studied the instability of the solutions of the sixth order nonlinear scalar delay differential equations given by

$$x^{(6)}(t)+a_1x^{(5)}(t)+f_2(x(t-r),\dot{x}(t-r),...,x^{(5)}(t-r))x^{(4)}(t)$$
$$+f_3(\ddot{x}(t))\dddot{x}(t)\quad+f_4(x(t-r),\dot{x}(t-r),...,x^{(5)}(t-r))\ddot{x}(t)$$
$$+f_5(\dot{x}(t-r))+f_6(x(t-r))=0$$

and

$$x^{(6)}(t)+a_1x^{(5)}(t)+a_2x^{(4)}(t)+g_3(\ddot{x}(t))\dddot{x}(t)+g_4(\dot{x}(t))\ddot{x}(t)$$
$$+g_5(x(t-r),\dot{x}(t-r),...,x^{(5)}(t-r))\dot{x}(t)+g_6(x(t-r)=0,$$

respectively.

On the other hand, Tunc (2004a) discussed the same subject for the sixth order nonlinear vector differential equations of the form:

$$X^{(6)}(t)+AX^{(5)}(t)+\Phi(X(t),\dot{X}(t),...,X^{(5)}(t))X^{(4)}(t)+\Psi(\ddot{X}(t))\dddot{X}(t)$$
$$+F(X(t),\dot{X}(t),...,X^{(5)}(t))\ddot{X}(t)+G(\dot{X}(t))+H(X(t))=0. \qquad (1)$$

We also refer the readers to the papers of Tunc (2004b, 2007, 2008a, 2008b, and 2012a) and Tunc and Tunc (2008) for some other related papers on the instability of the solutions of the various sixth order nonlinear differential equations.

In this paper, instead of equation (1), we consider its delay form given by

$$X^{(6)}(t)+AX^{(5)}(t)+\Phi(X(t),\dot{X}(t),...,X^{(5)}(t))X^{(4)}(t)+\Psi(\ddot{X}(t))\dddot{X}(t)$$
$$+F(X(t),\dot{X}(t),...,X^{(5)}(t))\ddot{X}(t)+G(\dot{X}(t-\tau))+H(X(t-\tau))=0, \qquad (2)$$

where $X\in\Re^n$, $\tau>0$ is the constant retarded argument, A is a constant $n\times n$-symmetric matrix, Φ, Ψ and F are continuous $n\times n$-symmetric matrix functions for the arguments displayed explicitly, $G:\Re^n\to\Re^n$ and $H:\Re^n\to\Re^n$ with $G(0)=H(0)=0$, and G and H are continuous functions for the arguments displayed explicitly. The existence and uniqueness of the solutions of equation (2) [Èl'sgol'ts (1966)] are assumed.

Equation (2) is the vector version for systems of real nonlinear differential equations of the sixth order:

$$x_i^{(6)}+\sum_{k=1}^{n}a_{ik}x_k^{(5)}+\sum_{k=1}^{n}\phi_{ik}(x_1,,...,x_k,...,x_1^{(5)},,...,x_1^{(5)})x_k^{(4)}+\sum_{k=1}^{n}\psi_{ik}(x_1'',,...,x_k'')x_k'''$$
$$+\sum_{k=1}^{n}f_{ik}(x_1,,...,x_k,...,x_1^{(5)},,...,x_1^{(5)})x_k''+g_i(x_1'(t-\tau),...,x_n'(t-\tau))$$
$$+h_i(x_1(t-\tau),...,x_n(t-\tau))=0,\ (i=1,2,...,n).$$

The sequel, equation (1), is stated in system form as follows

$$\dot{X}=Y,\ \dot{Y}=Z,\quad \dot{Z}=W,\quad \dot{W}=U,\quad \dot{U}=T,$$

$$\dot{T}=-AT-\Phi(X,Y,Z,W,U,T)U-\Psi(Z)W-F(X,Y,Z,W,U,T)Z-G(Y)-H(X)$$
$$+\int_{t-\tau}^{t}J_G(Y(s))Z(s)ds+\int_{t-\tau}^{t}J_H(X(s))Y(s)ds, \qquad (3)$$

which was obtained by setting $\dot{X}=Y$, $\ddot{X}=Z$, $\dddot{X}=W$, $X^{(4)}=U$ and $X^{(5)}=U$ from equation (2).

