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Difference Cordial Labeling of Graphs Obtained from Triangular Snakes

R. Ponraj and S. Sathish Narayanan
Department of Mathematics
Sri Paramakalyani College
Alwarkurichi 627412, India
ponrajmaths@gmail.com; sathishrvss@gmail.com

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Abstract

In this paper, we investigate the difference cordial labeling behavior of corona of triangular snake with the graphs of order one and order two and also corona of alternative triangular snake with the graphs of order one and order two.

Keywords: Corona; triangular snake; complete graph

MSC 2010 No.: 05C78; 05C38

1. Introduction:

Throughout this paper we have considered only simple and undirected graph. Let $G = (V, E)$ be a (p, q) graph. The cardinality of V is called the order of G and the cardinality of E is called the size of G. The corona of the graph G with the graph H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. Graph labeling are used in several areas like communication network, radar, astronomy, database management, see Gallian (2011). Rosa (1967) introduced graceful labeling of graphs which was the foundation of the graph labeling. Consequently Graham (1980) introduced harmonious labeling, Cahit (1987) initiated the concept of cordial labeling, and k-product cordial labeling by Ponraj et al. (2012). Recently Ponraj et al. (2012) introduced k- Total product cordial labeling of graphs. Ebrahim Salehi (2010) defined the notion of product cordial set. On analogous of this, the notion of difference cordial labeling has been introduced by Ponraj et al. (2013). Ponraj et al. (2013) studied the Difference cordial labeling behavior of quite a lot of graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs . In this paper we investigate the difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot K_2$ and $A(T_n) \odot K_2$, where T_n and K_n respectively denotes the triangular snake and complete graph. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary (2001).

2. Difference Cordial Labeling

Definition 2.1.

Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1,2, \dots, p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. Let P_n be the path $u_1 u_2 \dots u_n$. Let

$$V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$$

and

$$E(T_n) = E(P_n) \cup \{u_i v_i , v_i u_{i+1} : 1 \leq i \leq n-1\}.$$

We now investigate the difference cordiality of corona of triangular snake T_n with K_1, $2K_1$ and K_2.

Theorem 2.2.

$T_n \odot K_1$ is difference cordial.

Proof:

Clearly, $T_n \odot K_1$ has $4n-2$ vertices and $5n-4$ edges. Let

$$V(T_n \odot K_1) = V(T_n) \cup \{w_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq n-1\}$$

and

$$E(T_n \odot K_1) = E(T_n) \cup \{u_i w_i : 1 \leq i \leq n\} \cup \{v_i z_i : 1 \leq i \leq n-1\}.$$

Case 1. n is even

Define $f: V(T_n \odot K_1) \to \{1,2,3,\dots,4n-2\}$ as follows:

$$\begin{aligned}
f(u_{2i-1}) &= \left\lfloor \frac{5n-6}{2} \right\rfloor + 2i, && 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil,\\
f(u_{2i}) &= 5i-2\,, && 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,\\
f(v_{2i-1}) &= 5i-3, && 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil,\\
f(v_{2i}) &= 5i-1\,, && 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,\\
f(w_{2i-1}) &= \left\lfloor \frac{5n-4}{2} \right\rfloor + 2i\,, && 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil,\\
f(w_{2i}) &= \left\lceil \frac{7n-4}{2} \right\rceil + i\,, && 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,\\
f(z_{2i-1}) &= 5i-4\,, && 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil,\\
f(z_{2i}) &= 5i. && 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor.
\end{aligned}$$

$$f(u_{n-1}) = \frac{5n-8}{2}, \qquad f(u_n) = \frac{7n-6}{2},$$

$$f(w_{n-1}) = 4n-2, \qquad f(w_n) = \frac{7n-4}{2},$$

$$f(v_{n-1}) = \frac{5n-6}{2} \text{ and } \qquad f(z_{n-1}) = \frac{5n-4}{2}.$$

Case 2. n is odd

Label the vertices u_i, v_i, w_i and z_i $(1 \leq i \leq n-2)$ as in case (i). Now, define,

$$f(u_{n-1}) = \frac{5n-9}{2}, \qquad f(u_n) = \frac{7n-5}{2},$$
$$f(w_{n-1}) = 4n-2, \qquad f(w_n) = \frac{7n-3}{2},$$
$$f(v_{n-1}) = \frac{5n-7}{2} \quad \text{and} \quad f(z_{n-1}) = \frac{5n-5}{2}.$$

Table 1 shows that f is a difference cordial labeling.

