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Effects of time and diffusion phase-lags in a thick circular plate due to a ring load with axisymmetric heat supply

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Abstract

The purpose of this paper is to depict the effect of time, thermal, and diffusion phase lags due to axisymmetric heat supply in a ring. The problem is discussed within the context of DPLT and DPLD models. The upper and lower surfaces of the ring are traction-free and subjected to an axisymmetric heat supply. The solution is found by using Laplace and Hankel transform techniques. The analytical expressions of displacements, stresses and chemical potential, temperature and mass concentration are computed in transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerically simulated results are depicted graphically. The effect of time, diffusion, and thermal phase-lags are shown on the various components. Some particular results are also deduced from the present investigation.

Keywords: Dual phase lag; isotropic thermoelastic; Laplace Transform; Hankel Transform; plane axisymmetric; diffusion

MSC 2010 No.: 74B15, 80A20, 74H15

1. Introduction

Classical Fourier heat conduction law implies an infinitely fast propagation of a thermal signal which is violated in ultra-fast heat conduction system due to its very small dimensions and short timescales. Catteno (1958) and Vernotte (1958) proposed a thermal wave with a single phase lag in which the temperature gradient after a certain elapsed time was given by $\boldsymbol{q}+\tau_q\frac{\partial \boldsymbol{q}}{\partial t}=-k\nabla T$, where τ_q denotes the relaxation time required for thermal physics to take account of hyperbolic effect within the medium. Here when $\tau_q>0$,the thermal wave propagates through the medium with a finite speed of $\sqrt{\alpha/\tau_q}$, where α is thermal diffusivity. When τ_q approaches zero, the thermal wave has an infinite speed and thus the single phase lag model reduces to traditional Fourier model .

The dual phase lag model of heat conduction was proposed by Tzou (1996) as $\boldsymbol{q}+\tau_q\frac{\partial \boldsymbol{q}}{\partial t}=-k(\nabla T+\tau_t\frac{\partial}{\partial t}\nabla T)$,where the temperature gradient ∇T at a point P of the material at time $t+\tau_t$ corresponds to the heat flux vector q at the same time which is $t+\tau_q$. Here K is thermal conductivity of the material. The delay time τ_t is interpreted as that caused by the microstructural interactions and is called the phase lag of temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of thermal inertia and is called the phase lag of heat flux. This universal model is claimed to be able to bridge the gap between microscopic and macroscopic approaches, covering a wide range of heat transfer models.

If $\tau_t=0$, Tzou (1996) refers to the model as single phase model. Numerous efforts have been invested in the development of an explicit mathematical solution to the heat conduction equation under dual phase lag model. Quintanilla (2006) compared two different mathematical hyperbolic models proposed by Tzou. Kumar and Mukhopadhaya (2010) investigated the propagation of harmonic waves of assigned frequency by employing the thermoelasticity theory with three phase lags. Chou and Yang (2009) discussed two dimensional dual phase lag thermal behavior in single-/multi-layer structures using CESE method.

Zhou, Zhang and Chen (2009) proposed an axisymmetric dual-phase-lag bioheat model for laser heating of living tissues. Abdallah (2009) used uncoupled thermoelastic model based on dual phase lag to investigate the thermoelastic properties of a semi-infinite medium. Kumar, Chawla and Abbas (2012) discussed the effect of viscosity on wave propagation in anisotropic thermoelastic medium with three phase lag model. Bhattacharya and Kanoria (2014) investigated the problem of elasto-thermo-diffusion inside a spherical shell.

Rukolaine (2014) investigated Unphysical effects of the dual-phase-lag model of heat conduction. Liu, Cheng and Wang (2015) analyzed thermal damage in a laser-Irradiated based on non-Fourier model. Ying and Yun (2015) built a fractional dual-phase-lag model and the corresponding bio-heat transfer equation. Tripathi et al. (2015) analyzed generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply. Many active researchers such as Abbas (2012, 2014a, 2014b, 2014c, 2015), Abbas and Kumar (2015), Abbas, Kumar and Rani (2015), Abbas, Marin and Kumar (2015), Zenkour and Abbas (2014) have worked and contributed in this area.

Diffusion is defined as the spontaneous movement of the particles from high concentration region to the low concentration region, and it occurs in response to a concentration gradient expressed as the change in concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Dual phase lag diffusion model was considered by Kumar and Gupta (2014). Abbas (2015) proposed a dual phase lag model on thermoelastic interaction in an infinite fiber-reinforced anisotropic medium with a circular hole.

Here in this investigation, a generalized form of mass diffusion equation is introduced instead of classical Fick's diffusion theory by using two diffusion phase-lags in axisymmetric form. One phase-lag of diffusing mass flux vector, represents the delayed time required for the diffusion of the mass flux and the other phase-lag of chemical potential, represents the delayed time required for the establishment of the potential gradient. The basic equations for the isotropic thermoelastic diffusion medium in the context of dual-phase-lag heat transfer (DPLT) and dual-phase-lag diffusion (DPLD) models in axisymmetric form are presented. The components of displacements, stresses and chemical potential, temperature and mass concentration are computed numerically. Numerically computed results are depicted graphically. The effect of diffusion and thermal phase-lags are shown on the various components.

2. Basic Equations

The basic equations of motion, heat conduction and mass diffusion in a homogeneous isotropic thermoelastic solid with DPLT and DPLD models in the absence of body forces, heat sources and mass diffusion sources are

$$(\lambda+\mu)\nabla(\nabla.u)+\mu\nabla^2 u-\beta_1\nabla T-\beta_2\nabla C=\rho\ddot{u}, \tag{1}$$

$$\left(1+\tau_t\frac{\partial}{\partial t}\right)KT_{,ii}=\left(1+\tau_q\frac{\partial}{\partial t}+{\tau_q}^2\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\dot{T}+\beta_1 T_0\dot{e}_{kk}+aT_0\dot{C}\right], \tag{2}$$

$$\left(1+\tau_p\frac{\partial}{\partial t}\right)\left(D\beta_2\nabla^2(\nabla.u)+Da\nabla^2 T-Db\nabla^2 C\right)+\frac{\partial}{\partial t}\left(1+\tau_\eta\frac{\partial}{\partial t}+{\tau_\eta}^2\frac{\partial^2}{\partial t^2}\right)C=0, \tag{3}$$

and the constitutive relations are

$$\sigma_{ij}=2\mu e_{ij}+\delta_{ij}(\lambda e_{kk}-\beta_1 T-\beta_2 C), \tag{4}$$

$$\rho T_0 S=\left(1+\tau_q\frac{\partial}{\partial t}+{\tau_q}^2\frac{\partial^2}{\partial t^2}\right)(\rho C_E T+\beta_1 T_0 e_{kk}+aT_0 C), \tag{5}$$

$$P=-\beta_2 e_{kk}-aT-bC\ , \tag{6}$$

where λ , μ are Lame's constants , ρ is the density assumed to be independent of time, D is the diffusivity, P is the chemical potential per unit mass, C is the concentration, u_i are components of displacement vector u, K is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, $T = \vartheta - T_0$ is small temperature increment, ϑ is the absolute temperature of the medium, T_0 is the reference temperature of the body such that $\left|\frac{T}{T_0}\right| \ll 1$, a and b are the coefficients describing the measure of thermodiffusion, and mass diffusion effect respectively, σ_{ij} and e_{ij} are the components of stress and strain respectively, e_{kk} is dilatation, S is the entropy per unit mass, $\beta_1 = (3\lambda + 2\mu)\alpha_t$, $\beta_2 = (3\lambda + 2\mu)\alpha_c$, α_c is the coefficient of linear diffusion expansion and α_t is the coefficient of thermal linear expansion. $\tau_t, \tau_q, \tau_p, \tau_\eta$ are respectively, phase lag of temperature gradient, the phase lag of heat flux, the phase lag of chemical potential, and phase lag of diffusing mass flux vector. In the above equations, a comma followed by suffix denotes spatial derivative and a superposed dot denotes derivative with respect to time.