Let $J_H(X)$ and $J_G(Y)$ denote the linear operators from $H(X)$ and $G(Y)$ to

$$J_H(X)=\left(\frac{\partial h_i}{\partial x_j}\right)$$

and

$$J_G(Y) = \left(\frac{\partial g_i}{\partial y_j}\right), \ (i, \ j = 1,2,...,n),$$

where $(x_1,...,x_n)$, $(y_1,...,y_n)$, $(h_1,...,h_n)$ and $(g_1,...,g_n)$ are the components of X, Y, H and G, respectively. Throughout what follows, it is assumed that $J_H(X)$ and $J_G(Y)$ exist and are symmetric and continuous. However, a review to date of the literature indicates that the instability of solutions of vector differential equations of the sixth order with delay has not been investigated till now. This paper is the first known work regarding the instability of solutions for the nonlinear vector differential equations of the sixth order with delay. The motivation of this paper comes from the above papers done on scalar nonlinear differential equations of the sixth order without and with delay and the vector differential equations of the sixth order without delay. By this work, we improve the result in Tunc (2004a) to a vector differential equation of the sixth order with delay. Based on Krasovskiĭ's (1963) criterions, we prove our main result, and an example is also provided to illustrate the feasibility of the proposed result. The result to be obtained is new and makes a contribution to the topic.

The symbol $\langle X, Y\rangle$ corresponding to any pair X, Y in $\Re^n$ stands for the usual scalar product $\sum_{i=1}^{n} x_i y_i$, that is,

$$\langle X, Y\rangle = \sum_{i=1}^{n} x_i y_i;$$

Thus, $\langle X, X\rangle = \|X\|^2$, and $\lambda_i(\Omega)$, $(i = 1,2,...,n)$, are the eigenvalues of the real symmetric $n \times n$-matrix Ω. The matrix Ω is said to be negative-definite, when $\langle \Omega X, X\rangle \le 0$, for all nonzero X in $\Re^n$.

2. Main Result

In order to achieve our main result, we introduce the following well known algebraic results.

Lemma.

Let D be a real symmetric $n \times n$-matrix. Then for any $X \in \Re^n$

$$\delta_d \|X\|^2 \le \langle DX, X\rangle \le \Delta_d \|X\|^2,$$

where δ_d and Δ_d are the least and greatest eigenvalues of D, respectively [Bellman (1997)].

Let $r \geq 0$ be given, and let $C = C([-r,0], \Re^n)$ with

$$\|\phi\| = \max_{-r \leq s \leq 0} |\phi(s)|, \ \phi \in C.$$

For $H > 0$ define $C_H \subset C$ by

$$C_H = \{\phi \in C : \|\phi\| < H\}.$$

If $x : [-r, A) \to \Re^n$ is continuous, $0 < A \leq \infty$, then, for each t in $[0, A)$, x_t in C is defined by

$$x_t(s) = x(t+s), -r \leq s \leq 0, \ t \geq 0.$$

Let G be an open subset of C and consider the general autonomous delay differential system with finite delay

$$\dot{x} = F(x_t), \ F(0) = 0, \ x_t = x(t+\theta), \ -r \leq \theta \leq 0, \ t \geq 0,$$

where $F : G \to \Re^n$ is continuous and maps closed and bounded sets into bounded sets. It follows from these conditions on F that each initial value problem

$$\dot{x} = F(x_t), \ x_0 = \phi \in G$$

has a unique solution defined on some interval $[0, A)$, $0 < A \leq \infty$. This solution will be denoted by $x(\phi)(.)$ so that $x_0(\phi) = \phi$.

Definition.

The zero solution, $x = 0$, of $\dot{x} = F(x_t)$ is stable if for each $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon) > 0$ such that $\|\phi\| < \delta$ implies that $\|x(\phi)(t)\| < \varepsilon$ for all $t \geq 0$. The zero solution is said to be unstable if it is not stable.

The main result of this paper is the following theorem.

Theorem.

