A Ranking Method Based on Common Weights and Benchmark Point

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Abstract

The highest efficiency score 1 (100% efficiency) is regarded as a common benchmark for Decision Making Units (DMUs). This brings about the existence of more than one DMU with the highest score. Such a case normally occurs in all Data Envelopment Analysis (DEA) models and also in all the Common Set of Weights (CSWs) methods and it may lead to the lack of thorough ranking of DMUs. And ideal DMU based on its specific structure is a unit that no unit would do better than. Therefore, it can be utilized as a benchmark for other units. We are going to take advantage of this feature to introduce a linear programming problem that will produce CSWs. The proposed method assures that the efficiency of all the units is less than that of the benchmark unit. As a result, it provides a comprehensive ranking of DMUs. Moreover, the proposed method is also noteworthy regarding computation. A numerical example is suggested to clarify and explain the proposed method and compare it to two other CSWs methods. Finally, 33 universities in Iran were ranked and compared using the proposed method.

1. Introduction

Data Envelopment Analysis (DEA) is an acceptable tool for measuring the efficiency of homogeneous Decision Making Units (DMUs) using mathematical programming problems. DMUs are homogeneous systems that make use of several inputs to produce several outputs. For example, the universities in a country can be considered as DMUs in which the numbers of students, faculties or staff are considered as inputs while outputs may include the number of students who graduate or publish papers etc. The first research on DEA was conducted by Charnes et al. (1978). The proposed model called the CCR was applied to evaluate the program followed all over the U.S. Education system. In this approach, DMUs are divided into two groups: efficient and inefficient units. In the CCR model, efficient units have the efficiency score one (the highest score) and inefficient units have efficiency scores smaller than one. There is no difference between the performances of efficient units per score, even though it's unrealistic to assume them equal in actual practice for them. To deal with this issue, several approaches have been presented to distinguish among the efficient units. These are known as ranking methods.

The ranking methods are divided into two basic groups. In one group, there are methods that can only rank vertex efficient units, i.e. units that cannot be generated by using any combination of other decision making units. The main idea behind this method was presented by Andersen and Petersen (1993) known as super-efficiency method. The gist of this approach is to measure the efficiency of the evaluating unit, while it is excluded from the set of observed DMUs. The obtained super-efficiency of the evaluating unit is more than unity and in this situation vertex efficient units can be ranked. In the other group, all efficient DMUs can be ranked. They are divided into three basic subgroups: 1) cross-efficiency methods such as Doyle and Green's work (1994) in which the obtained multipliers or weights for a unit are used to evaluate the other DMUs, 2) Multiple Criteria Decision Making (MCDM) methods such as Li and Reeves' (1999) work in which a set of multipliers or weights are obtained to enable us to evaluate DMUs realistically, and 3) interval DEA methods such as what Wang and Yang (2007) did. Their work involved lower and upper bounds of interval efficiency provided for the units. A full review of ranking methods was presented by Adler et al. (2002).

Extension of the MCDM approach led to Common Set of Weights (CSWs) methods. Considering the literature on DEA, CSW methods have been used both for ranking efficient DMUs and for evaluating all units. In this approach, the efficiency of DMUs is simultaneously measured by a fiedx set of weights. The concept of CSWs was first introduced by Cook et al. (1990). Jahanshahloo et al. (2005) proposed a multiple objective linear fractional programming problem and presented a nonlinear program to determine the CSWs. Liu and Peng (2008) suggested a model for obtaining CSWs which is a Linear Programming Problem (LPP), and then provided a three-stage process to rank the DMUs. Chiang et al. (2011) introduced a linear model including a separation vector to obtain the CSWs.