Table 1. The edge conditions of difference cordial labeling of $T_n \odot K_1$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 2)$	$\frac{5n-4}{2}$	$\frac{5n-4}{2}$
$n \equiv 1(mod\ 2)$	$\frac{5n-3}{2}$	$\frac{5n-5}{2}$

Example. A difference cordial labeling of $T_4 \odot K_1$ is given in Figure 1.

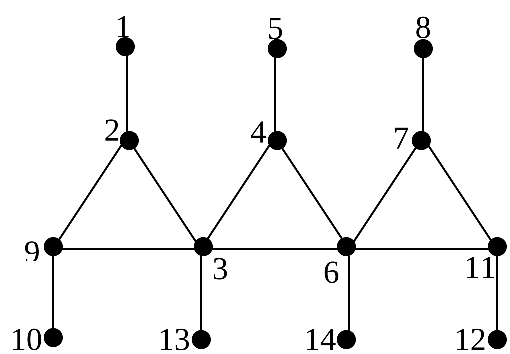

Figure 1. $T_4 \odot K_1$

Theorem 2.3.

$T_n \odot 2K_1$ is difference cordial.

Proof:

Clearly, the order and size of $T_n \odot 2K_1$ are $6n-3$ and $7n-5$, respectively. Let

$$V(T_n \odot 2K_1) = V(T_n) \cup \{w_i, w_i' : 1 \leq i \leq n\} \cup \{z_i, z_i' : 1 \leq i \leq n-1\}$$

and

$$E(T_n \odot 2K_1) = E(T_n) \cup \{u_i w_i, u_i w_i' : 1 \leq i \leq n\} \cup \{v_i z_i, v_i z_i' : 1 \leq i \leq n-1\}.$$

Define an injective map from the vertices of $T_n \odot 2K_1$ to the set $\{1,2,3, \ldots, 6n-3\}$ as follows:

$$f(u_i) = 3i - 1, \qquad 1 \leq i \leq n,$$
$$f(w_i) = 3i - 2, \qquad 1 \leq i \leq n,$$
$$f(w_i') = 3i - 1, \qquad 1 \leq i \leq n.$$

$$f(z_i) = 3n + 3i - 2, \qquad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$
$$f\left(z_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 1, \qquad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil,$$
$$f(z_i') = 3n + 3i, \qquad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$
$$f\left(z'_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i, \qquad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil,$$
$$f(v_i) = 3n + 3i - 1, \qquad 1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor,$$
$$f\left(v_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 3n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 2, \qquad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil.$$

Table 2. The conditions of difference cordial labeling of $T_n \odot 2K_1$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 2)$	$\frac{7n-6}{2}$	$\frac{7n-4}{2}$
$n \equiv 1(mod\ 2)$	$\frac{7n-5}{2}$	$\frac{7n-5}{2}$

Theorem 2.4.

$T_n \odot K_2$ is difference cordial.

Proof:

Clearly, the order and size of $T_n \odot K_2$ are $6n-3$ and $9n-6$, respectively. Let

$$V(T_n \odot K_2) = V(T_n) \cup \{w_i, w_i' : 1 \le i \le n\} \cup \{z_i, z_i' : 1 \le i \le n-1\}$$

and

$$E(T_n \odot K_2) = E(T_n) \cup \{u_i w_i, u_i w_i', w_i w_i' : 1 \le i \le n\} \cup \{v_i z_i, v_i z_i', z_i z_i' : 1 \le i \le n-1\}.$$

Case 1. n is even.