We suppose that there exists a constant a_2 and positive constants δ, ε, a_1, a_4, a_5 and a_6 such that the following conditions hold:

A, $\Phi(.)$, $F(.)$, $\Psi(.)$, $J_G(Y)$ and $J_H(X)$ are symmetric, $\lambda_i(A) \geq a_1$,

$H(0) = 0$, $H(X) \neq 0$, $(X \neq 0)$, $\lambda_i(J_H(X)) \leq -a_6$,

$$G(0)=0,\ \ G(Y)\neq 0,\ \ (Y\neq 0),\ \left|\lambda_i(J_G(Y))\right|\leq a_5,$$

$$\lambda_i(F(.))\geq a_4+\varepsilon,\ \lambda_i(\Phi(.))\leq a_2,\quad (i=1,2,...,n),$$

and

$$a_4-\frac{1}{4}a_2^2\geq\delta.$$

If

$$\tau<\min\left\{\frac{2}{\sqrt{n}},\frac{2(\varepsilon+\delta)}{2\sqrt{n}a_5+\sqrt{n}a_6}\right\},$$

then the zero solution of equation (2) is unstable.

Remark.

It should be noted that there is no sign restriction on eigenvalues of Ψ, and it is clear that our assumptions have a very simple form and the applicability can be easily verified.

Proof:

We define a Lyapunov-Krasovskii functional $V=V(X_t,Y_t,Z_t,W_t,U_t,T_t)$:

$$V(.)=-\langle T,Z\rangle-\langle Z,AU\rangle+\langle W,U\rangle+\frac{1}{2}\langle W,AW\rangle-\langle H(X),Y\rangle$$
$$-\int_0^1\langle G(\sigma Y),Y\rangle d\sigma-\int_0^1\langle\Psi(\sigma Z)Z,Z\rangle d\sigma-\lambda\int_{-\tau}^{0}\int_{t+s}^{t}\|Y(\theta)\|^2 d\theta ds-\mu\int_{-\tau}^{0}\int_{t+s}^{t}\|Z(\theta)\|^2 d\theta ds,$$

where λ and μ are certain positive constants; that will be determined later in the proof.

It is clear that $V(0,0,0,0,0,0)=0$ and

$$V(0,0,0,\varepsilon,\varepsilon,0)=\langle\varepsilon,\varepsilon\rangle+\frac{1}{2}\langle\varepsilon,A\varepsilon\rangle=\|\varepsilon\|^2+\frac{1}{2}a_1\|\varepsilon\|^2>0,$$

for all arbitrary $\varepsilon\neq 0,\ \ \varepsilon\in\Re^n$, which verifies the property (P_1) of Krasovskiĭ (1963).

Using a basic calculation, the time derivative of V in the solutions of system (3) results in

$$\frac{d}{dt}V(.)=\langle U,U\rangle+\langle\Phi(.)U,Z\rangle+\langle F(.)Z,Z\rangle-\langle J_H(X)Y,Y\rangle+\langle G(Y),Z\rangle+\langle\Psi(Z)W,Z\rangle$$
$$+<\int_{t-\tau}^{t}J_H(X(s))Y(s)ds,Z>+<\int_{t-\tau}^{t}J_G(Y(s))Z(s)ds,Z>$$
$$-\frac{d}{dt}\int_0^1\langle G(\sigma Y),Y\rangle d\sigma-\frac{d}{dt}\int_0^1\langle\sigma\Psi(\sigma Z)Z,Z\rangle d\sigma$$
$$-\lambda\frac{d}{dt}\int_{-\tau}^{0}\int_{t+s}^{t}\|Y(\theta)\|^2d\theta ds-\mu\frac{d}{dt}\int_{-\tau}^{0}\int_{t+s}^{t}\|Z(\theta)\|^2d\theta ds.$$

