Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics: An International Journal (AAM)

Vol. 11, Issue 1 (June 2016), pp. 266 - 284

Effect of Fluid Suction on an Oscillatory MHD Channel Flow with Heat Transfer

N. Ahmed[1], A. H. Sheikh[2] and D. P. Barua[3]
Department of Mathematics
Gauhati University
Guwahati-781014
Assam, India
[1]saheel_nazib@yahoo.com; [2]alihussainmath@gmail.com;[3]math_byte@yahoo.com

Received: May 3, 2015; Accepted: March 1, 2016

Abstract

Magnetohydrodynamics (MHD) is generally concerned with the study of the magnetic properties (behaviour) of electrically conducting fluids (plasmas, liquid metals etc.) moving in an electromagnetic field. The importance of the concept of MHD in various fields such as astrophysics, bio-medical research, missile technology and geophysics motivates the modelling and investigation of MHD flow and transport problems. The role of fluid suction is paramount in laminar flow control and has wide applications in fields such as aeronautical engineering, automobile engineering and rocket science. This fact inspires the study of the effects of fluid suction in flow and transport models. Time dependent flows are widely encountered in engineering applications such as turbines and in physiological studies such as flow of bio-fluid (blood etc.). In the present paper, an attempt has been made to investigate analytically the problem of a time dependent channel flow with heat transfer, where the channel is bounded by two infinite parallel porous walls. The pressure gradient is assumed to be oscillatory in nature. A magnetic field of uniform strength is assumed to be applied normal to the walls. After necessary idealization of the momentum and energy equations, the governing equations of our problem are solved by adopting the regular perturbation technique. The effects of magnetic field, suction velocity, viscous dissipation, Reynolds number, Prandtl number etc. on the flow and heat transfer are studied and demonstrated graphically. It is seen that magnetic field, fluid suction, viscous dissipation, Reynolds number, Prandtl number have a significant effect on the flow and heat transfer characteristic. For instance, the imposition of the magnetic field enhances the rates of heat transfer at the walls and the fluid suction decreases the temperature and aids in laminar flow control.

Keywords: MHD, Magnetohydrodynamics; Injection/suction; Ohmic dissipation; Induced magnetic field

MSC 2010 No.: 76D05, 80A20, 78A25

Nomenclature

C_p Specific heat at constant pressure, $\frac{Joule}{kg \times K}$
Ec Eckert number
H_0 Applied magnetic field, $\frac{Ampere}{m}$
$\bar{H}_x$ Induced magnetic field
h Non-dimensional induced magnetic field
$\bar{J}_x$ Current density
J Magnitude of current density, Ampere/m^2
k Thermal conductivity
L Wave length of the periodic suction
M Hartmann number
$\bar{p}$ Dimensional pressure gradient, *Pascal/m*
p Non-dimensional pressure
Pr Prandtl number
Re Reynolds number
Rm Magnetic Reynolds number
$\bar{t}$ Dimensional time, s
$\bar{T}$ Temperature of the fluid
$\bar{T}_1$ Fluid temperature at the lower plate, K
$\bar{T}_2$ Fluid temperature at the upper plate, K
ΔT Temperature difference between two plates, K
$(\bar{u},\bar{v})$ Components of the fluid velocity, m
(u,v) Non-dimensional components of the fluid velocity
$(\bar{X},\bar{Y})$ Coordinate system, m
(X,Y) Non-dimensional coordinate system
V_0 Suction velocity, *m/s*

Greek symbols

μ Coefficient of viscosity, $\frac{kg}{ms}$
μ_e Magnetic permeability, *henry/m*
σ Electrical conductivity, $(Ohm/m)^{-1}$
ρ Fluid density, $\frac{kg}{m^3}$
ω Frequency parameter
$\bar{\omega}$ Dimensional frequency parameter
ν Kinematic viscosity, $m^2 s^{-1}$
θ Non-dimensional temperature
η Magnetic diffusivity, $m^2 s^{-1}$
ε Small reference parameter
The other symbols have their usual meanings.

1. Introduction

MHD (Magnetohydrodynamics) is generally regarded as the science of motion of electrically conducting fluids, such as plasmas and liquid metals, in electromagnetic fields. In such situations, the currents generated in the fluid due to induction alter the field, thereby resulting in the coupling of the field and dynamics equations. MHD treats, especially, conductive fluids, whether liquid or gaseous, in which certain idealizations (simplifying assumptions) are accepted. Studies in MHD channel flows are of wide interest among many workers due to their great importance in the field of industrial applications such as MHD generator, MHD pump, Nuclear reactors, etc. Significant works on various topics concerning MHD situations abound in the works of Chang and Yen (1965), Cowling (1957), Hughes and Young (1960), Sutton and Sherman (1965), Sengupta (2015), Oahimire and Olajuwon (2014) to cite a few.

Fluid suction plays a paramount role in laminar flow control and finds many applications in areas such as aeronautical engineering, automobile engineering and rocket science. The importance of laminar flow control using fluid suction was pointed out by several research workers like Muhuri (1963), Ramamoorty (1962), Rathy (1963), Verma and Bansal (1966), Govindarajulu (1976), and Shukla (1963), Dessie and Kishan (2014), Masthanrao et al. (2013) through their works.

Time dependent (unsteady or transient) flows are of great importance in engineering applications such as turbines and in physiological investigations such as flow of bio-fluid (blood, blood plasma, etc.).

Channel flows are encountered in several situations such as natural drainage of water through river systems, flow in canals and sewers, flow in pipes, etc. Modelling of channel flows through porous media are of great significance in the investigation of underground water resources, oil and natural gas reservoirs, flow of fluid in geothermal regions, chemical purification techniques, cooling techniques in electronic devices, etc. Clearly, channel flows are of considerable interest in engineering geophysics, chemical engineering, electronics, etc. The works by Jain and Gupta (2006) and Ahmed and Barua (2008) may be cited in this regard. The analysis of MHD channel flow problems become meaningful when the combined influence of magnetic field and viscous thermal energy dissipation are taken into account for investigating high speed flows, and such attempts were made by researchers like Hitesh Kumar (2009), Soundalgekar and Bhatt (1976), Cookey Israel (2003), Ahmed and Kalita (2010) and Manjulatha et al. (2014) to mention a few.