3. Formulation and solution of the problem

Consider a thick circular plate of thickness $2b$ occupying the space D defined by $0 \leq r \leq \infty$, $-b \leq z \leq b$. Let the plate be subjected to an axisymmetric heat supply and chemical potential source with stress free boundary depending on the radial and axial directions of the cylindrical co-ordinate system. The initial temperature in the thick plate is given by a constant temperature T_0. The heat flux and chemical potential sources of unit magnitude are prescribed along with vanishing of stress components on the upper and lower boundary surfaces along with traction free boundary $z = \pm d$. We take a cylindrical polar co-ordinate system (r, θ, z) with symmetry about z –axis. As the problem considered is plane axisymmetric, the field component $u_\theta = 0$, and u_r, u_z, T and C are independent of θ. The components of displacement vector $\vec{u}$ for the two dimensional axisymmetric problem take the form

$$\vec{u} = (u_r, 0, u_z). \tag{7}$$

Equations (1) - (6) with the aid of (7) take the form:

$$(\lambda + \mu)\frac{\partial e}{\partial r} + \mu\left(\nabla^2 - \frac{1}{r^2}\right)u_r - \beta_1\frac{\partial T}{\partial r} - \beta_2\frac{\partial C}{\partial r} = \rho\frac{\partial^2 u_r}{\partial t^2}\,, \tag{8}$$

$$(\lambda + \mu)\frac{\partial e}{\partial z} + \mu\nabla^2 u_z - \beta_1\frac{\partial T}{\partial z} - \beta_2\frac{\partial C}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2}\,, \tag{9}$$

$$(1+\tau_t\frac{\partial}{\partial t})\mathrm{K}\nabla^2 T = \left(1 + \tau_q\frac{\partial}{\partial t} + \frac{{\tau_q}^2}{2}\frac{\partial^2}{\partial t^2}\right)[\rho C_E\dot{T} + \beta_1 T_0\frac{\partial}{\partial t}div\,u + aT_0\frac{\partial C}{\partial t}]\,, \tag{10}$$

$$(1+\tau_p\frac{\partial}{\partial t})(D\beta_2\nabla^2 div\,u + Da\nabla^2 T - Db\nabla^2 C) + \frac{\partial}{\partial t}\left(1 + \tau_\eta\frac{\partial}{\partial t} + \frac{{\tau_\eta}^2}{2}\frac{\partial^2}{\partial t^2}\right)C = 0\,. \tag{11}$$

We define the dimensionless quantities

$$r' = \frac{\omega_1}{c_1} r \, , z' = \frac{\omega_1}{c_1} z \, , (u'_r, u'_z) = \frac{\omega_1}{c_1} (u_r, u_z), t' = \omega_1 t \, ,$$
$$(\sigma'_{rr}, \sigma'_{\theta\theta}, \sigma'_{zz}, \sigma'_{rz}) = \frac{1}{\beta_1 T_0} (\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}), T' = \frac{\beta_1}{\rho c_1^2} T,$$
$$C' = \frac{\beta_2}{\rho c_1^2} C, \ \left(\tau'_q, \tau'_t, \tau'_p, \tau'_\eta\right) = \omega_1 (\tau_q, \tau_t, \tau_p, \tau_\eta) \, , P' = \frac{P}{\beta_2}, \tag{12}$$

where

$$\omega_1 = \frac{\rho C_E c_1^2}{K} \, , c_1^2 = \frac{\lambda + 2\mu}{\rho} \, .$$

Using the dimensionless quantities defined by (12) in Equations (8) - (11) and suppressing the primes for convenience yield

$$\frac{(\lambda+\mu)}{\rho c_1^2} \frac{\partial e}{\partial r} + \frac{\mu}{\rho c_1^2} \left(\nabla^2 - \frac{1}{r^2} \right) u_r - \frac{\partial T}{\partial r} - \frac{\partial C}{\partial r} = \frac{\partial^2 u_r}{\partial t^2}, \tag{13}$$

$$\frac{\mu}{\rho c_1^2} \frac{\partial e}{\partial z} + \frac{\mu}{\rho c_1^2} \nabla^2 u_z - \frac{\partial T}{\partial z} - \frac{\partial C}{\partial z} = \frac{\partial^2 u_z}{\partial t^2} \, , \tag{14}$$

$$(1+\tau_t \frac{\partial}{\partial t}) K \nabla^2 T = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{{\tau_q}^2}{2} \frac{\partial^2}{\partial t^2} \right) \frac{1}{\omega_1} \left[\rho C_E c_1^2 \dot{T} + \frac{{\beta_1}^2 T_0}{\rho} \frac{\partial}{\partial t} div \, u + \frac{a T_0 \beta_1 c_1^2}{\beta_2} \frac{\partial C}{\partial t} \right], \tag{15}$$

$$(1+\tau_p \frac{\partial}{\partial t}) \left(D \beta_2 \nabla^2 div \, u + D a \nabla^2 T \frac{\rho c_1^2}{\beta_1} - D b \nabla^2 C \frac{\rho c_1^2}{\beta_1} \right)$$
$$+ \frac{\partial}{\partial t} \left(1 + \tau_\eta \frac{\partial}{\partial t} + \frac{{\tau_\eta}^2}{2} \frac{\partial^2}{\partial t^2} \right) \frac{\rho c_1^4}{\beta_1 \omega_1} C = 0 \, . \tag{16}$$

The stress components and chemical potential source in dimensionless form are

$$\sigma_{rr} = \mu^1 \frac{\partial u_r}{\partial r} + \lambda^1 e - \frac{\rho c_1^2}{\beta_1^2 T_0} T - \frac{\beta_2 \rho c_1^2}{\beta_1 T_0} C, \tag{17}$$

$$\sigma_{\theta\theta} = \mu^1 \frac{u_r}{r} + \lambda^1 e - \frac{\rho c_1^2}{\beta_1^2 T_0} T - \frac{\beta_2 \rho c_1^2}{\beta_1 T_0} C \, , \tag{18}$$

$$\sigma_{zz} = \mu^1 \frac{\partial u_z}{\partial z} + \lambda^1 e - \frac{\rho c_1^2}{\beta_1^2 T_0} T - \frac{\beta_2 \rho c_1^2}{\beta_1 T_0} C, \tag{19}$$

$$\sigma_{rz} = \frac{\mu^1}{4} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right), \tag{20}$$

$$\sigma_{r\theta} = 0 = t_{z\theta}, \tag{21}$$

$$P = -e - \frac{a \rho c_1^2}{\beta_2 \beta_1} T - \frac{b \rho c_1^2}{\beta_2^2} C, \tag{22}$$

where

$$\mu^1 = \frac{2\mu}{\beta_1 T_0}, \lambda^1 = \frac{\lambda}{\beta_1 T_0} \, .$$