Almost all CSWs approaches consider the number one as the highest common benchmark level for DMUs. In this situation, after calculating the efficiency of DMUs by CSWs, more than one DMU may have the efficiency score one. This means that the presented benchmark is attainable for more than one DMU, and so a full ranking of DMUs is not obtained. This is because the

CSWs used for ranking DMUs are also optimal in DEA models to evaluate more than one efficient DMU. Therefore we need a benchmark for efficient DMUs which is more difficult to attain than benchmark level one. The present paper deals with this issue and offers a new CSW method to overcome the above-mentioned difficulty. The main idea of the proposed method is to use a benchmark point instead of the benchmark level for deriving CSWs. In a geometrical interpretation, in Section 3, details of the suggested method are discussed.

This paper essentially follows these steps: Section 1 introduces DMU's and the difficulty with benchmarks. Section 2 briefly introduces the background of DEA. Section 3 presents our proposed method and states some facts about it. Numerical examples are given in Section 4, and Section 5 contains our remarks and conclusions.

2. DEA Background

Considering n DMUs for evaluating, each DMU has m inputs to produce s outputs. Let $x_{ij} > 0 \ (i = 1, ..., m)$ and $y_{rj} > 0 \ (r = 1, ..., s)$ be respectively the values of the ith input and the rth output of $DMU_j \ (j = 1, ..., n)$. Charnes et al. (1978) define the efficiency of $DMU_j \ (j = 1, ..., n)$ as a fraction of the weighted sum of its outputs to that of its inputs. Therefore, we have:

$$E_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \qquad j = 1, ..., n,$$

where $u_r \ (r = 1, ..., s)$ and $v_i \ (i = 1, ..., m)$ are the weights or multipliers of the rth output and the ith input, respectively. The relative efficiency of evaluating DMU_o is the optimal value of the following fractional program which is known as the CCR model as follows:

$$E_o^* = Max \ E_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}},$$

$$s.t. E_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, ..., n,$$

$$u_r \ge \varepsilon, \qquad r = 1, ..., s,$$

$$v_i \ge \varepsilon, \qquad i = 1, ..., m, \tag{1}$$

where ε is the non-Arithmetic number. DMU_o is called CCR efficient if the optimal value of the objective function in (1) is equal to one. By using appropriate transformations, an equivalent linear program is obtained as:

$$E_o^* = Max \sum_{r=1}^{s} u_r y_{ro},$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n,$$

$$u_r \geq \varepsilon, \quad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \ldots, m. \tag{2}$$

3. Acquiring CSWS by a Benchmark Point

An ideal DMU (IDMU) may be defined as follows:

Definition 1.

A virtual DMU is called *IDMU* if it uses the least inputs to generate the most outputs.

Based on the above definition, if we denote by x_i^{min} $(i = 1, \ldots, m)$ and y_r^{max} $(r = 1, \ldots, s)$ the inputs and outputs of the IDMU, then

$$x_i^{min} = \min_{1 \leq j \leq n}\{x_{ij}\}, \quad i = 1, \ldots, m, \qquad y_r^{max} = \max_{1 \leq j \leq n}\{y_{rj}\}, \quad r = 1, \ldots, s.$$

The efficiency of the IDMU is denoted by E_{IDMU}^* and can be determined by the following model:

$$E_{IDMU}^* = Max \sum_{r=1}^{s} u_r y_r^{max},$$

$$s.t. \sum_{i=1}^{m} v_i x_i^{min} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n,$$

$$u_r \geq \varepsilon, \quad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \ldots, m. \tag{3}$$

We know $E_{IDMU}^* \geq 1$ and $E_j^* \leq 1$ $(j = 1, \ldots, n)$, so the IDMU has the maximal efficiency score. We take this maximal efficiency score as the highest benchmark level for DMUs. We will make use of this benchmark level to generate a set of weights to suit the inputs and outputs. We begin our discussion with a geometrical interpretation. In Figure 1, the vertical and horizontal axes are

set to be the virtual output (weighted sum of outputs $\equiv \sum_{r=1}^{s} u_r\, y_{rj}$) and virtual input (weighed sum of inputs $\equiv \sum_{i=1}^{m} v_i\, x_{ij}$) for a given set of weights $(v_1, \ldots, v_m, u_1, \ldots, u_s)$. The highest benchmark level is considered as a straight line that passes through the origin with the slope of E^*_{IDMU}. A virtual DMU corresponding to DMU_j $(j = 1, \ldots, n)$ is represented by the pair