The main objective of our present study is to extend the work done by Soundalgekar and Bhatt (1976), by considering the effect of suction/injection on the flow and transport characteristics. In the current work an attempt has been made to study analytically the problem of a transient channel flow with heat transfer, where the channel is bounded by two infinite parallel porous walls. We assume an oscillatory pressure gradient for this pressure driven flow. A uniform magnetic field is imposed on the flow, normal to the walls. After necessary idealization of the momentum and energy equations, the equations governing our flow and transport model are solved by employing the regular perturbation technique. The influence of magnetic field, suction velocity, viscous dissipation, Reynolds number, Prandtl number etc. on the flow and heat transfer are analysed and demonstrated graphically.

2. Mathematical Analysis

Consider a fully developed flow of an electrically conducting viscous incompressible fluid between two infinite parallel plates. The origin is taken along the centre line of the channel with $\overline{X}$ axis taken along the direction of the flow and $\overline{Y}$ axis taken normal to the plate which is also the direction of the applied uniform magnetic field. The plates are assumed to be electrically non-conducting. We consider an oscillatory pressure gradient in the form

$$-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} = A\left(1+\varepsilon e^{i\overline{\omega}\overline{t}}\right),\ \varepsilon \le 1, \text{ where } A \text{ is constant.}$$

Then, the fully developed unsteady flow is governed by the following equations:

Equation of continuity:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0$$

$$=> \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \quad (\text{since } \overline{u} \text{ is free of } \overline{x})$$

$$=> \overline{v} = \text{a constant} = -V_0, \text{ the suction velocity.} \tag{1}$$

Momentum equation:

$$\frac{\partial \overline{u}}{\partial \overline{t}} - V_0 \frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \nu \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + \frac{\mu_e H_0}{4\pi\rho}\frac{\partial \overline{H}_x}{\partial \overline{y}}\ . \tag{2}$$

Magnetic diffusion equation:

$$\frac{\partial \overline{H}_x}{\partial \overline{t}} = H_0 \frac{\partial \overline{u}}{\partial \overline{y}} + \eta \frac{\partial^2 \overline{H}_x}{\partial \overline{y}^2}\ . \tag{3}$$

Energy equation:

$$\rho c_p\left(\frac{\partial \overline{T}}{\partial \overline{t}} - V_0 \frac{\partial \overline{T}}{\partial \overline{y}}\right) = k\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \mu\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^2 + \frac{\overline{J}_x^2}{\sigma}\ . \tag{4}$$

The last two terms on the right hand side of the Equation (4) represent the viscous and Joule dissipations.

The relevant boundary conditions are:

$$\overline{u}(\pm L) = 0,\ \ \overline{H}_x(\pm L) = 0,\ \ \overline{T}(+L) = \overline{T}_2,\ \ \overline{T}(-L) = \overline{T}_1\ . \tag{5}$$

In order to make the mathematical model normalized, we introduce the following non-dimensional quantities:

$$y=\frac{\bar{y}}{L},\ \ h=\frac{\bar{H}_x}{H_0 Rm},\ \ u=\frac{\bar{u}}{V_0},\ \ \omega=\frac{\bar{\omega}L^2}{\nu},\ \ Rm=4\pi\mu_e\sigma LV_0,\ M^2=\frac{\mu_e^2H_0^2L^2\sigma}{\mu},$$
$$Re=\frac{LV_0}{\nu},\ \ V_0=\frac{AL^2}{\nu},\ \ Pr=\frac{\mu c_p}{k},Ec=\frac{V_0^2}{c_p\Delta T},\ \ J=\frac{\bar{J}_x}{\sigma\mu_e H_0V_0},\ \ \theta=\frac{\bar{T}-\bar{T}_1}{\bar{T}_2-\bar{T}_1}. \tag{6}$$

All the physical quantities are defined in the Nomenclature.

The non-dimensional governing equations are:

$$\frac{\partial u}{\partial t}-Re\frac{\partial u}{\partial y}=1+\varepsilon e^{i\omega t}+\frac{\partial^2 u}{\partial y^2}+M^2\frac{\partial h}{\partial y}\ , \tag{7}$$
$$\frac{\partial h}{\partial t}=\frac{Re}{Rm}\frac{\partial u}{\partial y}+\frac{\eta}{\nu}\frac{\partial^2 h}{\partial y^2}\ , \tag{8}$$
$$\frac{\partial \theta}{\partial t}-Re\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2+M^2J^2Ec\,. \tag{9}$$

The relevant boundary conditions in non-dimensional forms are:

$$\left.\begin{aligned} &y=-1: u=0,\ h=0,\ \theta=0,\\ &y=1:\ u=0,\ h=0,\ \theta=0.\end{aligned}\right\} \tag{10}$$

To solve the Equations (7), (8) and (9), we take $Re=Rm$ and

$$\left.\begin{aligned} u&=u_0(y)+\varepsilon e^{i\omega t}u_1(y),\\ h&=h_0(y)+\varepsilon e^{i\omega t}h_1(y),\\ \theta&=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y),\\ J&=J_0(y)+\varepsilon e^{i\omega t}J_1(y),\end{aligned}\right\} \tag{11}$$

where

$$J_0=-\frac{\partial h_0}{\partial y},\ \ J_1=-\frac{\partial h_1}{\partial y}.$$

Substituting the transformation (11) in (7), (8) and (9), we derive the following set of differential equations:

$$\frac{d^2u_0}{dy^2}+M^2\frac{dh_0}{dy}=-1-Re\frac{du_0}{dy}\ , \tag{12}$$
$$\frac{d^2u_1}{dy^2}+M^2\frac{dh_1}{dy}+1=i\omega u_1-Re\frac{du_1}{dy}\ , \tag{13}$$

$$\frac{du_0}{dy}+\frac{d^2h_0}{dy^2}=0, \tag{14}$$

$$\frac{du_1}{dy}+\frac{d^2h_1}{dy^2}=i\omega N h_1, \text{ where } N=\frac{Rm}{Re}, \tag{15}$$

$$\frac{d^2\theta_0}{dy^2}+Pr\,Re\frac{d\theta_0}{dy}=-Pr\,Ec\left(\frac{du_0}{dy}\right)^2-M^2J_0^2Pr\,Ec\ , \tag{16}$$

$$\frac{d^2\theta_1}{dy^2}+PrRe\frac{d\theta_1}{dy}-iPr\,\omega\theta_1=-2PrEc\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right)-2M^2PrEcJ_0J_1\ , \tag{17}$$

with the boundary conditions:

$$\left.\begin{aligned} u_0(\pm1)=0,\ u_1(\pm1)=0,\\ h_0(\pm1)=0,\ h_1(\pm1)=0,\\ \theta_0(\pm1)=0,\ \theta_1(\pm1)=0.\end{aligned}\right\} \tag{18}$$