Define an injective map from the vertices of $T_n \odot K_2$ to the set $\{1,2,3,\dots,6n-3\}$ as follows:

$$f(u_{2i-1}) = 6i-3, \qquad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor,$$

$$f(u_{2i}) = 6i-2, \qquad 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil,$$

$$f(w_{2i-1}) = 6i-4, \qquad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor,$$

$$f(w_{2i}) = 6i, \qquad 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil,$$

$$f(w'_{2i-1}) = 6i-5, \qquad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor,$$

$$f(w'_{2i}) = 6i-1, \qquad 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil,$$

$$f(v_i) = 3n+3i-3, \qquad 1 \le i \le n-1,$$

$$f(z_i) = 3n+3i-1, \qquad 1 \le i \le n-1,$$

$$f(z_i') = 3n+3i-2, \qquad 1 \le i \le n-1,$$

$$f(u_n) = 3n-2, f(w_n) = 6n-3 \text{ and } f(w_n') = 3n-1.$$

Case 2. n is odd

Label the vertices u_i, w_i' $(1 \le i \le n)$ and w_i $(1 \le i \le n-1)$ as in case 1. Define,

$$f(v_i) = 3n+3i-2, \qquad 1 \le i \le n-1,$$

$$f(z_i) = 3n+3i, \qquad 1 \le i \le n-1,$$

$$f(z_i') = 3n+3i-1, \qquad 1 \le i \le n-1.$$

$$\text{and } f(w_n) = 3n.$$

Table 3. The edge conditions of difference cordial labeling of $T_n \odot K_2$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 2)$	$\frac{9n-6}{2}$	$\frac{9n-6}{2}$
$n \equiv 1(mod\ 2)$	$\frac{9n-7}{2}$	$\frac{9n-5}{2}$

Example.

The graph $T_5 \odot K_2$ with a difference cordial labeling is shown in figure 2.

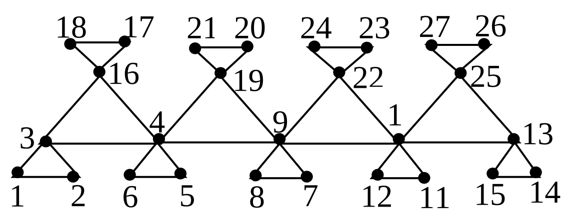

Figure 2. $T_5 \odot K_2$

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is, every alternate edge of a path is replaced by C_3.

Theorem 2.5.

$A(T_n) \odot K_1$ is difference cordial.

Proof:

Case 1.

Let the first triangle start from u_1 and the last triangle ends with u_n. Here, n is even. Let

$$V(A(T_n) \odot K_1) = V\big(A(T_n)\big) \cup \{x_i: 1 \le i \le n\} \cup \left\{w_i: 1 \le i \le \frac{n}{2}\right\}$$

and

$$E(A(T_n) \odot K_1) = E\big(A(T_n)\big) \cup \{u_ix_i: 1 \le i \le n\} \cup \left\{v_iw_i: 1 \le i \le \frac{n}{2}\right\}.$$

In this case, the order and size of $A(T_n) \odot K_1$ are $3n$ and $\frac{7n-2}{2}$, respectively. Define a map $f: V(A(T_n) \odot K_1) \to \{1,2,\dots,3n\}$ as follows:

$$
\begin{aligned}
f(v_i) &= 2n+2i-1, & 1 \le i \le \frac{n}{2},\\
f(w_i) &= 2n+2i\,, & 1 \le i \le \frac{n}{2},\\
f(x_i) &= 4i\ \ , & 1 \le i \le \frac{n}{2},\\
f(u_{2i}) &= 4i-1, & 1 \le i \le \frac{n}{2},\\
f(u_{2i-1}) &= 4i-2\,, & 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f(x_{2i-1}) &= 4i-3, & 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f\left(u_{2\left\lfloor\frac{n}{4}\right\rfloor-1+2i}\right) &= 4\left\lfloor\frac{n}{4}\right\rfloor+4i-3, & 1 \le i \le \left\lceil\frac{n}{4}\right\rceil,\\
f\left(x_{2\left\lfloor\frac{n}{4}\right\rfloor-1+2i}\right) &= 4\left\lfloor\frac{n}{4}\right\rfloor+4i-2, & 1 \le i \le \left\lceil\frac{n}{4}\right\rceil.
\end{aligned}
$$

Table 4. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 4)$	$\frac{7n-4}{4}$	$\frac{7n}{4}$
$n \equiv 2(mod\ 4)$	$\frac{7n-2}{4}$	$\frac{7n-2}{4}$

Case 2.