$$D_j = \left(\sum_{i=1}^{m} v_i\, x_{ij}, \sum_{r=1}^{s} u_r\, y_{rj} \right)$$

as it is shown in Figure 1.
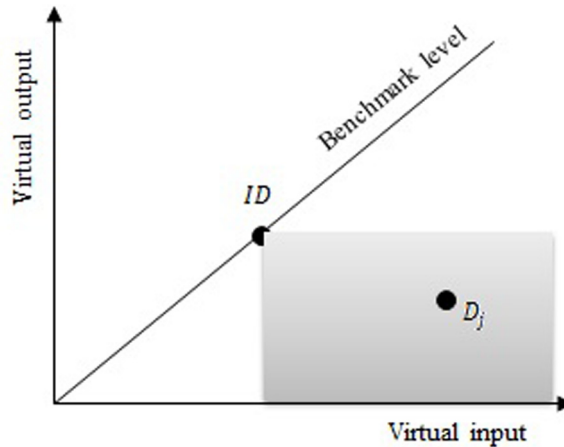
Figure 1. Interpretation of a virtual DMU (D_j), benchmark point (ID), and the benchmark level

Based on the definition of the highest benchmark level, virtual IDMU

$$ID = \left(\sum_{i=1}^{m} v_i\, x_i^{min}, \sum_{r=1}^{s} u_r\, y_r^{max} \right)$$

for a given set of weights $(v_1, \ldots, v_m, u_1, \ldots, u_s)$ must be on this benchmark level, which is shown in Figure 2. With any set of weights as $(v_1, \ldots, v_m, u_1, \ldots, u_s)$, we have

$$\sum_{i=1}^{m} v_i\, x_i^{min} \leq \sum_{i=1}^{m} v_i\, x_{ij} \text{ and } \sum_{r=1}^{s} u_r\, y_{rj} \leq \sum_{r=1}^{s} u_r\, y_r^{max} \quad (j = 1, \ldots, n).$$

Therefore, virtual DMUs are brought in the shaded region in Figure 1. Another result may be

$$\left(\sum_{r=1}^{s} u_r\, y_{rj} \right) \Big/ \left(\sum_{i=1}^{m} v_i\, x_{ij} \right) \leq \left(\sum_{r=1}^{s} u_r\, y_r^{max} \right) \Big/ \left(\sum_{i=1}^{m} v_i\, x_i^{min} \right) = E^*_{IDMU} \quad (j = 1, \ldots, n).$$

Thus, point D_j is under the highest benchmark level.
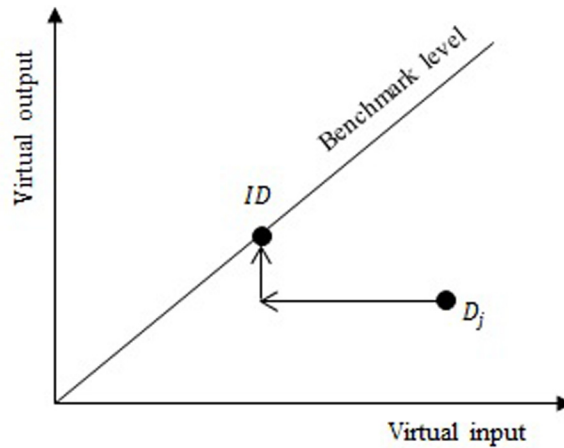
Figure 2. Analysis of moving DMUs toward benchmark point

It is evident that a DMU has a better performance in comparison to another DMU when its corresponding virtual DMU is closer to benchmark level than that of another DMU. Virtual DMUs are in the shaded region in Figure.1 and virtual IDMU is on benchmark level. Therefore, a closer distance to benchmark level is equivalent to a closer distance to the virtual IDMU. Consequently, we name the virtual IDMU as benchmark point. In this case, the aim is to find CSWs such that virtual IDMU lies on the benchmark level, and other virtual DMUs get as close to the benchmark level (Benchmark point) as possible.