Equations (12) to (15) are solved subject to the boundary conditions (18) and the solutions are as follows:

$$u_0(y)=-c_2m_2e^{m_2y}-c_3m_3e^{m_3y}+c_4\ , \tag{19}$$

$$h_0(y)=c_1+c_2e^{m_2y}+c_3e^{m_3y}-\frac{y}{M^2}\ , \tag{20}$$

$$u_1(y)=c_5e^{m_5y}+c_6e^{m_6y}+c_7e^{m_7y}+c_8e^{m_8y}+\frac{1}{i\omega}\ , \tag{21}$$

$$h_1(y)=\frac{b}{M^2}c_5e^{m_5y}+\frac{b}{M^2}c_6e^{m_6y}-\frac{a}{M^2}c_7e^{m_7y}-\frac{a}{M^2}c_8e^{m_8y}\ . \tag{22}$$

The expression for the velocity field is given by

$$\begin{aligned} u(y)&=u_0(y)+\varepsilon e^{i\omega t}u_1(y)\\ &=-c_2m_2e^{m_2y}-c_3m_3e^{m_3y}+c_4+\varepsilon e^{i\omega t}\left(c_5e^{m_5y}+c_6e^{m_6y}+c_7e^{m_7y}+c_8e^{m_8y}+\frac{1}{i\omega}\right), \end{aligned} \tag{23}$$

where $c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,m_2,m_3,m_5,m_6,m_7,m_8$ are defined in the Appendix.

The solutions of the Equations (16) and (17) under (18) are:

$$\theta_0(y)=c_9+c_{10}e^{-PrRey}-PrEc\left(P_{11}e^{2m_2y}+P_{12}e^{2m_3y}-P_{13}e^{m_2y}-P_{14}e^{m_3y}+P_{15}y\right). \tag{24}$$

$$\begin{aligned}\theta_1(y)=c_{11}e^{m_{11}y}+c_{12}e^{m_{12}y}+2PrEc\Big(&P_{16}e^{(m_2+m_5)y}+P_{17}e^{(m_2+m_6)y}+P_{18}e^{(m_2+m_7)y}+P_{19}e^{(m_2+m_8)y}\\ &+P_{20}e^{(m_3+m_5)y}+P_{21}e^{(m_3+m_6)y}-P_{22}e^{m_5y}-P_{23}e^{m_6y}-P_{24}e^{m_7y}-P_{25}e^{m_8y}\Big).\end{aligned} \tag{25}$$

The expression for the temperature field is given by:

$$\theta(y) = \theta_0(y) + \varepsilon e^{i\omega t}\theta_1(y)$$
$$= c_9 + c_{10}e^{-PrRey} - PrEc\left(P_{11}e^{2m_2y} + P_{12}e^{2m_3y} - P_{13}e^{m_2y} - P_{14}e^{m_3y} + P_{15}y\right)$$
$$+ \varepsilon e^{i\omega t}\{c_{11}e^{m_{11}y} + c_{12}e^{m_{12}y} + 2PrEc(P_{16}e^{(m_2+m_5)y} + P_{17}e^{(m_2+m_6)y} + P_{18}e^{(m_2+m_7)y}$$
$$+ P_{19}e^{(m_2+m_8)y} + P_{20}e^{(m_3+m_5)y} + P_{21}e^{(m_3+m_6)y} - P_{22}e^{m_5y} - P_{23}e^{m_6y} - P_{24}e^{m_7y} - P_{25}e^{m_8y})\}, \quad (26)$$

where

c_9, c_{10}, c_{11}, c_{12}, m_2, m_3, m_5, m_6, m_7, m_8, m_{11}, m_{12}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} are defined in the Appendix.

3. Skin frictions

The skin frictions τ_1 and τ_2 at the plates $y = 1$ and $y = -1$ respectively are as follows:

$$\tau_1 = \tau_1^0 + \varepsilon|B|\cos(\omega t + \alpha)\ , \quad (27)$$

where

$$\tau_1^0 = \left[\frac{du_0}{dy}\right]_{y=1},\ |B| = \sqrt{\left(B_r^2 + B_i^2\right)},\ \alpha = \tan^{-1}\left(\frac{B_i}{B_r}\right),$$

$$B_r = \text{Real part of }\left[\frac{du_1}{dy}\right]_{y=1},\ B_i = \text{Imaginary part of }\left[\frac{du_1}{dy}\right]_{y=1},$$

and

$$\tau_2 = \tau_2^0 + \varepsilon|C|\cos(\omega t + \beta)\ , \quad (28)$$

where

$$\tau_2^0 = \left[\frac{du_0}{dy}\right]_{y=-1},\ |C| = \sqrt{\left(C_r^2 + C_i^2\right)},\ \beta = \tan^{-1}\left(\frac{C_i}{C_r}\right),$$

$$C_r = \text{Real part of }\left[\frac{du_1}{dy}\right]_{y=-1},\ C_i = \text{Imaginary part of }\left[\frac{du_1}{dy}\right]_{y=-1}.$$

4. Nusselt number

The coefficient of the rates of the heat transfer at the plates $y = 1$ and $y = -1$ in terms of the Nusselt number are as follows:

$$Nu_1 = Nu_1^0 + \varepsilon|D|\cos(\omega t + \varphi), \quad (29)$$

where

$$Nu_1^0 = \left[\frac{d\theta_0}{dy}\right]_{y=1},\ |D| = \sqrt{\left(D_r^2 + D_i^2\right)},\ \varphi = \tan^{-1}\left(\frac{D_i}{D_r}\right),$$

$$D_r = \text{Real part of } \left[\frac{d\theta_1}{dy}\right]_{y=1}, \quad D_i = \text{Imaginary part of } \left[\frac{d\theta_1}{dy}\right]_{y=1},$$

and

$$Nu_2 = Nu_2^0 + \varepsilon|F|\cos(\omega t + \delta), \tag{30}$$

where

$$Nu_2^0 = \left[\frac{d\theta_0}{dy}\right]_{y=-1}, \quad |F| = \sqrt{(F_r^2 + F_i^2)}, \quad \delta = \tan^{-1}\left(\frac{F_i}{F_r}\right),$$

$$F_r = \text{Real part of } \left[\frac{d\theta_1}{dy}\right]_{y=-1}, \quad F_i = \text{Imaginary part of } \left[\frac{d\theta_1}{dy}\right]_{y=-1}.$$

5. Results and discussion

In order to get the physical insight into this problem, data tabulation was carried out and the subsequent figures for the velocity and temperature fields, skin-friction and Nusselt number are presented. The influence of the magnetic field, the viscous dissipation and the suction on the flow and heat transfer have been depicted graphically and discussed through the study of the effect of non-dimensional parameters viz. Hartmann number (*M*), Prandtl number (*Pr*), Eckert number (*Ec*) and Reynolds number (*Re*) on the dimensionless velocity *u* and temperature *Q*, non-dimensional skin frictions τ_1, τ_2 and the dimensionless rates of heat transfer Nu_1 and Nu_2 at the walls $y = 1$ and $y = -1$.