The Laplace Transform of a function $f = f(x_1, x_3, t)$ with respect to time variable t , with s as a Laplace Transform variable is defined as

$$\overline{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t) e^{-st} dt. \tag{23}$$

The Hankel transform of order n of $\bar{f}(r, z, s)$ with respect to the variable r is defined by

$$H(\bar{f}(r, z, s)) = \bar{f}^*(\xi, z, s) = \int_0^\infty \bar{f}(r, z, s) r J_n(r\xi)\, dr. \tag{24}$$

Applying the Laplace Transform defined by (23) on Equations (13) - (16), and simplifying, we obtain

$$\nabla^2 \bar{T} + \nabla^2 \bar{C} - (\nabla^2 - \mathrm{s}^2)\bar{e} = 0, \tag{25}$$

$$(\nabla^2 - \frac{\tau_q{}^1}{\tau_t{}^1} ks)\bar{T} - \frac{K a T_0 \tau_q{}^1 s}{\rho C_E \beta_1 \tau_t{}^1} \bar{C} - \frac{K {\beta_1}^2 T_0}{\rho^2 C_E c_1^2} \frac{\tau_q{}^1}{\tau_t{}^1} s\, \bar{e} = 0, \tag{26}$$

$$D\beta_2 a \nabla^2 \frac{\rho c_1^2}{\beta_1} \bar{T} - \left(Db \frac{\rho c_1^2}{\beta_1} \nabla^2 - \frac{sk\tau_\eta{}^1 c_1^2}{\tau_p{}^1 C_E}\right) \bar{C} + D\beta_2 \nabla^2 \bar{e} = 0, \tag{27}$$

where

$$\tau_q{}^1 = 1 + s\tau_q + \frac{s^2 {\tau_q}^2}{2}, \tau_\eta{}^1 = 1 + s\tau_\eta + \frac{s^2 {\tau_\eta}^2}{2}, \tau_p{}^1 = 1 + s\tau_p\,,\ \tau_t{}^1 = 1 + s\tau_t.$$

Applying Hankel Transform defined by (24) on the system of Equations (25) - (27), we obtain

$$\left(-\xi^2 + \frac{d^2}{dz^2}\right) \bar{T}^* + \left(-\xi^2 + \frac{d^2}{dz^2}\right) \bar{C}^* - \left(-\xi^2 + \frac{d^2}{dz^2} - \mathrm{s}^2\right) \bar{e}^* = 0, \tag{28}$$

$$(-\xi^2 + \frac{d^2}{dz^2} - \frac{\tau_q{}^1}{\tau_t{}^1} ks)\bar{T}^* - \frac{K a T_0 \tau_q{}^1 s}{\rho C_E \beta_1 \tau_t{}^1} \bar{C}^* - \frac{K {\beta_1}^2 T_0}{\rho^2 C_E c_1^2} \frac{\tau_q{}^1}{\tau_t{}^1} s\, \bar{e}^* = 0, \tag{29}$$

$$D\beta_2 a \left(-\xi^2 + \frac{d^2}{dz^2}\right) \frac{\rho c_1^2}{\beta_1} \bar{T}^* - \left(Db \frac{\rho c_1^2}{\beta_1} \left(-\xi^2 + \frac{d^2}{dz^2}\right) - \frac{sk\tau_\eta{}^1 c_1^2}{\tau_p{}^1 C_E}\right) \bar{C}^*$$
$$+ D\beta_2 \left(-\xi^2 + \frac{d^2}{dz^2}\right) \bar{e}^* = 0. \tag{30}$$

Solving Equations (28) - (30), we obtain

$$(M \frac{d^6}{dz^6} + Q \frac{d^4}{dz^4} + R \frac{d^2}{dz^2} + S)(\bar{T}^*, \bar{C}^*, \bar{e}^*) = 0, \tag{31}$$

where

$$M = \frac{Db\rho c_1^2}{\beta_1} - D\beta_2 ,$$

$$Q = Q' - 3\xi^2 , Q' = \frac{\tau_q^1}{\tau_t^1}\left(\frac{KaT_0D\beta_2}{\rho C_E\beta_1} + \frac{KbT_0D\beta_1}{\rho C_E} + KsD\beta_2 - \frac{Db\rho c_1^2}{\beta_1}\right) - \frac{sK\tau_\eta^1}{\tau_t^1 C_E},$$

$$R = 3P\xi^4 - 2Q'\xi^2 + R' ,R' = \frac{K\tau_q^1}{\tau_t^1}\left(\frac{-K\beta_1^2T_0\tau_\eta^1}{\rho^2\tau_p^1{C_E}^2} + \frac{Db\rho c_1^2s^3}{\beta_1} + \frac{D\beta_2aT_0c_1^2s^2}{C_E{\beta_1}^2}\right) + \frac{K\tau_\eta^1c_1^2s^2(K+s)}{\tau_p^1C_E},$$

$$S = -P^{\xi^6} + Q'^{\xi^4} - R'^{\xi^2} - S' ,\ S' = \frac{-sK\tau_\eta^1c_1^4D\beta_2a\rho}{\tau_p^1C_E\beta_1}.$$

The solution of Equation (31) is assumed to be of the form

$$\bar{T}^* = \sum_{i=1}^3 A_i\cosh(q_iz), \tag{32}$$

$$\bar{C}^* = \sum_{i=1}^3 d_iA_i\cosh(q_iz), \tag{33}$$

$$\bar{e}^* = \sum_{i=1}^3 f_iA_i\cosh(q_iz), \tag{34}$$

where q_i (i = 1,2,3) are the roots of the polynomial equation

$$P\frac{d^6}{dz^6} + Q\frac{d^4}{dz^4} + R\frac{d^2}{dz^2} + S = 0,$$

and the coupling constants d_i and f_i are given by

$$d_i = \frac{\zeta_{10}q_i^4 + q_i^2\zeta_{10}(-2\xi^2 - \zeta_{14} + \zeta_{13}) + \zeta_{10}(\xi^4 + \zeta_{14}s\xi^2 - \zeta_{15})}{(-q_i^2+\xi^2)(\zeta_{11}+\zeta_{12}) + \zeta_{12}},$$

$$f_i = \frac{\zeta_{16}q_i^4 + (q_i^2-\xi^2)(-\zeta_{16}\zeta_{14} + \zeta_{13}\zeta_{17} + \zeta_{18}) - \zeta_{14}\zeta_{18}}{(-q_i^2+\xi^2)(\zeta_{11}+\zeta_{12}) + \zeta_{12}},$$