One way to minimize the distance between virtual DMU_J $(J = 1, ..., n)$ and the benchmark point is to determine weights such that the virtual input decreases (moves to left side of D_j) and the virtual output increases (moves to upper of D_j), as it is shown in Figure 2.
For having the smallest total possible distance, we can consider a model as:

$$Max \left(\frac{\sum_{j=1}^{n} \sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{j=1}^{n} \sum_{i=1}^{m} v_i\, x_{ij}} = \frac{\sum_{r=1}^{s} u_r\, y_r}{\sum_{i=1}^{m} v_i\, x_i} \right),$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r\, y_r^{max}}{\sum_{i=1}^{m} v_i\, x_i^{min}} = E_{IDMU}^*,$$

$$u_r \geq \varepsilon, \quad r = 1, ..., s,$$

$$v_i \geq \varepsilon, \quad i = 1, ..., m, \tag{4}$$

where the optimal value of this problem is the efficiency of aggregate DMU, a DMU which the inputs and outputs are respectively the summation of inputs and outputs of all DMUs. Moreover, the objective function of problem (4) can be considered as a weighted sum of the efficiency

scores of DMUs. By some changes that seem appropriate, this problem is transformed to a problem as:

$$Max \sum_{r=1}^{s} u_r y_{r'}$$

$$s.t. \sum_{i=1}^{m} v_i x_i = 1,$$

$$\sum_{r=1}^{s} u_r y_r^{max} - E_{IDMU}^* \sum_{i=1}^{m} v_i x_i^{min} = 0,$$

$$u_r \geq \varepsilon, \quad r = 1, \dots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \dots, m. \tag{5}$$

We consider the optimal solution of this problem as the CSWs. It is notable that if in model (5) constraint

$$\sum_{r=1}^{s} u_r y_r^{max} - E_{IDMU}^* \sum_{i=1}^{m} v_i x_i^{min} = 0$$

is replaced with

$$\sum_{r=1}^{s} u_r y_r^{max} - E_{IDMU}^* \sum_{i=1}^{m} v_i x_i^{min} \leq 0,$$

this constraint is binding in optimality, so the IDMU is automatically considered on the benchmark level. With this substitution, model (5) is rewritten as:

$$Max \sum_{r=1}^{s} u_r y_{r'}$$

$$s.t. \sum_{i=1}^{m} v_i x_i = 1,$$

$$\sum_{r=1}^{s} u_r y_r^{max} - E_{IDMU}^* \sum_{i=1}^{m} v_i x_i^{min} \leq 0,$$

$$u_r \geq \varepsilon, \quad r = 1, \dots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \dots, m. \tag{6}$$

This method clarifies that there is one set of weights in which a virtual DMU coincides with the benchmark point unless its corresponding DMU is the IDMU. Considering that IDMU is not an observed DMU in practice, there is no DMU that is virtual DMU for any feasible set of weights

in (6) or equivalent models can reach the benchmark point. In other words, the efficiency of DMUs in this method is less than the defined common benchmark level. Thus there is no the problem of ranking DMUs with the highest score, so DMUs can be fully ranked.

4. Numerical Examples

Two data sets are introduced here in order to illustrate the proposed CSWs.

4.1. Example 1.

In DEA when the number of indices is more than half of DMUs, the number of efficient DMUs is usually very large. In Table 1, there are seven DMUs with three inputs and three outputs having this structure. As shown in the last column of Table 1, all of DMUs are CCR efficient. This data was previously examined by Liu and Peng (2008).