Let the first triangle be starts from u_2 and the last triangle ends with u_{n-1}. Here, also n is even. In this case, the order and size of $A(T_n) \odot K_1$ are $3n-2$ and $\frac{7n-8}{2}$, respectively. Label the vertices v_i, w_i $\left(1 \le i \le \frac{n-2}{2}\right)$ and u_{2i}, x_{2i} $\left(1 \le i \le \frac{n}{2}\right)$ and u_{2i-1}, x_{2i-1} $\left(1 \le i \le \left\lfloor\frac{n-2}{4}\right\rfloor\right)$ as in case 1 and define,

$$
\begin{aligned}
f\left(u_{2\left\lfloor\frac{n-2}{4}\right\rfloor-1+2i}\right) &= 4\left\lfloor\frac{n-2}{4}\right\rfloor+4i-3, & 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil,\\
f\left(x_{2\left\lfloor\frac{n-2}{4}\right\rfloor-1+2i}\right) &= 4\left\lfloor\frac{n-2}{4}\right\rfloor+4i-2, & 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil.
\end{aligned}
$$

Table 5. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 4)$	$\frac{7n-8}{4}$	$\frac{7n-8}{4}$
$n \equiv 2(mod\ 4)$	$\frac{7n-10}{4}$	$\frac{7n-6}{4}$

Case 3.

Let the first triangle be starts from u_2 and the last triangle ends with u_n. Here, n is odd. In this case, the order and size of $A(T_n) \odot K_1$ are $3n-1$ and $\frac{7n-5}{2}$, respectively. Label the vertices v_i, w_i $\left(1 \leq i \leq \frac{n-1}{2}\right)$ and u_{2i}, x_{2i} $\left(1 \leq i \leq \frac{n-1}{2}\right)$ and u_{2i-1}, x_{2i-1} $\left(1 \leq i \leq \left\lfloor\frac{n-1}{4}\right\rfloor\right)$ as in case (i) and define,

$$f\left(u_{2\left\lfloor\frac{n-1}{4}\right\rfloor-1+2i}\right) = 4\left\lfloor\frac{n-1}{4}\right\rfloor + 4i - 3, \qquad 1 \leq i \leq \left\lfloor\frac{n+1}{4}\right\rfloor + 1,$$
$$f\left(x_{2\left\lfloor\frac{n-1}{4}\right\rfloor-1+2i}\right) = 4\left\lfloor\frac{n-1}{4}\right\rfloor + 4i - 2, \qquad 1 \leq i \leq \left\lfloor\frac{n+1}{4}\right\rfloor + 1.$$

Table 6. The conditions of difference cordial labeling of $A(T_n) \odot K_1$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1(mod\ 4)$	$\frac{7n-7}{4}$	$\frac{7n-3}{4}$
$n \equiv 3(mod\ 4)$	$\frac{7n-5}{4}$	$\frac{7n-5}{4}$

□

Theorem 2.6.

$A(T_n) \odot 2K_1$ is difference cordial.

Proof:

Case 1.

