Difference Cordial Labeling of Graphs Obtained from Triangular Snakes

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Abstract

In this paper, we investigate the difference cordial labeling behavior of corona of triangular snake with the graphs of order one and order two and also corona of alternative triangular snake with the graphs of order one and order two.

Keywords: Corona; triangular snake; complete graph

MSC 2010 No.: 05C78; 05C38

1. Introduction:

Throughout this paper we have considered only simple and undirected graph. Let \( G = (V,E) \) be a \((p,q)\) graph. The cardinality of \( V \) is called the order of \( G \) and the cardinality of \( E \) is called the size of \( G \). The corona of the graph \( G \) with the graph \( H \), \( G \odot H \) is the graph obtained by taking one copy of \( G \) and \( p \) copies of \( H \) and joining the \( i^{th} \) vertex of \( G \) with an edge to every vertex in the \( i^{th} \) copy of \( H \). Graph labeling are used in several areas like communication network, radar, astronomy, database management, see Gallian (2011). Rosa (1967) introduced graceful labeling of graphs which was the foundation of the graph labeling. Consequently Graham (1980)
introduced harmonious labeling, Cahit (1987) initiated the concept of cordial labeling, and k-product cordial labeling by Ponraj et al. (2012). Recently Ponraj et al. (2012) introduced k- Total product cordial labeling of graphs. Ebrahim Salehi (2010) defined the notion of product cordial set. On analogous of this, the notion of difference cordial labeling has been introduced by Ponraj et al. (2013). Ponraj et al. (2013) studied the Difference cordial labeling behavior of quite a lot of graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs. In this paper we investigate the difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot K_2$ and $A(T_n) \odot K_2$, where $T_n$ and $K_n$ respectively denotes the triangular snake and complete graph. Let $x$ be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to $x$ and $\lceil x \rceil$ stands for the smallest integer greater than or equal to $x$. Terms and definitions not defined here are used in the sense of Harary (2001).

2. Difference Cordial Labeling

**Definition 2.1.**

Let $G$ be a $(p, q)$ graph. Let $f$ be a map from $V(G)$ to $\{1, 2, \ldots, p\}$. For each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called difference cordial labeling if $f$ is $1-1$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

The triangular snake $T_n$ is obtained from the path $P_n$ by replacing each edge of the path by a triangle $C_3$. Let $P_n$ be the path $u_1u_2 \ldots u_n$. Let

$$V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n - 1\}$$

and

$$E(T_n) = E(P_n) \cup \{u_iv_i, v_iu_{i+1} : 1 \leq i \leq n - 1\}.$$  

We now investigate the difference cordiality of corona of triangular snake $T_n$ with $K_1$, $2K_1$ and $K_2$.

**Theorem 2.2.**

$T_n \odot K_1$ is difference cordial.
Proof:

Clearly, $T_n \circ K_1$ has $4n - 2$ vertices and $5n - 4$ edges. Let

$$V(T_n \circ K_1) = V(T_n) \cup \{w_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq n - 1\}$$

and

$$E(T_n \circ K_1) = E(T_n) \cup \{u_iw_i : 1 \leq i \leq n\} \cup \{v_i z_i : 1 \leq i \leq n - 1\}.$$

Case 1. $n$ is even

Define $f: V(T_n \circ K_1) \to \{1, 2, 3, \ldots, 4n - 2\}$ as follows:

$$f(u_{2i-1}) = \left\lceil \frac{5n - 6}{2} \right\rceil + 2i, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(u_{2i}) = 5i - 2, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(v_{2i-1}) = 5i - 3, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(v_{2i}) = 5i - 1, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(w_{2i-1}) = \left\lceil \frac{5n - 4}{2} \right\rceil + 2i, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(w_{2i}) = \left\lceil \frac{7n - 4}{2} \right\rceil + i, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(z_{2i-1}) = 5i - 4, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f(z_{2i}) = 5i. \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor.$$

$$f(u_{n-1}) = \frac{5n-8}{2}, \quad f(u_n) = \frac{7n-6}{2},$$

$$f(w_{n-1}) = 4n - 2, \quad f(w_n) = \frac{7n - 4}{2},$$

$$f(v_{n-1}) = \frac{5n - 6}{2} \quad \text{and} \quad f(z_{n-1}) = \frac{5n - 4}{2}.$$
Case 2. $n$ is odd