Table 2 shows the efficiency scores of our proposed method and also depicts a comparison with the results of two popular CSWs methods. The second and third column of Table 2 report the efficiency score of DMUs according to the obtained CSWs by methods of Jahanshahloo et al. (2005) and Liu and Peng (2008). Based on these results, DMUs B, C, D, and F by Jahanshahloo et al.'s method (2005), and DMUs C, D, E, and F by Liu and Peng's method (2008) have the score equal to one. Thus, more than a DMU has the highest efficiency score with these two different methods, while by our proposed method DMUs have different scores. The obtained score for DMUs is less than 6.989862 which is the efficiency of IDMU. According to these scores, DMUs can fully ranked and we have $E \gg D \gg F \gg A \gg C \gg B \gg G$.

Table 1. An example of a CCR efficient set of DMUs

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	CCR Efficiency
A	1621	436	205	174	497	22	1
B	2718	314	221	172	497	22	1
C	1523	345	215	160	443	22	1
D	5514	1314	553	487	1925	63	1
E	1941	507	309	220	521	36	1
F	1496	321	339	109	699	38	1
G	932	158	200	37	431	19	1

Table 2. Efficiency scores and ranking outcomes of Table 1

DMU	Jahanshahloo et al. (2005)	Liu and Peng (2008)	Our result
A	0.9423738	0.9767531	2.380506
B	1.000000	0.9114037	2.208041
C	1.000000	1.000000	2.269985
D	1.000000	1.000000	2.526859
E	0.9260117	1.000000	2.584472
F	1.000000	1.000000	2.486968
G	0.9264960	0.8629335	2.107744

4.2. Example 2

In this section, we are going to illustrate the use of the proposed procedure with a new example in universities in Iran. Table 3 reports the input and output data and efficiency scores of 33 universities as DMUs. Eight factors are considered for evaluating the universities. We intend to utilize faculty rate (I_1), student rate (I_2), budget rate (I_3), space per capita rate (I_4) as inputs, and graduation rate (O_1), published papers' rate (O_2), published books' rate (O_3) and the percentage of graduates accepted for postgraduate course (O_4) as outputs. We can see that 18 universities are CCR efficient, whereas the remaining fifteen DMUs are inefficient.

As shown in the last column of Table 3, the large numbers of DMUs are CCR efficient. Thus, a method for comparing them is needed. Table 4 exhibits the results of applying our proposed method for ranking efficient DMUs. According to the second column of Table 4, U_6 has the highest score of among efficient DMUs, and it also the best rank. Against, U_{32} with the score of 1.208787 has the last rank among efficient universities. The ranking of efficient universities is prepared in the last column of Table 4.

5. Remarks and Conclusion

In this paper, by defining the IDMU as a benchmark for efficient DMUs and presenting a geometrical interpretation, a model for finding the CSWs was obtained. The method in this study includes the following main features. First, the model to obtain the CSWs is an LPP. Second, the critical factor in solving an LPP is the number of constraints, and in this model the constraints are as small as possible. So, from the point of view of computation, this method is economical. Third, a full ranking for efficient DMUs is derived by the proposed method. The case of having fuzzy or stochastic parameters in the proposed model (DMUs with fuzzy or stochastic data) will be investigated in future studies. For control weights in the suggested method, we will consider weight restrictions in the method.