Figures 1 and 2 depict how the magnetic field and the suction affect the flow. It is seen that a growth in the magnetic field strength accelerates the flow whereas the suction retards the fluid flow. This is obvious from the fact that the velocity *u* increases as *M* rises and *u* decreases as *Re* rises. Hence, the magnetic field and the fluid suction helps in controlling the flow field. It is a well-known fact that electric currents are induced whenever a moving electrically conducting fluid is subjected to the presence of a magnetic field. When these electric currents interact with the applied magnetic field, an electromagnetic force known as the Lorentz force is generated. Generally, the Lorentz force slows down the motion of the fluid, particularly in the core (middle part) of the flow. This impediment to the flow field in the core region is quite evident from the Figures 1 and 2. It can be seen from Figures 1 and 2 that the velocity profiles are slightly flattened in the middle when compared to the edges. The edges seem to bulge out slightly. Thus, the magnetic field as well as the fluid suction helps in decelerating the flow in the core region of the flow and hence the magnetic field and fluid suction can control the core velocity. Consequently, the flow velocities are comparatively higher at the edges of the flow profiles.

The effects of the magnetic field, viscous dissipation, and fluid suction on the fluid temperature have been portrayed in Figures 3, 4, 5 and 6, respectively. It is inferred that the fluid temperature *Q* rises with the increase in each of magnetic parameter / Hartmann number (*M*), Prandtl number (*Pr*) and the Eckert number (*Ec*). Clearly, an increase in each of magnetic field strength, electrical conductivity, and viscous dissipative energy leads to a growth in the fluid temperature. The thermal energy dissipated on account of fluid viscosity (internal fluid friction) causes the temperature to rise. But the influence of viscous energy dissipation is significant at the edges of the flow region and trivial in the middle of the flow region. This is obvious from the somewhat flattened temperature profiles at the core region, as can be seen from the Figures 3 to 6. Subsequently, the temperature is slightly higher at the edges of the profiles. However, the temperature drops as the suction Reynolds number (*Re*) rises i.e. as the fluid suction increases. Again, as in the case of velocity profiles, we also observe from the Figures 3 to 6 that the magnetic field (through *M*) and the fluid suction (through *Re*) help in reducing the temperature in the core region of flow. This is apparent from the slightly flattened temperature profiles at the middle region, as can be seen from the Figures 3 to 6. Consequently, the temperature is slightly higher at the edges of the profiles. Thus, the magnetic field and the fluid suction aid in regulating the temperature field. Figures 1 to 6 show that the flow and temperature profiles and hence the velocity and thermal boundary layers are analogous.

Figures 7 and 8 portray the influence of fluid suction (through the Reynolds number *Re*) and the magnetic field (through the Hartmann number M) on the skin frictions τ_1, τ_2 at the walls $y = 1$ and $y = -1$, respectively. We note that $\tau_1 < 0$ and $\tau_2 > 0$. Thus, for our model, the upper wall ($y = 1$) exerts drag on the moving fluid whereas the moving fluid exerts drag on the lower wall ($y = -1$). It is observed that a growth in fluid suction (i.e. suction Reynolds number *Re*) causes the *magnitude* of skin friction τ_1 to decrease and the *magnitude* of skin friction τ_2 to increase. Further, the same figures also depict that the imposition of the magnetic field (i.e. increase in *M*) augments the *magnitudes* of the skin frictions at the walls. Clearly, the suction and the magnetic field act as regulatory mechanisms for controlling the skin-frictions at the walls.

Figures 9, 10, 11, 12, 13, and 14 illustrate the behaviours of the rates of heat transfer at the walls under the effects of fluid suction, and viscous energy dissipation. It may also be noted from Figures 9 to 14 that Nu_1 is negative whereas Nu_2 is positive. Thus, in our model, heat flows from the upper wall towards the fluid and the lower wall receives heat from the fluid. It follows from Figures 9 and 10 that an augmentation in the suction Reynolds number (*Re*) leads to a fall in the *magnitude* of Nu_1 at the wall $y = 1$ and a growth in the *magnitude* of Nu_2 at the wall $y = -1$. Clearly, Figures 9 and 10 illustrate that as *Re* increases i.e. as the suction V_0 increases, the *magnitude* of heat transfer at the upper wall ($y = 1$) falls and that at the lower wall ($y = -1$) rises. Subsequently, the heat transfer from the upper wall to the fluid tends to fall and that from the fluid to the lower wall tends to rise, on account of increasing suction.

Thus, suction plays a vital role in regulating heat transfer at the walls. The same Figures 9 and 10 indicate that the *magnitudes* of Nu_1 and Nu_2 increase as *M* increases i.e. as the applied magnetic field strength (H_0) or the electrical conductivity (σ) of the fluid increases. Recalling that Nu_1 is negative whereas Nu_2 is positive, it may be inferred that the heat transfer from the upper wall ($y = 1$) to the fluid increases as the applied magnetic field strength increases. Similarly, a rise in the magnetic field strength leads to enhanced heat transfer from the fluid to the lower wall ($y = -1$). Hence, the imposition of the magnetic field enhances the magnitudes of heat flux at the walls. Evidently, the imposition of a magnetic field is beneficial for effective heat transfer at the walls. Hence, the application of the magnetic field and the fluid suction facilitates an efficient control of the heat flux at the walls. From Figures 11 to 14, it is apparent that an increase in the Prandtl number (*Pr*) and the Eckert number (*Ec*) enhance the magnitudes of the rates of heat transfer at the walls. It may be noted that a rise in Eckert number *Ec* indicates a growth in the viscous dissipative energy and this consequently leads to greater heat transfer at the walls.

6. Conclusions

In view of the facts presented in the preceding 'Results and discussion' section, the following conclusions are drawn:

- An increase in the magnetic field strength accelerates the flow whereas a growth in suction impedes the flow. Also, the imposition of the magnetic field as well as the fluid suction assists in reducing the core velocity i.e. the flow velocity in the core region of flow. This is of practical importance concerning flows such as those of liquid metals through rectangular channels in fusion power reactors, and flows of coolant fluids through channels in micro fluidic devices etc.