Label the vertices $u_i, v_i, w_i$ and $z_i$ ($1 \leq i \leq n - 2$) as in case (i). Now, define,

\[
\begin{align*}
    f(u_{n-1}) &= \frac{5n-9}{2}, & f(u_n) &= \frac{7n-5}{2}, \\
    f(w_{n-1}) &= 4n - 2, & f(w_n) &= \frac{7n-3}{2}, \\
    f(v_{n-1}) &= \frac{5n-7}{2} \quad \text{and} \quad f(z_{n-1}) &= \frac{5n-5}{2}.
\end{align*}
\]

Table 1 shows that $f$ is a difference cordial labeling.

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 \pmod{2}$</td>
<td>$5n - 4$</td>
<td>$5n - 4$</td>
</tr>
<tr>
<td>$n \equiv 1 \pmod{2}$</td>
<td>$\frac{5n - 3}{2}$</td>
<td>$\frac{5n - 5}{2}$</td>
</tr>
</tbody>
</table>

Example. A difference cordial labeling of $T_4 \odot K_1$ is given in Figure 1.

![Figure 1. $T_4 \odot K_1$](image)

Theorem 2.3.

$T_n \odot 2K_1$ is difference cordial.

Proof:

Clearly, the order and size of $T_n \odot 2K_1$ are $6n - 3$ and $7n - 5$, respectively. Let
Define an injective map from the vertices of $T_n \odot 2K_1$ to the set $\left\{1, 2, 3, \ldots, 6n-3\right\}$ as follows:

$$f(u_i) = 3i - 1, \quad 1 \leq i \leq n,$$

$$f(w_i) = 3i - 2, \quad 1 \leq i \leq n,$$

$$f(w'_i) = 3i - 1, \quad 1 \leq i \leq n.$$

$$f(z_i) = 3n + 3i - 2, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f\left(z_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3 \left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 1, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor,$$

$$f(z'_i) = 3n + 3i, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f\left(z'_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3 \left\lfloor \frac{n-2}{2} \right\rfloor + 3i, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor,$$

$$f(v_i) = 3n + 3i - 1, \quad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$

$$f\left(v_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3 \left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 2, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor.$$
\[ V(T_n \odot K_2) = V(T_n) \cup \{w_i, w'_i : 1 \leq i \leq n\} \cup \{z_i, z'_i : 1 \leq i \leq n - 1\} \]

and

\[ E(T_n \odot K_2) = E(T_n) \cup \{u_iw_i, u_iw'_i, w_iw'_i : 1 \leq i \leq n\} \cup \{v_i z_i, v'_i z'_i, z_i z'_i : 1 \leq i \leq n - 1\}. \]

**Case 1.** \( n \) is even.

Define an injective map from the vertices of \( T_n \odot K_2 \) to the set \( \{1, 2, 3, \ldots, 6n - 3\} \) as follows:

\[
\begin{align*}
  f(u_{2i-1}) &= 6i - 3, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor, \\
  f(u_{2i}) &= 6i - 2, \quad 1 \leq i \leq \left\lfloor \frac{n - 2}{2} \right\rfloor, \\
  f(w_{2i-1}) &= 6i - 4, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor, \\
  f(w_{2i}) &= 6i, \quad 1 \leq i \leq \left\lfloor \frac{n - 2}{2} \right\rfloor, \\
  f(w'_{2i-1}) &= 6i - 5, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor, \\
  f(w'_{2i}) &= 6i - 1, \quad 1 \leq i \leq \left\lfloor \frac{n - 2}{2} \right\rfloor, \\
  f(v_i) &= 3n + 3i - 3, \quad 1 \leq i \leq n - 1, \\
  f(z_i) &= 3n + 3i - 1, \quad 1 \leq i \leq n - 1, \\
  f(z'_i) &= 3n + 3i - 2, \quad 1 \leq i \leq n - 1, \\
  f(u_n) &= 3n - 2, \quad f(w_n) = 6n - 3 \quad \text{and} \quad f(w'_n) = 3n - 1.
\]