Table 3. Data for 33 universities

University	Inputs				Outputs				CCR efficiency
	I_1	I_2	I_3	I_4	O_1	O_2	O_3	O_4	
U_1	0.101611	0.731501	0.23245	0.491012	0.379061	0.125056	0.047826	0.261364	1.000000
U_2	0.259958	0.741573	0.35353	0.553676	0.42614	0.420128	0.333913	0.449134	1.000000
U_3	0.124642	0.394132	0.220106	0.275565	0.237296	0.131383	0.031304	0.176948	0.8578998
U_4	0.115943	0.535891	0.222901	0.521206	0.262316	0.11925	0.131304	0.263528	0.9303116
U_5	0.161411	0.610235	0.432597	0.699741	0.419233	0.174557	0.053043	0.528139	1.000000
U_6	1	0.972289	1	0.650796	1	1	1	1	1.000000
U_7	0.075912	0.603423	0.230866	0.497328	0.286067	0.085455	0.142609	0.258117	1.000000
U_8	0.072057	0.982127	0.403974	0.420717	0.237825	0.036996	0.053913	0.215909	0.8531757
U_9	0.165563	0.726669	0.266848	0.620105	0.472866	0.174259	0.049565	0.360931	1.000000
U_{10}	0.188989	0.395529	0.486262	0.326512	0.415216	0.231502	0.281739	0.398268	1.000000
U_{11}	0.202629	0.601094	0.418971	0.322818	0.389985	0.086646	0.087826	0.372835	0.9726094
U_{12}	0.202629	0.540548	0.593542	0.760028	0.441363	0.69272	0.046087	0.488636	1.000000
U_{13}	0.201443	0.915468	0.468757	0.586233	0.598245	0.266637	0.22	0.606602	1.000000
U_{14}	0.157359	0.408977	0.21251	0.491979	0.336317	0.082254	0.093913	0.312771	1.000000
U_{15}	0.100326	1	0.303024	0.488777	0.396857	0.083073	0.045217	0.349567	1.000000
U_{16}	0.126223	0.41183	0.188436	0.746134	0.308373	0.065729	0.06	0.285173	1.000000
U_{17}	0.02995	0.315829	0.108622	0.396391	0.098139	0.021215	0.011304	0.138528	1.000000
U_{18}	0.042305	0.30896	0.112557	0.299246	0.044224	0.011836	0.018261	0.046537	0.3366303
U_{19}	0.060492	0.294056	0.114463	0.533763	0.241913	0.028808	0.012174	0.132035	0.9782245
U_{20}	0.076307	0.361239	0.114478	0.416427	0.116957	0.044886	0.015652	0.116342	0.6953815
U_{21}	0.047247	0.197823	0.103792	0.590153	0.068187	0.019949	0.006957	0.079004	0.5675762
U_{22}	0.108333	0.853234	0.162112	0.222335	0.315244	0.009305	0.011304	0.089286	1.000000
U_{23}	0.075615	0.367468	0.14641	0.508417	0.178237	0.037889	0.016522	0.13474	0.7057036
U_{24}	0.041712	0.281306	0.137573	0.384564	0.118296	0.027468	0.053043	0.094156	0.7583645
U_{25}	0.024414	0.268207	0.116057	0.983312	0.055007	0.070493	0.011304	0.055195	0.9452449
U_{26}	0.067214	0.249811	0.083533	0.538912	0.093735	0.013622	0.037391	0.056818	0.5679866
U_{27}	0.064347	0.269954	0.114152	0.290976	0.139404	0.062677	0.033913	0.172619	1.000000
U_{28}	0.083523	0.279618	0.110412	0.560604	0.332793	0.042057	0.034783	0.131494	1.000000
U_{29}	0.058614	0.578506	0.121898	0.277916	0.148742	0.028212	0.025217	0.112554	0.7362502
U_{30}	0.055451	0.314665	0.092478	0.621466	0.281204	0.053819	0.05913	0.097944	1.000000
U_{31}	0.036967	0.258602	0.094538	0.377869	0.061174	0.004615	0.031304	0.064935	0.5742116
U_{32}	0.027182	0.219596	0.078952	0.363513	0.083868	0.007593	0.067826	0.042749	1.000000
U_{33}	0.033903	0.215288	0.081766	0.405941	0.079463	0.034018	0.006957	0.050866	0.6047586

Table 4. Ranking of efficient universities

University	Scores for ranking	Ranking
U_1	4.044689	6
U_2	4.032413	7
U_5	3.138981	10
U_6	8.050354	1
U_7	3.013664	13
U_9	3.995235	8
U_{10}	6.662534	3
U_{12}	3.042548	12
U_{13}	5.346571	4
U_{14}	3.581567	9
U_{15}	4.253914	5
U_{16}	2.165373	16
U_{17}	1.297151	17
U_{22}	7.428392	2
U_{27}	2.510091	14
U_{28}	3.110220	11
U_{30}	2.370707	15
U_{32}	1.208787	18

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