- Growth in fluid suction reduces the temperature field. An increase in the magnetic field increases the fluid temperature due to greater amount of Joule heating. However, the fluid suction and the magnetic field have a damping effect on the temperature in the core region of flow.

- The viscous energy dissipation raises the fluid temperature i.e. high speed flows or highly viscous fluid flows will lead to a growth in fluid temperature. In many engineering flow and transport situations, high temperature is undesirable because it causes wear and tear to material surfaces.

- An increase in the strength of the magnetic field leads to a corresponding growth in the magnitudes of the skin frictions at the walls. Consequently, the application of strong magnetic field is undesirable. Growth in fluid suction causes the *magnitude* of skin friction at the upper wall to decrease and that at the lower wall to increase. Clearly large suction may be avoided in order to minimize the adverse effects of skin friction at the lower wall. The influence of suction on the wall-skin frictions is marked in presence of relatively strong magnetic fields.

- The heat transfer from the upper wall to the fluid exhibits a drop and that from the fluid to the lower wall exhibits a growth due to a rise in fluid suction. Moreover, a growth in specific heat of the fluid (through Prandtl number) leads to a growth in channel fluid temperature as well as increased heat transfer at the walls.

- The thermal energy dissipation due to viscosity increases the magnitudes of the rates of heat transfer at the walls. Since viscous thermal energy dissipation unduly raises the fluid temperature, it is necessary to maintain low flow speeds in order to achieve laminar flow control.

- The imposition of the magnetic field boosts the rates of heat transfer at the walls by facilitating increased heat transfer from the upper wall to the fluid and from the fluid to the lower wall.

GRAPHS

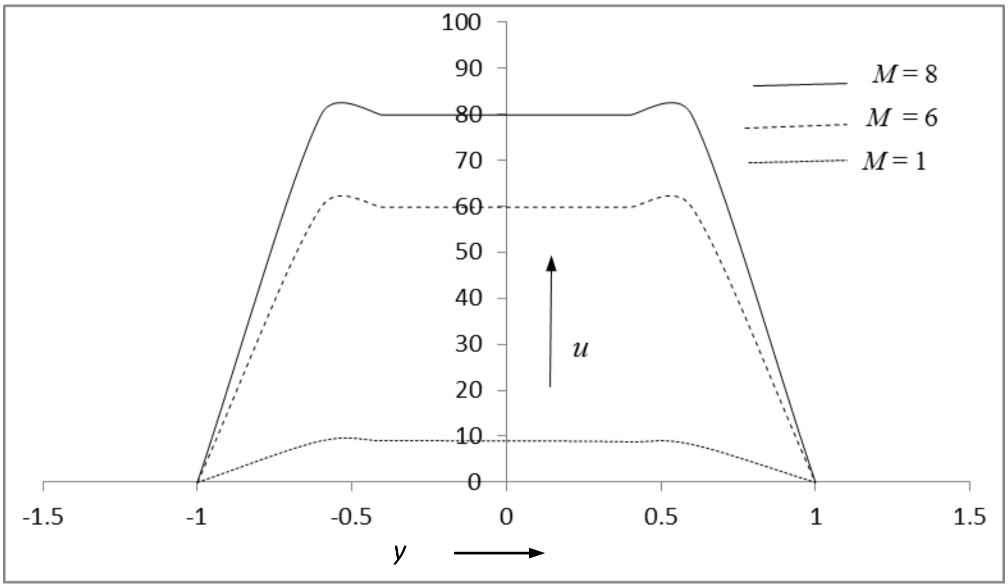

Figure 1. Velocity *u* against *y*, under *M* for *Re*=1, *t*=1, *Pr*=7, *Ec*=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

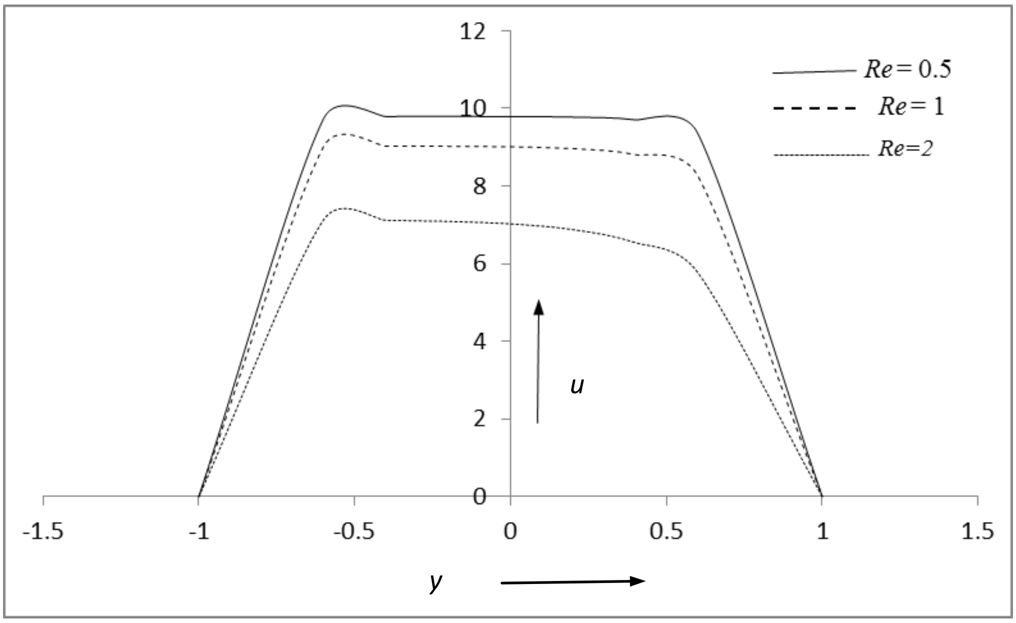

Figure 2. Velocity *u* against *y*, under *Re* for *Pr*=7, *Ec*=0.1, *M*=1, *t*=1, $\omega = 0.5$, $\varepsilon = 0.1$