where

$$\zeta_{11} = \frac{KaT_0{\tau_q}^1D\beta_2}{\rho C_E\beta_1{\tau_t}^1},\ \zeta_{12} = \frac{K{\beta_1}^2T_0}{\rho^2c_E^2\tau_p^1}\frac{{\tau_q}^1sK\tau_\eta^1}{{\tau_t}^1}s ,\ \ \zeta_{13} = \frac{KT_0{\tau_q}^1a\rho}{\rho^2C_E{\tau_t}^1} ,\zeta_{14} = \frac{{\tau_q}^1}{{\tau_t}^1}Ks ,$$

$$\zeta_{15} = \frac{K\beta_1T_0{\tau_q}^1\xi^2}{\rho C_E{\tau_t}^1}\ \zeta_{16} = \frac{-Db\rho c_1^2}{\beta_1}\ ,\ \zeta_{17} = \frac{D\beta_2c_1^2}{{\beta_1}^2}, \zeta_{18} = \frac{s{\tau_\eta}^1kc_1^2}{{\tau_p}^1\beta_1C_E}\ , \zeta_{10} = D\beta_2 .$$

Applying Laplace and Hankel Transforms defined by (23) and (24) on Equations (13), (14) and substituting the values of $\bar{T}^*$, $\bar{C}^*$ and $\bar{e}^*$ from (32) - (34), we obtain the components of displacement as

$$\bar{u}_r^* = Acosh(qz) + \sum_{i=1}^3\frac{\eta_iA_i}{m_i}\cosh(q_iz), \tag{35}$$

$$\bar{u}_Z^* = Asinh(qz) + \sum_{i=1}^{3} \frac{q_i \mu_i A_i \sinh(q_i z)}{m_i}, \tag{36}$$

where

$$\eta_i = \xi(-\frac{\lambda+\mu}{\rho c_1^2} f_i + 1 + d_i),$$

$$\mu_i = 1 + d_i + \mu f_i/\rho c_1^2,$$

$$m_i = \frac{\mu}{\rho c_1^2}(q_i^2 - \xi^2) - s^2.$$

Applying Laplace and Hankel Transforms defined by (23) and (24), on Equations (17) - (22) and substituting the values of $\bar{u}_r^*$, $\bar{u}_Z^*$ from (35) and (36), we obtain the values of stress components and chemical potential function in the transformed domain as

$$\bar{\sigma}_{zz}^* = \mu^1 Aqsinh(qz) + \sum_{i=1}^{3} \gamma_i A_i \cosh(q_i z), \tag{37}$$

$$\bar{\sigma}_{rz}^* = \frac{\mu^1}{2} Asinh(qz) + \sum_{i=1}^{3} \alpha_i A_i \sinh(q_i z), \tag{38}$$

$$\bar{\sigma}_{\theta\theta}^* = 2\mu^1 \xi Acosh(qz) + \sum_{i=1}^{3} \zeta_i A_i \sinh(q_i z), \tag{39}$$

$$\bar{P}^* = \sum_{i=1}^{3} \nu_i A_i \cosh(q_i z), \tag{40}$$

where

$$\gamma_i = \frac{q_i^2 \mu_i}{m_i \beta_1 T_0} + \frac{\lambda}{\beta_1 T_0} f_i - \frac{\rho c_1^2}{\beta_1 T_0} - \frac{\rho c_1^2}{\beta_1 T_0} d_i,$$

$$\alpha_i = \frac{\eta_i q_i}{m_i} + \frac{q_i \mu_i \xi}{m_i}, \nu_i = -f_i - a\frac{\rho c_1^2}{\beta_2 \beta_1} + \frac{b \rho c_1^2}{\beta_2^2} d_i,$$

$$\zeta_i = \frac{2\mu \xi \eta_i}{\beta_1 T_0 m_i} + \frac{\lambda f_i}{\beta_1 T_0} - \rho c_1^2 - \frac{\rho c_1^2}{\beta_1 T_0} d_i, q = sqrt(\xi^2 + \frac{s^2 \rho c_1^2}{\mu}).$$

4. Boundary Conditions

We consider a thermal source and chemical potential source along with vanishing of stress components at the stress free surface at $z = \pm d$. Mathematically, these can be written as

$$\frac{\partial T}{\partial z} = \pm g_0 F(r,z), \tag{41}$$
$$\sigma_{zz} = 0, \tag{42}$$
$$\sigma_{rz} = 0, \tag{43}$$
$$P = f(r,t). \tag{44}$$

Using the dimensionless quantities defined by (12) in the boundary conditions (41) - (44),and applying Laplace and Hankel Transforms defined by (23) and (24) on the resulting quantities, and substituting the values of $\bar{T}^*$, $\bar{\sigma}_{zz}^*$, $\bar{\sigma}_{rz}^*$ and $\bar{P}^*$, yields

$$\sum_{i=1}^{3} A_i \cosh(q_i z) = g_0 \bar{F}^*(\xi, d), \tag{45}$$

$$\mu^1 Aq sinh(qz) + \sum_{i=1}^{3} \gamma_i A_i \cosh(q_i z) = 0, \tag{46}$$

$$\frac{\mu^1}{2} A sinh(qz) + \sum_{i=1}^{3} \alpha_i A_i \sinh(q_i z) = 0, \tag{47}$$

$$\sum_{i=1}^{3} \nu_i A_i \cosh(q_i z) = \bar{f}^*(\xi, s). \tag{48}$$

Solving Equations (45) - (48), we obtain the values of A_1, A_2, A_3 and A as

$$A_1 = \frac{\Delta_1}{\Delta},\ A_2 = \frac{\Delta_2}{\Delta},\ A_3 = \frac{\Delta_3}{\Delta},\ A = \frac{\Delta_4}{\Delta}, \tag{49}$$

where

$$\Delta = \Delta_{24}\Delta_{11}(\Delta_{43}\Delta_{32} - \Delta_{33}\Delta_{42}) + \Delta_{24}\Delta_{12}(\Delta_{43}\Delta_{31} - \Delta_{41}\Delta_{33}) \\ - \Delta_{13}\Delta_{24}(\Delta_{31}\Delta_{42} - \Delta_{32}\Delta_{41}) + \Delta_{11}\Delta_{34}(\Delta_{22}\Delta_{43} - \Delta_{23}\Delta_{42}) \\ - \Delta_{34}\Delta_{12}(\Delta_{43}\Delta_{21} - \Delta_{41}\Delta_{23}) + \Delta_{34}\Delta_{13}(\Delta_{21}\Delta_{42} - \Delta_{22}\Delta_{41}),$$

$$\Delta_1 = g_0 \bar{F}^*(\xi, d)\Lambda_1 - \bar{f}^*(\xi, s)\Lambda^1,$$

$$\Delta_2 = -g_0 \bar{F}^*(\xi, d)\Lambda_2 + \bar{f}^*(\xi, s)\Lambda^2,$$

$$\Delta_3 = g_0 \bar{F}^*(\xi, d)\Lambda_3 - \bar{f}^*(\xi, s)\Lambda^3,$$

$$\Delta_4 = -g_0 \bar{F}^*(\xi, d)\Lambda_3 + \bar{f}^*(\xi, s)\Lambda^3,$$

where

$$\Lambda_1 = \Delta_{43}(\Delta_{24}\Delta_{32} - \Delta_{34}\Delta_{22}) + \Delta_{42}(\Delta_{23}\Delta_{34} - \Delta_{24}\Delta_{33}),$$