Let the first triangle be starts from u_1 and the last triangle ends with u_n. Here, n is even. Let

$$V(A(T_n) \odot 2K_1) = V\big(A(T_n)\big) \cup \{x_i, x_i' : 1 \leq i \leq n\} \cup \left\{w_i, w_i' : 1 \leq i \leq \frac{n}{2}\right\}$$

and

$$E(A(T_n) \odot 2K_1) = E\big(A(T_n)\big) \cup \{u_i x_i, u_i x_i' : 1 \leq i \leq n\} \cup \left\{v_i w_i, v_i w_i' : 1 \leq i \leq \frac{n}{2}\right\}.$$

In this case, the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n}{2}$ and $5n-1$, respectively. Define a map $f: V(A(T_n) \odot 2K_1) \to \left\{1,2,\dots,\frac{9n}{2}\right\}$ by

$$\begin{aligned}
f(u_i) &= 3i-1, && 1 \leq i \leq n,\\
f(x_i) &= 3i-2, && 1 \leq i \leq n,\\
f(x_i') &= 3i. && 1 \leq i \leq n,\\
f(v_i) &= 3n+3i-2. && 1 \leq i \leq \frac{n}{2},\\
f(w_i) &= 3n+3i-1, && 1 \leq i \leq \frac{n}{2},\\
f(w_i') &= 3n+3i, && 1 \leq i \leq \frac{n}{2}.
\end{aligned}$$

Since $e_f(1) = \frac{5n}{2}$ and $e_f(0) = \frac{5n-2}{2}$, f is a difference cordial labeling of $A(T_n) \odot 2K_1$.

Case 2.

Let the first triangle be starts from u_2 and the last triangle ends with u_{n-1}. Here n is even. In this case, the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n-6}{2}$ and $5n-5$, respectively. Define a one-one map f from the vertices of $A(T_n) \odot 2K_1$ to the set $\left\{1,2,\dots,\frac{9n-6}{2}\right\}$ as follows:

$$\begin{aligned}
f(u_{2i-1}) &= 4i-2, && 1 \leq i \leq \frac{n}{2},\\
f(u_{2i}) &= 4i-1\,, && 1 \leq i \leq \frac{n}{2},\\
f(x_{2i-1}) &= 4i-3, && 1 \leq i \leq \frac{n}{2},\\
f(x_{2i}) &= 4i, && 1 \leq i \leq \frac{n}{2},\\
f(x_i') &= \frac{7n-6}{2} + i, && 1 \leq i \leq n,\\
f(v_i) &= 2n+3i-1, && 1 \leq i \leq \frac{n-2}{2},\\
f(w_i) &= 2n+3i-2, && 1 \leq i \leq \frac{n-2}{2},\\
f(w_i') &= 2n+3i, && 1 \leq i \leq \frac{n-2}{2}.
\end{aligned}$$

Since $e_f(1) = \frac{5n-4}{2}$ and $e_f(0) = \frac{5n-6}{2}$, f is a difference cordial labeling of $A(T_n) \odot 2K_1$.

Case 3.

Let the first triangle be starts from u_2 and the last triangle ends with u_n. Here, n is odd. In this case, the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n-3}{2}$ and $5n-3$, respectively. Label the vertices $u_{2i-1}, x_{2i-1}, u_{2i}$ and x_{2i} $\left(1 \le i \le \frac{n-1}{2}\right)$ as in Case 2 and define $f(u_n) = 2n-1$, $f(x_n) = 2n$, $f(x'_n) = 2n+1$,

$$\begin{aligned} f(v_i) &= 2n + 3i, && 1 \le i \le \frac{n-1}{2}, \\ f(w_i) &= 2n + 3i - 1, && 1 \le i \le \frac{n-1}{2}, \\ f(w'_i) &= 2n + 3i + 1, && 1 \le i \le \frac{n-1}{2}. \end{aligned}$$

Since $e_f(1) = e_f(0) = \frac{5n-3}{2}$, f is a difference cordial labeling of $A(T_n) \odot 2K_1$. □

Theorem 2.7.

$A(T_n) \odot K_2$ is difference cordial.

Proof:

Case 1.