**Case 2.** \( n \) is odd

Label the vertices \( u_i, w'_i \ (1 \leq i \leq n) \) and \( w_i \ (1 \leq i \leq n - 1) \) as in case 1. Define,

\[
\begin{align*}
  f(v_i) &= 3n + 3i - 2, \quad 1 \leq i \leq n - 1, \\
  f(z_i) &= 3n + 3i, \quad 1 \leq i \leq n - 1, \\
  f(z'_i) &= 3n + 3i - 1, \quad 1 \leq i \leq n - 1, \\
  \text{and} \quad f(w_n) &= 3n.
\]
**Table 3.** The edge conditions of difference cordial labeling of $T_n \odot K_2$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 (\text{mod } 2)$</td>
<td>$\frac{9n-6}{2}$</td>
<td>$\frac{9n-6}{2}$</td>
</tr>
<tr>
<td>$n \equiv 1 (\text{mod } 2)$</td>
<td>$\frac{9n-7}{2}$</td>
<td>$\frac{9n-5}{2}$</td>
</tr>
</tbody>
</table>

**Example.**

The graph $T_5 \odot K_2$ with a difference cordial labeling is shown in figure 2.

![Figure 2. $T_5 \odot K_2$](image)

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2\ldots u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to new vertex $v_i$. That is, every alternate edge of a path is replaced by $C_3$.

**Theorem 2.5.**

$A(T_n) \odot K_1$ is difference cordial.

**Proof:**

**Case 1.**

Let the first triangle start from $u_1$ and the last triangle ends with $u_n$. Here, $n$ is even. Let

$$V(A(T_n) \odot K_1) = V(A(T_n)) \cup \{x_i: 1 \leq i \leq n\} \cup \{w_i: 1 \leq i \leq \frac{n}{2}\}$$

and

$$E(A(T_n) \odot K_1) = E(A(T_n)) \cup \{u_ix_i: 1 \leq i \leq n\} \cup \{v_iw_i: 1 \leq i \leq \frac{n}{2}\}.$$ 

In this case, the order and size of $A(T_n) \odot K_1$ are $3n$ and $\frac{7n-2}{2}$, respectively. Define a map $f: V(A(T_n) \odot K_1) \rightarrow \{1, 2, \ldots, 3n\}$ as follows:
Table 4. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 ,(mod,4)$</td>
<td>$\frac{7n-4}{4}$</td>
<td>$\frac{7n}{4}$</td>
</tr>
<tr>
<td>$n \equiv 2 ,(mod,4)$</td>
<td>$\frac{7n-2}{4}$</td>
<td>$\frac{7n-2}{4}$</td>
</tr>
</tbody>
</table>

Case 2.

Let the first triangle be starts from $u_2$ and the last triangle ends with $u_{n-1}$. Here, also $n$ is even. In this case, the order and size of $A(T_n) \odot K_1$ are $3n-2$ and $\frac{7n-8}{2}$, respectively. Label the vertices $v_i, w_i \ (1 \leq i \leq \frac{n-2}{2})$ and $u_{2i}, x_{2i} \ (1 \leq i \leq \frac{n}{2})$ and $u_{2i-1}, x_{2i-1} \ (1 \leq i \leq \frac{n-2}{4})$ as in case 1 and define,

\[
\begin{align*}
    f(v_i) &= 2n + 2i - 1, \quad 1 \leq i \leq \frac{n}{2}, \\
    f(w_i) &= 2n + 2i, \quad 1 \leq i \leq \frac{n}{2}, \\
    f(x_i) &= 4i, \quad 1 \leq i \leq \frac{n}{2}, \\
    f(u_{2i}) &= 4i - 1, \quad 1 \leq i \leq \frac{n}{2}, \\
    f(u_{2i-1}) &= 4i - 2, \quad 1 \leq i \leq \frac{n}{4}, \\
    f(x_{2i-1}) &= 4i - 3, \quad 1 \leq i \leq \frac{n}{4}, \\
    f\left(u_2\left|\frac{n}{4}\right|_{-1+2i}\right) &= 4\left|\frac{n}{4}\right| + 4i - 3, \quad 1 \leq i \leq \frac{n}{4}, \\
    f\left(x_2\left|\frac{n}{4}\right|_{-1+2i}\right) &= 4\left|\frac{n}{4}\right| + 4i - 2, \quad 1 \leq i \leq \frac{n}{4}.
\end{align*}
\]
Table 5. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
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<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 \pmod{4}$</td>
<td>$7n - 8$</td>
<td>$7n - 8$</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{4}$</td>
<td>$\frac{4}{4}$</td>
</tr>
<tr>
<td>$n \equiv 2 \pmod{4}$</td>
<td>$7n - 10$</td>
<td>$7n - 6$</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{4}$</td>
<td>$\frac{4}{4}$</td>
</tr>
</tbody>
</table>

Case 3.