$$\Lambda^1 = \Delta_{12}(\Delta_{23}\Delta_{34} - \Delta_{24}\Delta_{33}) - \Delta_{13}(\Delta_{22}\Delta_{34} - \Delta_{24}\Delta_{32}),$$

$$\Lambda_2 = \Delta_{24}(\Delta_{31}\Delta_{43} - \Delta_{23}\Delta_{41}) - \Delta_{34}(\Delta_{21}\Delta_{43} - \Delta_{23}\Delta_{41}),$$

$$\Lambda^2 = -\Delta_{24}(\Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31}) + \Delta_{34}(\Delta_{11}\Delta_{23} - \Delta_{13}\Delta_{21}),$$

$$\Lambda_3 = \Delta_{24}(\Delta_{31}\Delta_{43} - \Delta_{32}\Delta_{41}) - \Delta_{34}(\Delta_{21}\Delta_{43} - \Delta_{22}\Delta_{41}),$$

$$\Lambda^3 = \Delta_{24}(\Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}) - \Delta_{34}(\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}),$$

$$\Lambda_4 = \Delta_{21}(\Delta_{43}\Delta_{32} - \Delta_{33}\Delta_{42}) - \Delta_{22}(\Delta_{43}\Delta_{31} - \Delta_{41}\Delta_{33}) + \Delta_{23}(\Delta_{31}\Delta_{42} - \Delta_{32}\Delta_{41}),$$

$$\Lambda^4 = \Delta_{11}(\Delta_{22}\Delta_{33} - \Delta_{23}\Delta_{32}) - \Delta_{12}(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}) + \Delta_{13}(\Delta_{21}\Delta_{32} - \Delta_{31}\Delta_{22}),$$

$\Delta_{1i} = q_i \sinh(q_i d)$, i=1,2,3, $\Delta_{14} = 0$, $\Delta_{2i} = \gamma_i \cosh(q_i d)$, i=1,2,3,

$\Delta_{24} = 2\mu q \sinh(\mathrm{qd})$, $\Delta_{3i} = \alpha_i \sinh(q_i d)$, i =1,2,3,

$$\Delta_{34} = \mu(q+\xi)\text{sinh}(\text{qd})/2,\ \ \Delta_{4i} = \nu_i \cosh(q_i d),\ \ i\text{=1,2,3, } \Delta_{44} = 0.$$

Substituting the values of A_1, A_2, A_3 and A from (49), into Equations (32) - (34) and Equations (35) - (40), we obtain the components of displacement, stress components, chemical potential function, temperature change, mass concentration and cubic dilatation as

$$\bar{u}_r^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}\left(\frac{\eta_1}{m_1}\Lambda_1 \cosh(q_1 z) - \frac{\eta_2}{m_2}\Lambda_2 \cosh(q_2 z) + \frac{\eta_3}{m_3}\Lambda_3 \cosh(q_3 z) - \Lambda_4 \cosh(qz)\right)$$

$$- \frac{\bar{f}^*(\xi)}{\Delta}\left(\frac{\eta_1}{m_1}\Lambda^1 \cosh(q_1 z) - \frac{\eta_2}{m_2}\Lambda^2 \cosh(q_2 z) + \frac{\eta_3}{m_3}\Lambda^3 \cosh(q_3 z) - \Lambda^4 \cosh(qz)\right), \quad (50)$$

$$\bar{u}_z^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}$$

$$\times\left(\frac{q_1\mu_1}{m_1}\Lambda_1 \sinh(q_1 z) - \frac{q_2\mu_2}{m_2}\Lambda_2 \sinh(q_2 z) + \frac{q_3\mu_3}{m_3}\Lambda_3 \sinh(q_3 z) - \Lambda_4 \sinh(qz)\right)$$

$$- \frac{\bar{f}^*(\xi)}{\Delta}\ (\frac{q_1\mu_1}{m_1}\Lambda^1 \sinh(q_1 z)$$

$$\times\left(-\frac{q_2\mu_2}{m_2}\Lambda^2 \sinh(q_2 z) + \frac{q_3\mu_3}{m_3}\Lambda^3 \sinh(q_3 z) - \Lambda^4 \sinh(qz)\right), \quad (51)$$

$$\bar{\sigma}_{zz}^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}$$

$$\times\left(\gamma_1\Lambda_1 \cosh(q_1 z) - \gamma_2\Lambda_2 \cosh(q_2 z) + \gamma_3\Lambda_3 \cosh(q_3 z) - 2\mu q\Lambda_4 \sinh(qz)\right)$$

$$- \frac{\bar{f}^*(\xi)}{\Delta}(\gamma_1\Lambda^1 \cosh(q_1 z) - \gamma_2\Lambda^2 \cosh(q_2 z) + \gamma_3\Lambda^3 \cosh(q_3 z) - 2\mu q\Lambda^4 \cosh(qz)),$$

$$(52)$$

$$\bar{\sigma}_{rz}^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}$$

$$\times\left(\alpha_1\Lambda_1 \sinh(q_1 z) - \alpha_2\Lambda_2 \sinh(q_2 z) + \alpha_3\Lambda_3 \sinh(q_3 z) - \mu(q\ \ \xi)\ \Lambda_4 \sinh\frac{(qz)}{2}\right)$$

$$- \frac{\bar{f}^*(\xi)}{\Delta}(\alpha_1\Lambda^1 \sinh(q_1 z) - \alpha_2\Lambda^2 \sinh(q_2 z) + \alpha_3\Lambda^3 \sinh(q_3 z)$$

$$- \mu(q+\xi)\Lambda^4 \sinh\frac{(qz)}{2}, \quad (53)$$

$$\bar{\sigma}_{\theta\theta}^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}(\zeta_1\Lambda_1 \cosh(q_1 z) - \zeta_2\Lambda_2 \cosh(q_2 z) + \zeta_3\Lambda_3 \cosh(q_3 z) - 2\mu\xi\Lambda_4 \cosh(qz))$$

$$- \frac{\bar{f}^*(\xi)}{\Delta}(\zeta_1\Lambda^1 \cosh(q_1 z) - \zeta_2\Lambda^2 \cosh(q_2 z) + \zeta_3\Lambda^3 \cosh(q_3 z) - 2\mu\xi\Lambda^4 \cosh(qz)),$$

$$(54)$$

$$\bar{P}^* = \frac{g_0 \bar{F}^*(\xi\ ,d)}{\Delta}(\nu_1\Lambda_1 \cosh(q_1 z) - \nu_2\Lambda_2 \cosh(q_2 z) + \nu_3\Lambda_3 \cosh(q_3 z))$$

$$-\frac{\bar{f}^*(\xi)}{\Delta}(\nu_1\Lambda^1\cosh(q_1z)-\nu_2\Lambda^2\cosh(q_2z)+\nu_3\Lambda^3\cosh(q_3z)), \quad (55)$$