We can calculate that

$$\frac{d}{dt}\int_0^1\langle G(\sigma Y),Y\rangle d\sigma=\langle G(Y),Z\rangle,$$
$$\frac{d}{dt}\int_0^1\langle\sigma\Psi(\sigma Z)Z,Z\rangle d\sigma=\langle\Psi(Z)W,Z\rangle,$$
$$\frac{d}{dt}\int_{-\tau}^{0}\int_{t+s}^{t}\|Y(\theta)\|^2d\theta ds=\|Y\|^2\tau-\int_{t-\tau}^{t}\|Y(\theta)\|^2d\theta,$$
$$\frac{d}{dt}\int_{-\tau}^{0}\int_{t+s}^{t}\|Z(\theta)\|^2d\theta ds=\|Z\|^2\tau-\int_{t-\tau}^{t}\|Z(\theta)\|^2d\theta,$$
$$<\int_{t-\tau}^{t}J_G(Y(s))Z(s)ds,Z>\geq-\|Z\|\left\|\int_{t-\tau}^{t}J_G(Y(s))Z(s)ds\right\|$$
$$\geq-\sqrt{n}a_5\|Z\|\left\|\int_{t-\tau}^{t}Z(s)\right\|ds$$
$$\geq-\sqrt{n}a_5\|Z\|\int_{t-\tau}^{t}\|Z(s)\|ds$$
$$\geq-\frac{1}{2}\sqrt{n}a_5\tau\|Z\|^2-\frac{1}{2}\sqrt{n}a_5\int_{t-\tau}^{t}\|Z(s)\|^2ds,$$
$$<\int_{t-\tau}^{t}J_H(X(s))Y(s)ds,Z>\geq-\|Z\|\left\|\int_{t-\tau}^{t}J_H(X(s))Y(s)ds\right\|$$
$$\geq-\sqrt{n}a_6\|Z\|\left\|\int_{t-\tau}^{t}Y(s)\right\|ds$$
$$\geq-\sqrt{n}a_6\|Z\|\int_{t-\tau}^{t}\|Y(s)\|ds$$
$$\geq-\frac{1}{2}\sqrt{n}a_6\tau\|Z\|^2-\frac{1}{2}\sqrt{n}a_6\int_{t-\tau}^{t}\|Y(s)\|^2ds,$$

so that

$$\frac{d}{dt}V(.) \geq \langle U,U\rangle + \langle \Phi(.)U,Z\rangle + (a_6 - \lambda\tau)\|Y\|^2$$
$$+\{a_4 - (\mu + \frac{1}{2}\sqrt{n}a_5 + \frac{1}{2}\sqrt{n}a_6)\tau\}\|Z\|^2 + (\lambda - \frac{1}{2}\sqrt{n}a_6)\int_{t-\tau}^{t}\|Y(s)\|^2 ds$$
$$+(\mu - \frac{1}{2}\sqrt{n}a_5)\int_{t-\tau}^{t}\|Z(s)\|^2 ds.$$

Let

$$\lambda = \frac{1}{2}\sqrt{n}a_6$$

and

$$\mu = \frac{1}{2}\sqrt{n}a_5.$$

Hence,

$$\frac{d}{dt}V(.) \geq \langle U,U\rangle + \langle \Phi(.)U,Z\rangle + (a_6 - \frac{1}{2}\sqrt{n}a_6\tau)\|Y\|^2 + a_4\|Z\|^2 + \{\varepsilon - (\sqrt{n}a_5 + \frac{1}{2}\sqrt{n}a_6)\tau\}\|Z\|^2$$
$$= \|U + 2^{-1}\Phi(.)Z\|^2 - \frac{1}{4}\langle \Phi(.)Z,\Phi(.)Z\rangle + (a_6 - \frac{1}{2}\sqrt{n}a_6\tau)\|Y\|^2 + a_4\|Z\|^2$$
$$+\{\varepsilon - (\sqrt{n}a_5 + \frac{1}{2}\sqrt{n}a_6)\tau\}\|Z\|^2$$
$$\geq \|U + 2^{-1}\Phi(.)Z\|^2 - \frac{1}{4}\langle \Phi(.)Z,\Phi(.)Z\rangle$$
$$\geq (a_6 - \frac{1}{2}\sqrt{n}a_6\tau)\|Y\|^2 + (a_4 - \frac{1}{4}a_2^2)\|Z\|^2 + \{\varepsilon - (\sqrt{n}a_5 + \frac{1}{2}\sqrt{n}a_6)\tau\}\|Z\|^2$$
$$\geq (a_6 - \frac{1}{2}\sqrt{n}a_6\tau)\|Y\|^2 + \{(\varepsilon + \delta) - (\sqrt{n}a_5 + \frac{1}{2}\sqrt{n}a_6)\tau\}\|Z\|^2.$$