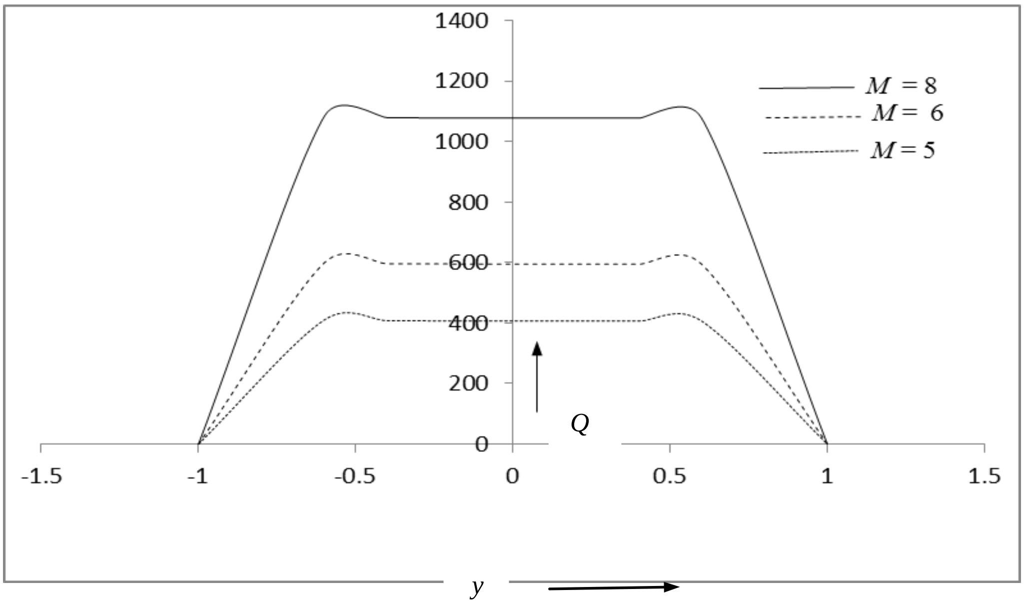

Figure 3. Temperature Q against y, under M for Re=1, t=1, Pr=7, Ec=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

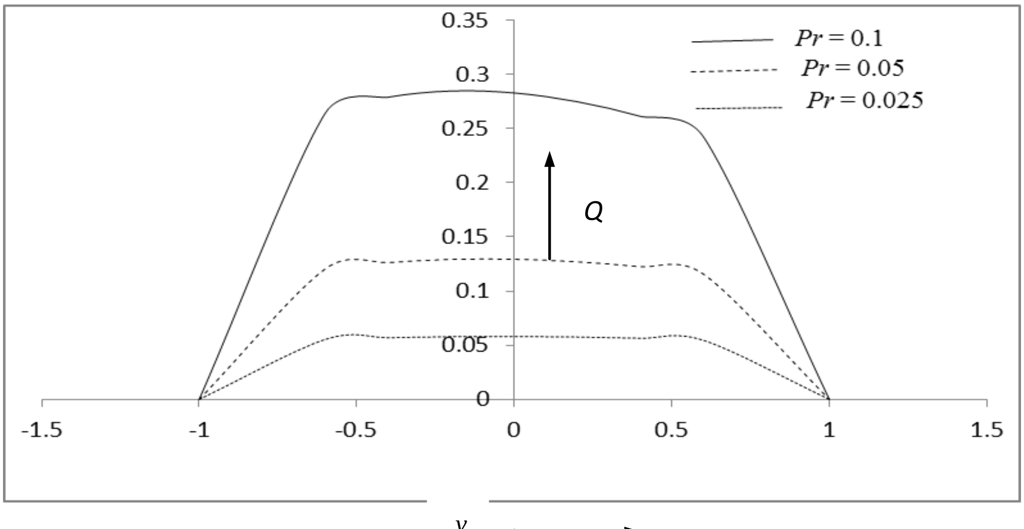

Figure 4. Temperature Q against y, under Pr for Re=1, t=1, M=1, Ec=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

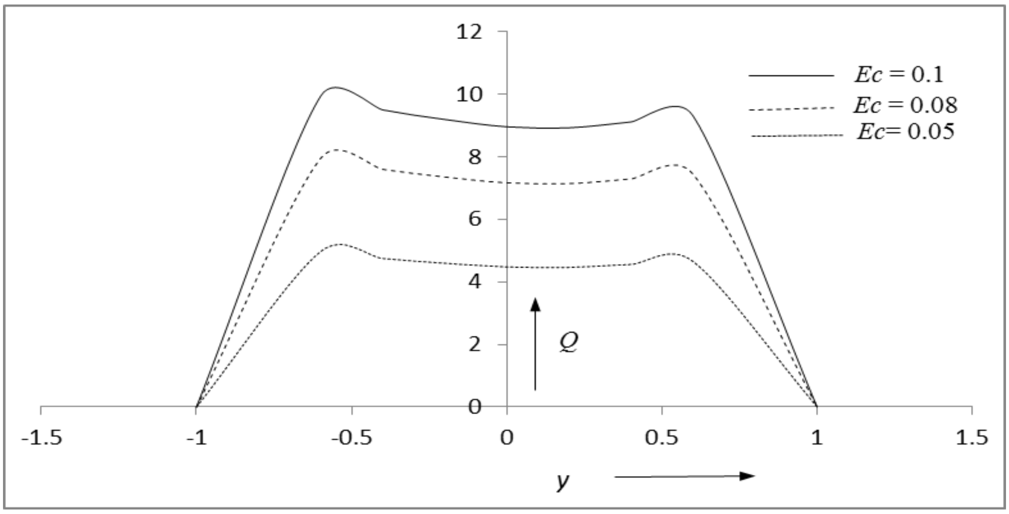

Figure 5. Temperature Q against y, under Ec for Re=1, t=1, M=1, Pr=7, $\omega = 0.5$, $\varepsilon = 0.1$

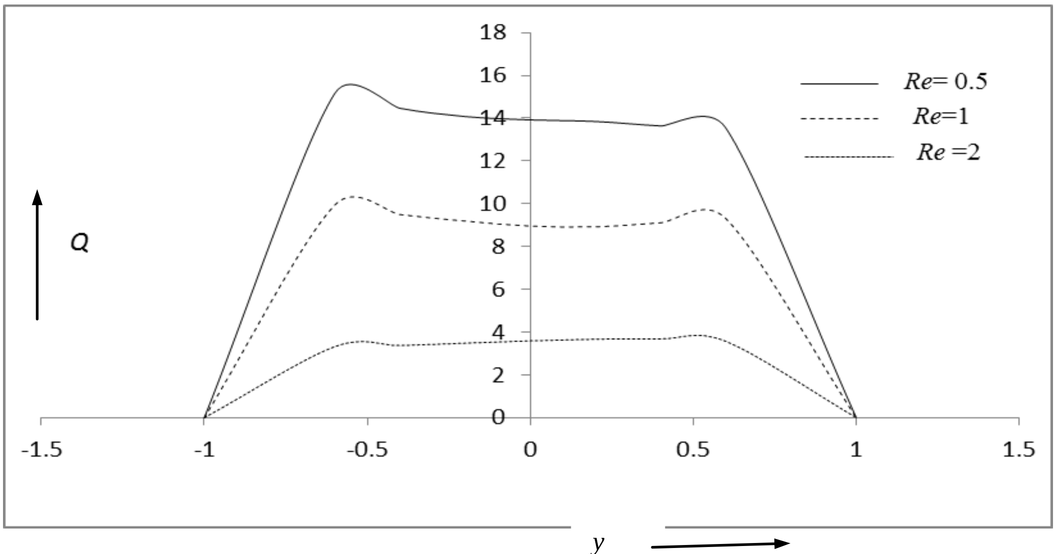

Figure 6. Temperature *Q* against *y*, under *Re* for *Pr* = 7, *t*=1, *M*=1, *Ec*=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