$$\bar{T}^*=\frac{g_0\bar{F}^*(\xi\,,d)}{\Delta}(\Lambda_1\cosh(q_1z)-\Lambda_2\cosh(q_2z)+\Lambda_3\cosh(q_3z))$$
$$-\frac{\bar{f}^*(\xi)}{\Delta}(\Lambda^1\cosh(q_1z)-\ \Lambda^2\cosh(q_2z)+\Lambda^3\cosh(q_3z)), \quad (56)$$

$$\bar{C}^*=\frac{g_0\bar{F}^*(\xi\,,d)}{\Delta}(d_1\Lambda_1\cosh(q_1z)-d_2\Lambda_2\cosh(q_2z)+d_3\Lambda_3\cosh(q_3z))$$
$$-\ \frac{\bar{f}^*(\xi)}{\Delta}(d_1\Lambda^1\cosh(q_1z)-d_2\Lambda^2\cosh(q_2z)+d_3\Lambda^3\cosh(q_3z)), \quad (57)$$

$$\bar{e}^*=\ \ \frac{g_0\bar{F}^*(\xi\,,d)}{\Delta}(f_1\Lambda_1\cosh(q_1z)-f_2\Lambda_2\cosh(q_2z)+f_3\Lambda_3\cosh(q_3z))$$
$$-\frac{\bar{f}^*(\xi)}{\Delta}(f_1\Lambda^1\cosh(q_1z)-f_2\Lambda^2\cosh(q_2z)+f_3\Lambda^3\cosh(q_3z)). \quad (58)$$

5. Applications

As an application of the problem, we take the source functions as

$$F(r,z)=z^2e^{-\omega r}, \quad (59)$$

$$f(r,t)=\frac{1}{2\pi r}\delta(ct-r). \quad (60)$$

Applying Laplace Transform and Hankel Transform, on Equations (59) and (60) , gives

$$\bar{F}^*(\xi,z)=\frac{z^2\omega}{(\xi^2+\omega^2)^{3/2}}\,, \quad (61)$$

$$\bar{f}^*(\xi,s)=\frac{1}{2\pi\sqrt{\xi^2+\frac{s^2}{c^2}}}\ . \quad (62)$$

The expressions of components of displacement, stress components, chemical potential function, temperature change, mass concentration and cubic dilatation can be obtained from Equations (50) - (58), by substituting the value of $\bar{F}^*(\xi,z)$ and $\bar{f}^*(\xi,s)$ from (61) and (62).

6. Particular cases

(i) If we neglect the diffusion effect (i.e. $\beta_2,a,b=0$) in Equations (50) - (58), we obtain the expressions for components of displacement, stress, chemical potential functions, temperature change, mass concentration and cubic dilatation for thermoelastic isotropic half space. In these expressions $\gamma_i,\ \mu_i,\ m_i,\ \alpha_i,\ \eta_i\ ,\nu_i$, and ζ_i take the form

$\gamma_i=\frac{q_i^2\mu_i}{m_i}+\lambda f_i-\rho c_1^2\ ,\mu_i=1+d_i+\mu f_i/\rho c_1^2,m_i=\frac{\mu}{\rho c_1^2}(q_i^2-\xi^2)-s^2,$

$$\alpha_i = \frac{\eta_i q_i}{m_i} + \frac{q_i \mu_i \xi}{m_i}, \eta_i = \xi(-\frac{\lambda+\mu}{\rho c_1^2} f_i + 1 + d_i), \nu_i = 0, \zeta_i = \frac{2\mu\xi\eta_i}{m_i} + \lambda f_i - \rho c_1^2,$$

$$d_i = 0 \ , \quad f_i = \frac{(q_i^2 - \xi^2)(\zeta_{18}) - \zeta_{14}\zeta_{18}}{(-q_i^2 + \xi^2)(\zeta_{12}) + \zeta_{12}},$$

where

$$\zeta_{11} = 0, \ \zeta_{12} = \frac{K{\beta_1}^2 T_0}{\rho^2 c_E^2 \tau_p^1} \frac{{\tau_q}^1 s K \tau_\eta^1}{{\tau_t}^1} s \ , \quad \zeta_{13} = 0 \ , \zeta_{14} = \frac{{\tau_q}^1}{{\tau_t}^1} K s \ ,$$

$$\zeta_{15} = \frac{K\beta_1 T_0 {\tau_q}^1 \xi^2}{\rho C_E {\tau_t}^1} \zeta_{16} = 0, \ \zeta_{17} = 0, \zeta_{18} = \frac{s {\tau_\eta}^1 k c_1^2}{{\tau_p}^1 \beta_1 C_E} \ , \zeta_{10} = 0.$$

(ii) If $\tau_q = \tau_t = 0$, in Equations (50) - (58), we obtain the expressions for components of displacement, stress components, chemical potential functions ,temperature change, mass concentration and cubic dilatation thermoelastic diffusive medium with dual phase- lag diffusion model . In these equations, ${\tau_q}^1$ and ${\tau_t}^1$ take the values

$${\tau_q}^1 = {\tau_t}^1 = 1.$$

(iii) If $\tau_\mathrm{p} = \tau_\eta = 0$, in Equations (50) - (58), we obtain the expressions for components of displacement, stress components, chemical potential functions, temperature change, mass concentration and cubic dilatation for thermoelastic diffusive isotropic half space with single -phase -lag heat (SPLT) model with the changed values of ${\tau_p}^1$ and ${\tau_\eta}^1$ as

$${\tau_p}^1 = {\tau_\eta}^1 = 1.$$

(iv) If $\tau_q = 0$ and $\tau_p = 0$, in Equations (50) - (58), we obtain the expressions for components of displacement, stress , chemical potential functions ,temperature change, mass concentration and cubic dilatation for single- phase-lag heat model (SPLT) and single-phase-lag diffusion model (SPLD) along with changed values of ${\tau_q}^1$ and ${\tau_p}^1$ as

$${\tau_q}^1 = {\tau_p}^1 = 1.$$

7. Inversion of double transform

To obtain the solution of the problem in physical domain, we must invert the transforms in Equations (50) - (58). These expressions are functions of z, the parameters of Laplace and Hankel Transforms s and ξ, respectively, and hence are of the form $\bar{f}^*$ (ξ, z, s). To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel Transform using

$$\bar{f}(r, z, s) = \int_0^\infty \xi \bar{f}^*(\xi, z, s) J_n(\xi r) d\xi. \tag{63}$$

Now for the fixed values of ξ , z and r the $\bar{f}(r, z, s)$ in the expression above can be considered as the Laplace Transform $\bar{g}(s)$ of $g(t)$. Following the method of Honig and Hirdes (1984), the Laplace Transform function $\bar{g}(s)$ can be inverted.