Let the first triangle be starts from $u_2$ and the last triangle ends with $u_n$. Here, $n$ is odd. In this case, the order and size of $A(T_n) \odot K_1$ are $3n - 1$ and $\frac{7n - 5}{2}$, respectively. Label the vertices $v_i, w_i \ (1 \leq i \leq \frac{n-1}{2})$ and $u_{2i}, x_{2i} \ (1 \leq i \leq \frac{n-1}{2})$ and $u_{2i-1}, x_{2i-1} \ (1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor)$ as in case (i) and define,

$$f\left(u_{2\lfloor \frac{n-1}{4}\rfloor-1+2i}\right) = 4\left\lfloor \frac{n-1}{4} \right\rfloor + 4i - 3, \quad 1 \leq i \leq \left\lfloor \frac{n+1}{4} \right\rfloor + 1,$$

$$f\left(x_{2\lfloor \frac{n-1}{4}\rfloor-1+2i}\right) = 4\left\lfloor \frac{n-1}{4} \right\rfloor + 4i - 2, \quad 1 \leq i \leq \left\lfloor \frac{n+1}{4} \right\rfloor + 1.$$

Table 6. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 1 \pmod{4}$</td>
<td>$7n - 7$</td>
<td>$7n - 3$</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{4}$</td>
<td>$\frac{4}{4}$</td>
</tr>
<tr>
<td>$n \equiv 3 \pmod{4}$</td>
<td>$7n - 5$</td>
<td>$7n - 5$</td>
</tr>
<tr>
<td></td>
<td>$\frac{4}{4}$</td>
<td>$\frac{4}{4}$</td>
</tr>
</tbody>
</table>

Theorem 2.6.

$A(T_n) \odot 2K_1$ is difference cordial.

Proof:

Case 1.

Let the first triangle be starts from $u_1$ and the last triangle ends with $u_n$. Here, $n$ is even. Let

$$V(A(T_n) \odot 2K_1) = V(A(T_n)) \cup \{x_i, x'_i : 1 \leq i \leq n\} \cup \{w_i, w'_i : 1 \leq i \leq \frac{n}{2}\}$$

and
\[ E(A(T_n) \odot 2K_1) = E(A(T_n)) \cup \{u_ix_i, u_ix'_i : 1 \leq i \leq n\} \cup \{v_iw_i, v_iw'_i : 1 \leq i \leq \frac{n}{2}\}. \]

In this case, the order and size of \(A(T_n) \odot 2K_1\) are \(\frac{9n}{2}\) and \(5n - 1\), respectively. Define a map \(f: V(A(T_n) \odot 2K_1) \to \left\{1, 2, \ldots, \frac{9n}{2}\right\}\) by

\[
\begin{align*}
  f(u_i) &= 3i - 1, \quad 1 \leq i \leq n, \\
  f(x_i) &= 3i - 2, \quad 1 \leq i \leq n, \\
  f(x'_i) &= 3i, \quad 1 \leq i \leq n, \\
  f(v_i) &= 3n + 3i - 2, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(w_i) &= 3n + 3i - 1, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(w'_i) &= 3n + 3i, \quad 1 \leq i \leq \frac{n}{2}.
\end{align*}
\]

Since \(e_f(1) = \frac{5n}{2}\) and \(e_f(0) = \frac{5n-2}{2}\), \(f\) is a difference cordial labeling of \(A(T_n) \odot 2K_1\).

**Case 2.**

Let the first triangle be starts from \(u_2\) and the last triangle ends with \(u_{n-1}\). Here \(n\) is even. In this case, the order and size of \(A(T_n) \odot 2K_1\) are \(\frac{9n-6}{2}\) and \(5n - 5\), respectively. Define a one-one map \(f\) from the vertices of \(A(T_n) \odot 2K_1\) to the set \(\left\{1, 2, \ldots, \frac{9n-6}{2}\right\}\) as follows:

\[
\begin{align*}
  f(u_{2i-1}) &= 4i - 2, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(u_{2i}) &= 4i - 1, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(x_{2i-1}) &= 4i - 3, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(x_{2i}) &= 4i, \quad 1 \leq i \leq \frac{n}{2}, \\
  f(x'_i) &= \frac{7n - 6}{2} + i, \quad 1 \leq i \leq n, \\
  f(v_i) &= 2n + 3i - 1, \quad 1 \leq i \leq \frac{n-2}{2}, \\
  f(w_i) &= 2n + 3i - 2, \quad 1 \leq i \leq \frac{n-2}{2}, \\
  f(w'_i) &= 2n + 3i, \quad 1 \leq i \leq \frac{n-2}{2}.
\end{align*}
\]
Since $e_f(1) = \frac{5n-4}{2}$ and $e_f(0) = \frac{5n-6}{2}$, $f$ is a difference cordial labeling of $A(T_n) \odot 2K_1$.

Case 3.

Let the first triangle be starts from $u_2$ and the last triangle ends with $u_n$. Here, $n$ is odd. In this case, the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n-3}{2}$ and $5n - 3$, respectively. Label the vertices $u_{2i-1}, x_{2i-1}, u_{2i}$ and $x_{2i}$ $(1 \leq i \leq \frac{n-1}{2})$ as in Case 2 and define $f(u_n) = 2n - 1, f(x_n) = 2n$, $f(x_n') = 2n + 1$,

\[
\begin{align*}
    f(v_i) &= 2n + 3i, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(w_i) &= 2n + 3i - 1, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(w_i') &= 2n + 3i + 1, \quad 1 \leq i \leq \frac{n-1}{2}.
\end{align*}
\]

Since $e_f(1) = e_f(0) = \frac{5n-3}{2}$, $f$ is a difference cordial labeling of $A(T_n) \odot 2K_1$. □

Theorem 2.7.

$A(T_n) \odot K_2$ is difference cordial.

Proof:

Case 1.

Let the first triangle be starts from $u_1$ and the last triangle ends with $u_n$. In this case $n$ is even. Let

\[
V(A(T_n) \odot K_1) = V(A(T_n)) \cup \{x_i,x'_i: 1 \leq i \leq n\} \cup \{w_i,w'_i: 1 \leq i \leq \frac{n}{2}\}
\]

and

\[
E(A(T_n) \odot 2K_1) = E(A(T_n)) \cup \{u_ix_i,u_ix'_i,x_ix'_i: 1 \leq i \leq n\} \cup \{v_iw_i,v_iw'_i,w_iw'_i: 1 \leq i \leq \frac{n}{2}\}.
\]

In this case, the order and size of $A(T_n) \odot K_2$ are $\frac{9n}{2}$ and $\frac{13n-2}{2}$, respectively. Define an injective map $f$ from the vertices of $A(T_n) \odot K_2$ to the set $\{1,2,\ldots,\frac{9n}{2}\}$ as follows:
\[ f(v_i) = 3n + 3i - 2, \quad 1 \leq i \leq \frac{n}{2}, \]
\[ f(w_i) = 3n + 3i - 1, \quad 1 \leq i \leq \frac{n}{2}, \]
\[ f(w'_i) = 3n + 3i, \quad 1 \leq i \leq \frac{n}{2}, \]
\[ f(u_{2i}) = 6i - 2, \quad 1 \leq i \leq \frac{n}{2}, \]
\[ f(u_{2i-1}) = 6i - 3, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f(x_{2i-1}) = 6i - 4, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f(x_{2i}) = 6i, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f(x'_{2i-1}) = 6i - 5, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f(x'_{2i}) = 6i - 1, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f \left( x_{\frac{n+3}{4} - 1 + 2i} \right) = 6 \lfloor \frac{n}{4} \rfloor + 6i - 5, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f \left( x_{\frac{n-2}{4} + 2i} \right) = 6 \lfloor \frac{n}{4} \rfloor + 6i - 1, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f \left( x'_{\frac{n-2}{4} - 1 + 2i} \right) = 6 \lfloor \frac{n}{4} \rfloor + 6i - 3, \quad 1 \leq i \leq \frac{n}{4}, \]
\[ f \left( x'_{\frac{n-2}{4} + 2i} \right) = 6 \lfloor \frac{n}{4} \rfloor + 6i, \quad 1 \leq i \leq \frac{n}{4}. \]

Table 7. The conditions of difference cordial labeling of \( A(T_n) \odot K_2 \)

<table>
<thead>
<tr>
<th>Nature of n ( \equiv 0 \pmod{4} )</th>
<th>( e_f(0) )</th>
<th>( e_f(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \equiv 0 \pmod{4} )</td>
<td>( 13n - 4 )</td>
<td>( 13n )</td>
</tr>
<tr>
<td>( n \equiv 2 \pmod{4} )</td>
<td>( 13n - 2 )</td>
<td>( 13n - 2 )</td>
</tr>
</tbody>
</table>

Case 2.