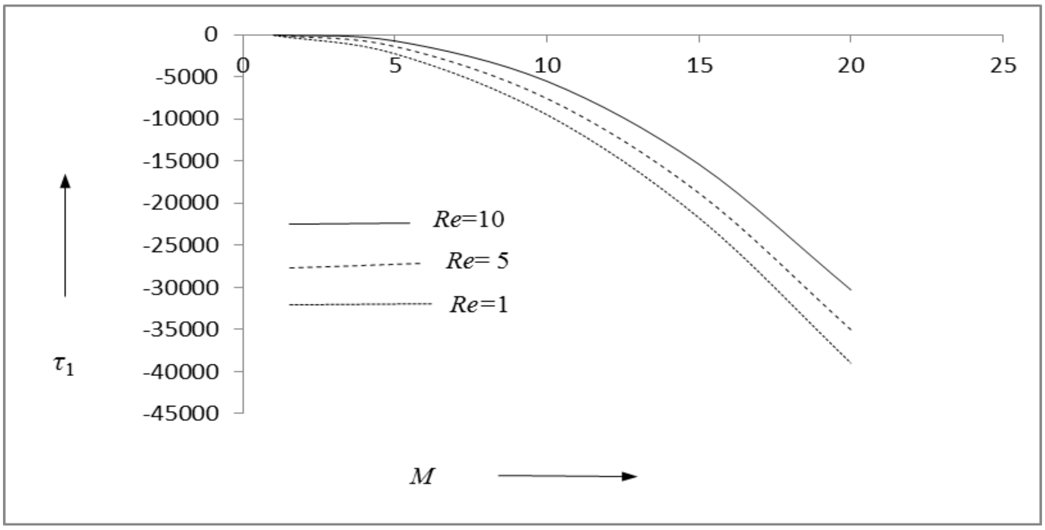

Figure 7. Skin-friction τ_1 against *M*, under *Re* for *t*=1, *Ec*=0.1, *Pr*=7, $\omega = 0.5$, $\varepsilon = 0.1$

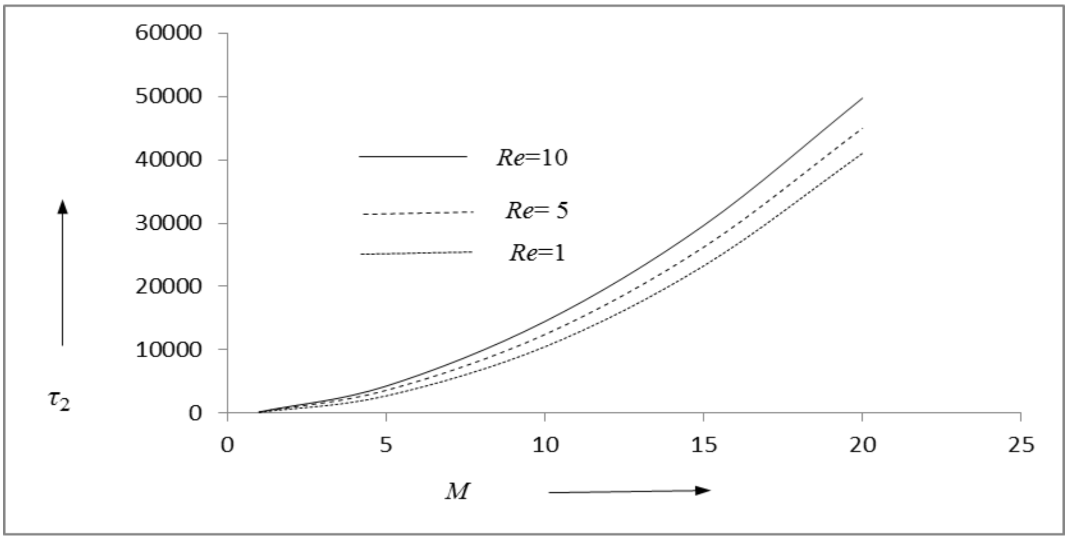

Figure 8. Skin-friction τ_2 against *M*, under *Re* for *t*=1, *Ec*=0.1, *Pr*=7, $\omega = 0.5$, $\varepsilon = 0.1$

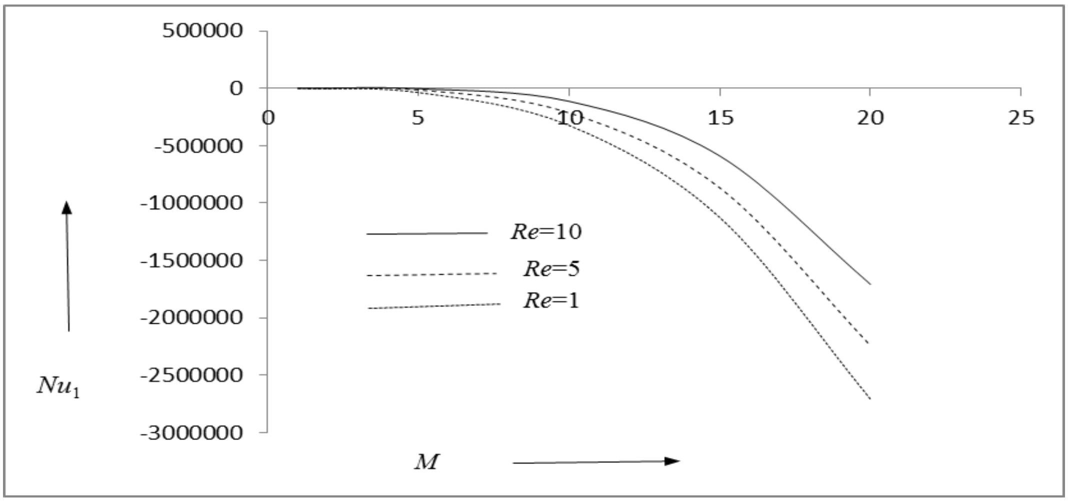

Figure 9. Nusselt number Nu_1 against *M*, under *Re* for t=1, *Pr* =7, *Ec*=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