The last step is to calculate the integral in Equation (63). The method for evaluating this integral is described in Press et al. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

The mathematical model is prepared with copper material for purposes of numerical computation. The material constants for the problem are taken from Youssef (2006) and are given by

$$\lambda = 7.76 \times 10^{10} Nm^{-2}, \ \mu = 3.86 \times 10^{10} Nm^{-2}, K = 386 JK^{-1}m^{-1}s^{-1}, \rho = 8954\ Kgm^{-3},$$
$$\beta_1 = 5.518 \times 10^6\ \ Nm^{-2}deg^{-1}, \beta_2 = 61.38 \times 10^7\ \ Nm^{-2}deg^{-1}, a = 1.2\ \times 10^4 m^2/s^2k,$$
$$b = 0.9\ \times 10^6 m^5/kgs^2, D = 0.88\ \times 10^{-8} kgs/m^3,\ \ , T_0 = 293\text{K}, C_E = 383.1\, Jkg^{-1}K^{-1}.$$

An investigation has been conducted to compare the effect of time on dual phase lag model in heat conduction and diffusion ***and*** single phase lag model in heat conduction and diffusion, and the graphs have been plotted in the range $0 \leq r \leq 10$. The phase lags are taken as

$\tau_p = .02$, $\tau_\eta = .08$, $\tau_t = .04$ and $\tau_q = .06$.

- In the figures a solid line corresponds to the dual-phase-lag of heat transfer and diffusion (DPL) with nonzero values $t = 0.1$
- A solid line with center symbol circle corresponds to single phase lag (SPL) with t=0.1 with $\tau_p = 0 = \tau_t$.
- A small dashed line corresponds to the dual phase lag of heat transfer and diffusion (DPL) with t = 0.2.
- A small dashed line with center symbol diamond corresponds single phase lag (SPL) with t = 0.2 with $\tau_p = 0 = \tau_t$.

Figure 1 exhibits variations of axial displacement u_z with distance r. Near the loading surface, there is a sharp decrease for the range $0 \leq r \leq 1$ and the behavior is oscillatory afterwards for all the cases. Figure 2 shows variations of temperature change T with distance r. We find that there is a sharp increase for the range $0 \leq r \leq 2$ corresponding to t = 0.1 and t = 0.2 for both the cases i.e. DPL and SPL and the behavior is ascending and opposite oscillatory for the rest. Variations of radial stress component σ_{rr} with displacement r are shown in Figure 3. Here ,we observe that values of DPL for the range $2 \leq r \leq 5$ are less than SPL. However, the trend is opposite for the rest. Figure 4 shows variations of mass concentration C with distance r . Here variations are similar as discussed in Figure 3 with change of amplitude. Figure 5 gives variations of shear stress σ_{rz} with displacement r. Here, for t = 0.1 and t = 0.2 trends corresponding to SPL and DPL are similar and there are only small variations near zero for the range $6 \leq \mathrm{r} \leq 10$. Variations of hoop stress component $\sigma_{\theta\theta}$ with distance r are shown in Figure 6. Here, variations are similar with change in amplitude as discussed in Figure 3. Variations of vertical stress component σ_{zz} with distance r are exhibited in Figure 7. Near the loading surface, there is a sharp decrease for the range $0 \leq r \leq 2$. Values corresponding to SPL are greater than DPL for the range $2 \leq r \leq 5$ and are opposite for the rest. Variations of chemical potential function P with distance r are given in Figure 8. Here, for t = 0.1 and for t = 0.2 , SPL and DPL follow similar trends. Variations for t = 0.2 are descending oscillatory.