Let the first triangle be starts from u_1 and the last triangle ends with u_n. In this case n is even. Let

$$V(A(T_n) \odot K_1) = V\big(A(T_n)\big) \cup \{x_i, x'_i : 1 \le i \le n\} \cup \left\{w_i, w'_i : 1 \le i \le \frac{n}{2}\right\}$$

and

$$E(A(T_n) \odot 2K_1) = E\big(A(T_n)\big) \cup \{u_i x_i, u_i x'_i, x_i x'_i : 1 \le i \le n\} \cup \left\{v_i w_i, v_i w'_i, w_i w'_i : 1 \le i \le \frac{n}{2}\right\}.$$

In this case, the order and size of $A(T_n) \odot K_2$ are $\frac{9n}{2}$ and $\frac{13n-2}{2}$, respectively. Define an injective map f from the vertices of $A(T_n) \odot K_2$ to the set $\left\{1, 2, \ldots, \frac{9n}{2}\right\}$ as follows:

$$
\begin{aligned}
f(v_i) &= 3n+3i-2, && 1 \le i \le \frac{n}{2},\\
f(w_i) &= 3n+3i-1, && 1 \le i \le \frac{n}{2},\\
f(w_i') &= 3n+3i, && 1 \le i \le \frac{n}{2},\\
f(u_{2i}) &= 6i-2, && 1 \le i \le \frac{n}{2},\\
f(u_{2i-1}) &= 6i-3, && 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f(x_{2i-1}) &= 6i-4, && 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f(x_{2i}) &= 6i, && 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f(x_{2i-1}') &= 6i-5, && 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f(x_{2i}') &= 6i-1, && 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor,\\
f\left(u_{2\left\lfloor\frac{n}{4}\right\rfloor-1+2i}\right) &= 6\left\lfloor\frac{n}{4}\right\rfloor+6i-5, && 1 \le i \le \left\lceil\frac{n}{4}\right\rceil,\\
f\left(x_{2\left\lfloor\frac{n}{4}\right\rfloor-1+2i}\right) &= 6\left\lfloor\frac{n}{4}\right\rfloor+6i-4, && 1 \le i \le \left\lceil\frac{n}{4}\right\rceil,\\
f\left(x'_{2\left\lfloor\frac{n}{4}\right\rfloor-1+2i}\right) &= 6\left\lfloor\frac{n}{4}\right\rfloor+6i-3, && 1 \le i \le \left\lceil\frac{n}{4}\right\rceil,\\
f\left(x_{2\left\lfloor\frac{n}{4}\right\rfloor+2i}\right) &= 6\left\lfloor\frac{n}{4}\right\rfloor+6i-1, && 1 \le i \le \left\lceil\frac{n}{4}\right\rceil,\\
f\left(x'_{2\left\lfloor\frac{n}{4}\right\rfloor+2i}\right) &= 6\left\lfloor\frac{n}{4}\right\rfloor+6i, && 1 \le i \le \left\lceil\frac{n}{4}\right\rceil.
\end{aligned}
$$

Table 7. The conditions of difference cordial labeling of $A(T_n) \odot K_2$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 4)$	$\frac{13n-4}{4}$	$\frac{13n}{4}$
$n \equiv 2(mod\ 4)$	$\frac{13n-2}{4}$	$\frac{13n-2}{4}$

Case 2.

Let the first triangle be starts from u_2 and the last triangle ends with u_{n-1}. Here, n is even. In this case, the order and size of $A(T_n) \odot K_2$ are $\frac{9n-6}{2}$ and $\frac{13n-12}{2}$, respectively. Label the vertices v_i, w_i', w_i $\left(1 \le i \le \frac{n-2}{2}\right)$, u_{2i} $\left(1 \le i \le \frac{n}{2}\right)$ and u_{2i-1}, x_{2i-1}, x'_{2i-1}, x_{2i}, x'_{2i} $\left(1 \le i \le \left\lfloor\frac{n-2}{4}\right\rfloor\right)$ as in case 1 and define

$$f\left(u_{2\left\lfloor\frac{n-2}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-2}{4}\right\rfloor + 6i - 5, \qquad 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil,$$
$$f\left(x_{2\left\lfloor\frac{n-2}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-2}{4}\right\rfloor + 6i - 4, \qquad 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil,$$
$$f\left(x'_{2\left\lfloor\frac{n-2}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-2}{4}\right\rfloor + 6i - 3, \qquad 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil,$$
$$f\left(x_{2\left\lfloor\frac{n-2}{4}\right\rfloor+2i}\right) = 6\left\lfloor\frac{n-2}{4}\right\rfloor + 6i - 1, \qquad 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil,$$
$$f\left(x'_{2\left\lfloor\frac{n-2}{4}\right\rfloor+2i}\right) = 6\left\lfloor\frac{n-2}{4}\right\rfloor + 6i, \qquad 1 \le i \le \left\lceil\frac{n+2}{4}\right\rceil.$$