Let the first triangle be starts from \( u_2 \) and the last triangle ends with \( u_{n-1} \). Here, \( n \) is even. In this case, the order and size of \( A(T_n) \odot K_2 \) are \( \frac{9n-6}{2} \) and \( \frac{13n-12}{2} \), respectively. Label the vertices \( v_i, w'_i, w_i \ (1 \leq i \leq \frac{n-2}{2}) \), \( u_{2i} \ (1 \leq i \leq \frac{n}{2}) \) and \( u_{2i-1}, x_{2i-1}, x'_{2i-1}, x_{2i}, x'_{2i} \ (1 \leq i \leq \frac{n-2}{4}) \) as in case 1 and define
Table 8. The conditions of difference cordial labeling of $A(T_n) \odot K_2$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 0 \pmod{4}$</td>
<td>$13n - 12\frac{4}{4}$</td>
<td>$13n - 12\frac{4}{4}$</td>
</tr>
<tr>
<td>$n \equiv 2 \pmod{4}$</td>
<td>$13n - 14\frac{4}{4}$</td>
<td>$13n - 10\frac{4}{4}$</td>
</tr>
</tbody>
</table>

Case 3.

Let the first triangle be starts from $u_2$ and the last triangle ends with $u_n$. Here, $n$ is odd. In this case, the order and size of $A(T_n) \odot K_2$ are $\frac{9n-3}{2}$ and $\frac{13n-7}{2}$ respectively. Label the vertices $v_i, w_i, w_i, u_{2i} \left(1 \leq i \leq \frac{n-1}{2}\right)$ and $u_{2i-1}, x_{2i-1}, x'_{2i-1}, x_{2i}, x'_{2i} \left(1 \leq i \leq \frac{n-1}{4}\right)$ as in case (i) and define

- $f \left( u_{2\lceil\frac{n-2}{4}\rceil-1+2i} \right) = 6 \left\lceil \frac{n-2}{4} \right\rceil + 6i - 5, \quad 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil,$
- $f \left( x_{2\lceil\frac{n-2}{4}\rceil-1+2i} \right) = 6 \left\lceil \frac{n-2}{4} \right\rceil + 6i - 4, \quad 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil,$
- $f \left( x'_{2\lceil\frac{n-2}{4}\rceil-1+2i} \right) = 6 \left\lceil \frac{n-2}{4} \right\rceil + 6i - 3, \quad 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil,$
- $f \left( x_{2\lceil\frac{n-2}{4}\rceil+2i} \right) = 6 \left\lceil \frac{n-2}{4} \right\rceil + 6i - 1, \quad 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil,$
- $f \left( x'_{2\lceil\frac{n-2}{4}\rceil+2i} \right) = 6 \left\lceil \frac{n-2}{4} \right\rceil + 6i, \quad 1 \leq i \leq \left\lceil \frac{n+2}{4} \right\rceil.$
Table 9. The conditions of difference cordial labeling of $A(T_n) \odot K_2$

<table>
<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \equiv 1 (mod\ 4)$</td>
<td>$\frac{13n - 9}{4}$</td>
<td>$\frac{13n - 5}{4}$</td>
</tr>
<tr>
<td>$n \equiv 3 (mod\ 4)$</td>
<td>$\frac{13n - 7}{4}$</td>
<td>$\frac{13n - 7}{4}$</td>
</tr>
</tbody>
</table>

Example.

A difference cordial labeling of $A(T_4) \odot K_2$ with the first triangle starts from $u_1$ and the last triangle ends with $u_n$ is given in Figure 3.

![Figure 3. $A(T_4) \odot K_2$](image)

3. Conclusions

In this paper we have studied about difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot K_2$ and $A(T_n) \odot K_2$. Investigation of difference cordiality of join, union and composition of two graphs are the open problems for future research.

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REFERENCES