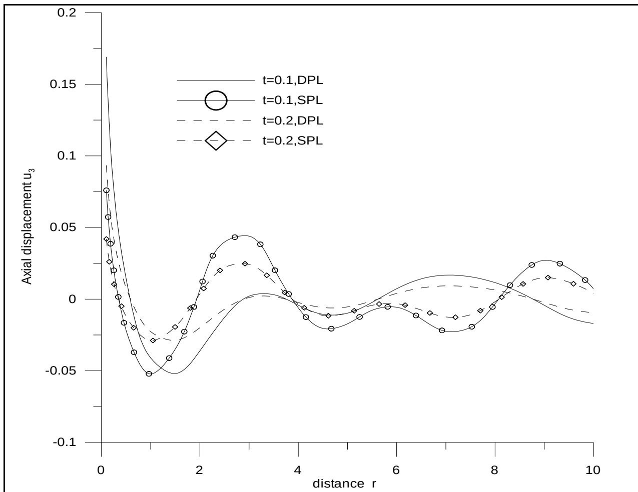

Figure1. Variations of axial displacement u_z with distance r

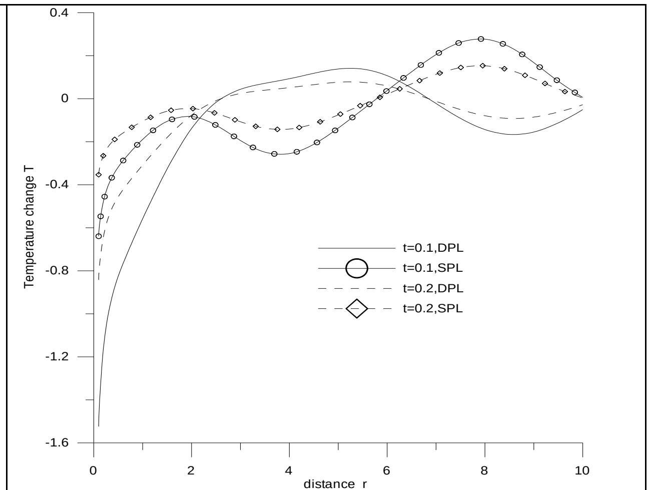

Figure 2. Variations of temperature change T with distance r

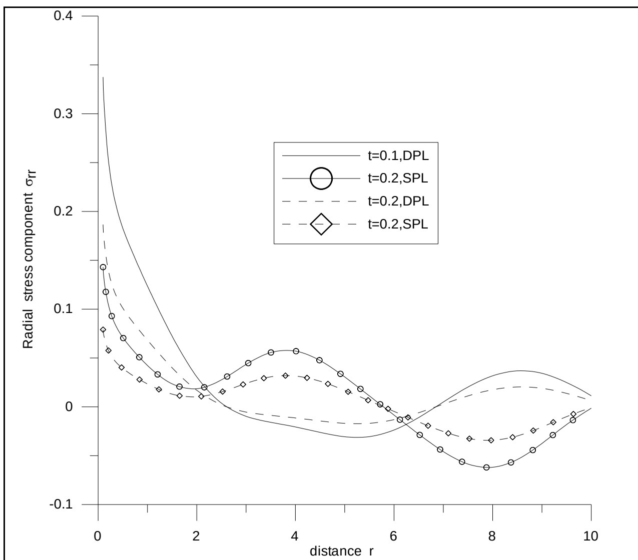

Figure 3. Variations of radial stress component σ_{rr} with displacement r

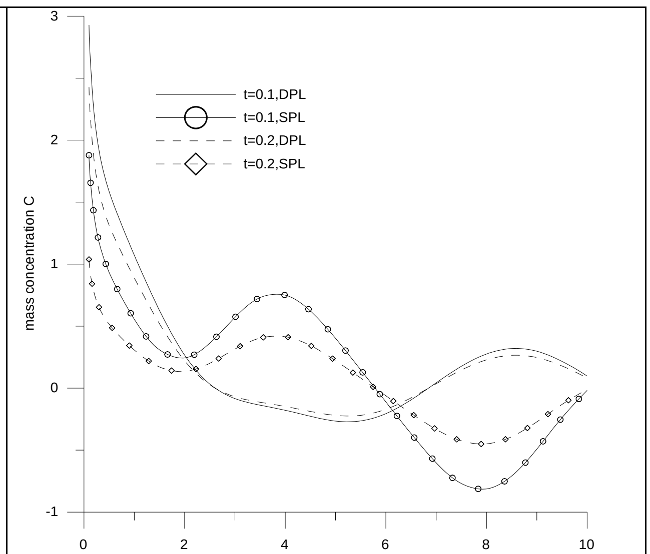

Figure 4. Variations of mass concentration C with distance r

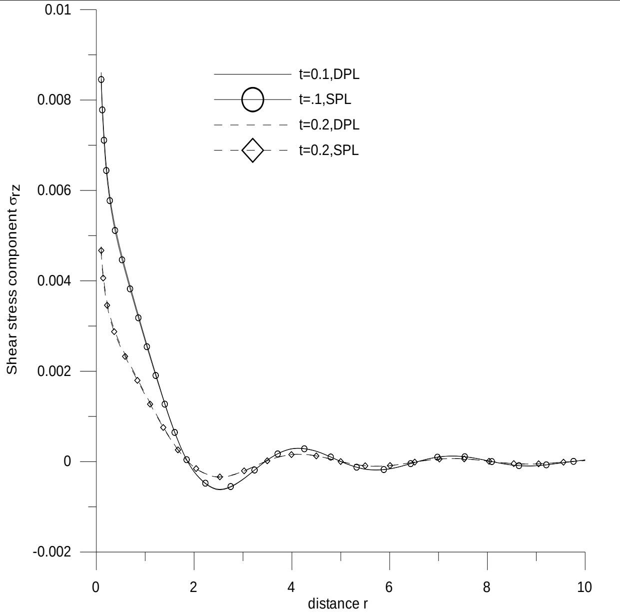

Figure 5. Variations of shear stress component σ_{rz} with displacement r

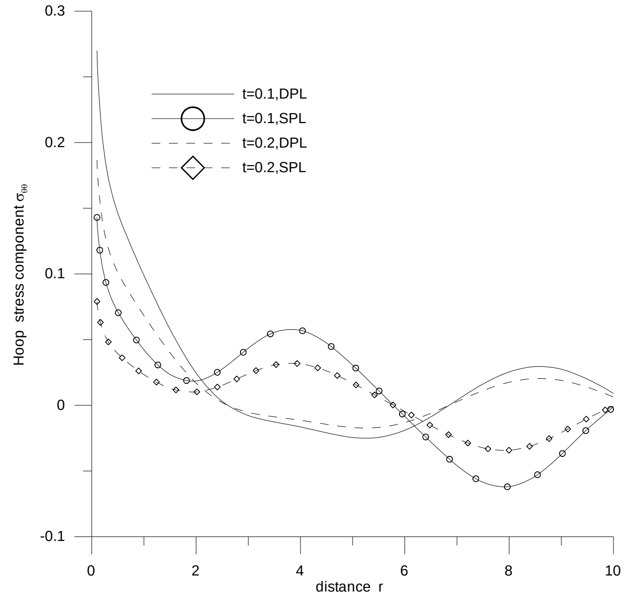

Figure 6. Variations of hoop stress component $\sigma_{\theta\theta}$ with distance r

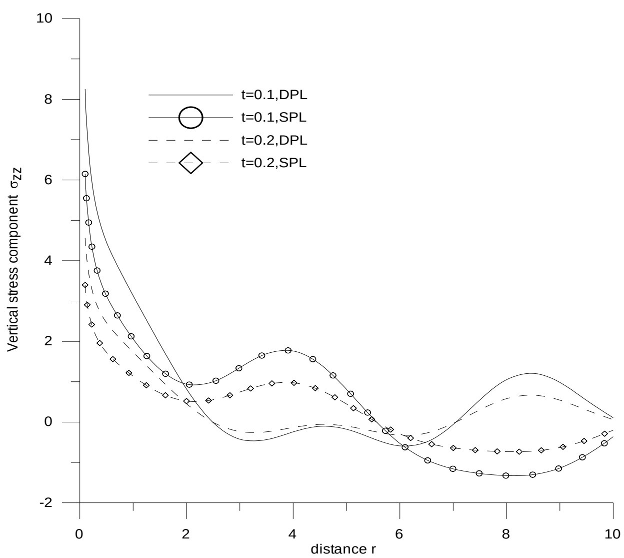

Figure 7. Variations of vertical stress component σ_{zz} with distance r

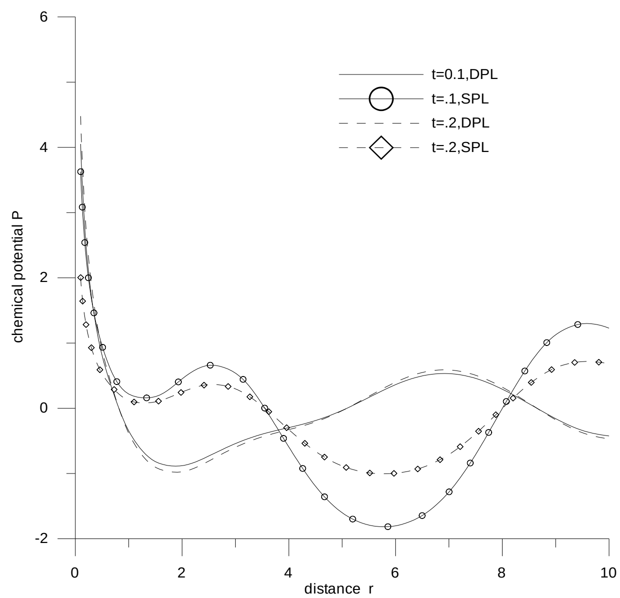

Figure 8. ariations of chemical potential function P with distance r

9. Conclusion

In this paper, we depicted the effect of time, thermal, and diffusion phase lags due to axisymmetric heat supply in a ring. We discussed the problem within the context of DPLT and DPLD models. The upper and lower surfaces of the ring are taken to be traction-free and subjected to an axisymmetric heat supply. The effect of time, diffusion and thermal phase-lags are shown on the various components.

From the graphs, we find that change in time changes the behavior of deformations of the various components of stresses, displacements, chemical potential function, temperature change and mass concentration. We also find that for t = 0.2, trends are oscillatory in all the cases whereas for t = 0.1, trends are quite different. Although being oscillatory, a big difference in the magnitudes is noticed.

A sound impact of diffusion and thermal phase-lags on the various quantities is found. A lot of difference in the trends of deformation while considering the single phase lag and dual phase lag is observed. The use of diffusion phase-lags in the equation of mass diffusion gives a more realistic model of thermoelastic diffusion media as it allows a delayed response between the relative mass flux vector and the potential gradient.

The result of the problem is useful in the two dimensional problem of dynamic response due to various sources of thermodiffusion which has various Geophysical and industrial applications.

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