Table 8. The conditions of difference cordial labeling of $A(T_n) \odot K_2$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 4)$	$\frac{13n-12}{4}$	$\frac{13n-12}{4}$
$n \equiv 2(mod\ 4)$	$\frac{13n-14}{4}$	$\frac{13n-10}{4}$

Case 3.

Let the first triangle be starts from u_2 and the last triangle ends with u_n. Here, n is odd. In this case, the order and size of $A(T_n) \odot K_2$ are $\frac{9n-3}{2}$ and $\frac{13n-7}{2}$ respectively. Label the vertices $v_i, w'_i, w_i, u_{2i}\left(1 \le i \le \frac{n-1}{2}\right)$ and $u_{2i-1}, x_{2i-1}, x'_{2i-1}, x_{2i}, x'_{2i}$ $\left(1 \le i \le \left\lfloor\frac{n-1}{4}\right\rfloor\right)$ as in case (i) and define

$$f\left(u_{2\left\lfloor\frac{n-1}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-1}{4}\right\rfloor + 6i - 5, \qquad 1 \le i \le \left\lfloor\frac{n+1}{4}\right\rfloor + 1,$$
$$f\left(x_{2\left\lfloor\frac{n-1}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-1}{4}\right\rfloor + 6i - 4, \qquad 1 \le i \le \left\lfloor\frac{n+1}{4}\right\rfloor + 1,$$
$$f\left(x'_{2\left\lfloor\frac{n-1}{4}\right\rfloor-1+2i}\right) = 6\left\lfloor\frac{n-1}{4}\right\rfloor + 6i - 3, \qquad 1 \le i \le \left\lfloor\frac{n+1}{4}\right\rfloor + 1,$$
$$f\left(x_{2\left\lfloor\frac{n-1}{4}\right\rfloor+2i}\right) = 6\left\lfloor\frac{n-1}{4}\right\rfloor + 6i - 1, \qquad 1 \le i \le \left\lfloor\frac{n+1}{4}\right\rfloor,$$
$$f\left(x'_{2\left\lfloor\frac{n-1}{4}\right\rfloor+2i}\right) = 6\left\lfloor\frac{n-1}{4}\right\rfloor + 6i, \qquad 1 \le i \le \left\lfloor\frac{n+1}{4}\right\rfloor.$$

Table 9. The conditions of difference cordial labeling of $A(T_n) \odot K_2$

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1(mod\ 4)$	$\frac{13n-9}{4}$	$\frac{13n-5}{4}$
$n \equiv 3(mod\ 4)$	$\frac{13n-7}{4}$	$\frac{13n-7}{4}$

Example.

A difference cordial labeling of $A(T_4) \odot K_2$ with the first triangle starts from u_1 and the last triangle ends with u_n is given in Figure 3.

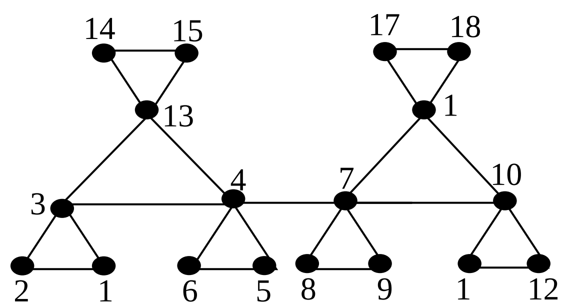

Figure 3. $A(T_4) \odot K_2$

3. Conclusions

In this paper we have studied about difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot K_2$ and $A(T_n) \odot K_2$. Investigation of difference cordiality of join, union and composition of two graphs are the open problems for future research.

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