If $\tau < \min\left\{\frac{2}{\sqrt{n}}, \frac{2(\varepsilon+\delta)}{2\sqrt{n}a_5 + \sqrt{n}a_6}\right\}$, then we have for some positive constants r_1 and r_2 that

$$\frac{d}{dt}V(.) \geq r_1\|Y\|^2 + r_2\|Z\|^2 \geq 0,$$

which verifies the property (P_2) of Krasovskiĭ (1963). On the other hand, it follows that

$$\frac{d}{dt}V(.)=0 \Leftrightarrow Y=\dot{X},\ Z=\dot{Y}=0,\ \ W=\dot{Z}=0,\ \ U=\dot{W}=0,\ T=\dot{U}=0 \ \text{ for all } t\geq 0.$$

Hence,

$$X=\xi,\ \ Y=Z=W=U=T=0.$$

Substituting these estimates in the system (3), we get that $H(\xi)=0,$ which necessarily implies that $\xi=0$ since $H(0)=0.$ Thus, we have

$$X=Y=Z=W=U=T=0 \text{ for all } t\geq 0.$$

Hence, the property (P_3) of Krasovskiĭ (1963) holds. The proof of the theorem is complete.

Example.

For case $n=2$ in equation (2), in particular, we choose

$$A=\begin{bmatrix}4 & 1\\ 1 & 4\end{bmatrix},$$

$$\Phi(.)=\begin{bmatrix}1+\dfrac{1}{1+x_1^2+...+t_1^2} & 0\\ 0 & 1+\dfrac{1}{1+x_2^2+...+t_2^2}\end{bmatrix},$$

$$F(.)=\begin{bmatrix}6+x_1^2+...+t_1^2 & 0\\ 0 & 6+x_2^2+...+t_2^2\end{bmatrix},$$

$$G(Y(t-\tau))=\begin{bmatrix}6y_1(t-\tau)\\ 6y_2(t-\tau)\end{bmatrix}$$

and

$$H(X(t-\tau))=\begin{bmatrix}-9x_1(t-\tau)\\ -9x_2(t-\tau)\end{bmatrix}.$$

Hence, an easy calculation results in

$$\lambda_1(A)=3,\ \lambda_2(A)=5,$$

$$\lambda_1(\Phi(.)) = 1 + \frac{1}{1 + x_1^2 + \ldots + t_1^2},$$

$$\lambda_2(\Phi(.)) = 1 + \frac{1}{1 + x_2^2 + \ldots + t_2^2},$$

$$\lambda_1(F(.)) = 6 + x_1^2 + \ldots + t_1^2,$$

$$\lambda_2(F(.)) = 6 + x_2^2 + \ldots + t_2^2,$$

$$J_G(Y) = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

and

$$J_H(X) = \begin{bmatrix} -9 & 0 \\ 0 & -9 \end{bmatrix}$$

so that

$$\lambda_i(A) \geq 3 = a_1 > 0,$$

$$\lambda_i(\Phi(.)) \leq 2 = a_2,$$

$$\lambda_i(F(.)) \geq 6 = 5 + 1 = a_4 + \varepsilon,$$

$$\left|\lambda_i(J_G(Y))\right| \leq 6 = a_5,$$

$$\lambda_i(J_H(X)) = -9 = -a_6 < 0, \ \ (i = 1, 2),$$

and

$$a_4 - \frac{1}{4}a_2^2 = 4 \geq \delta.$$

Thus, if

$$\tau < \min\left\{\frac{2}{\sqrt{2}}, \frac{10}{12\sqrt{2} + 9\sqrt{2}}\right\},$$

then all the assumptions of the theorem hold.

3. Conclusion

A kind of functional vector differential equation of the sixth order with constant delay has been considered. The instability of zero solution of this equation has been discussed by using the Lyapunov- Krasovskiĭ functional approach. The obtained result improves a well-known result in the literature and makes a contribution to the subject.

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