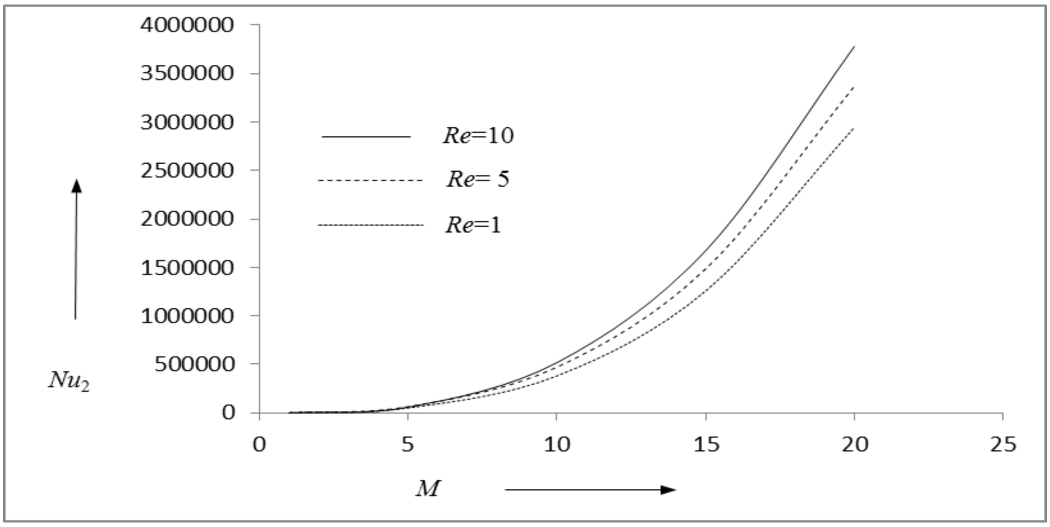

Figure 10. Nusselt number Nu_2 against *M*, under *Re* for t=1, *Pr* =7, *Ec*=0.1, $\omega = 0.5$, $\varepsilon = 0.1$

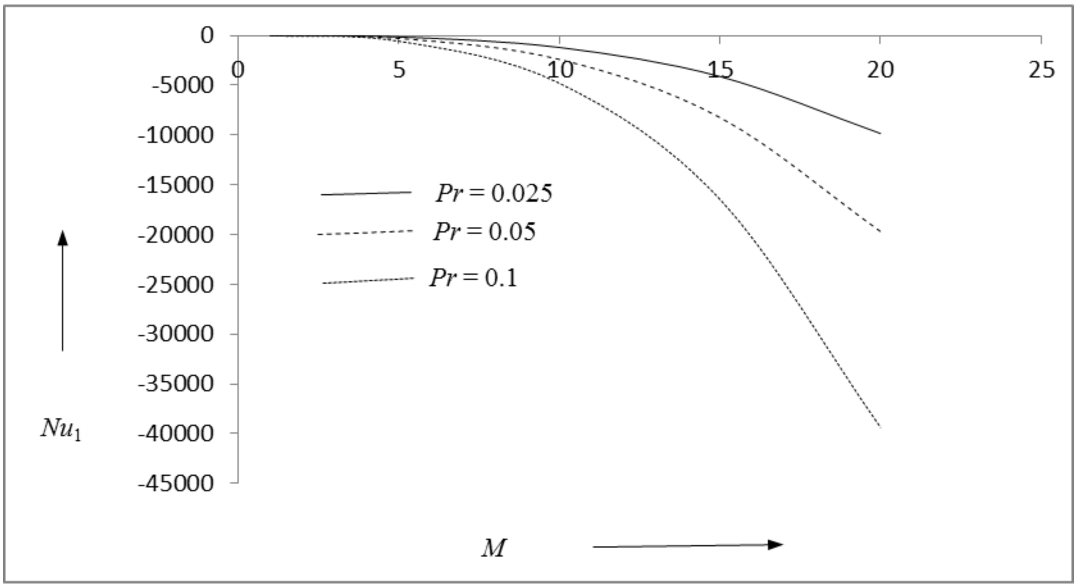

Figure 11. Nusselt number Nu_1 against *M*, under *Pr* for t=1, *Re*=1, *Ec*=0.1, $\omega = 0.5, \varepsilon = 0.1$

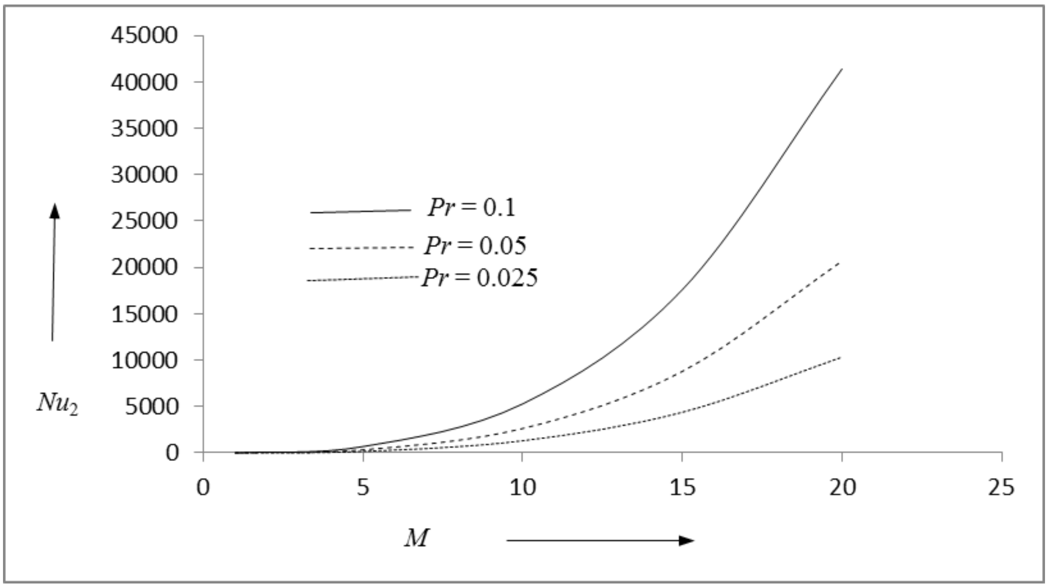

Figure 12. Nusselt number Nu_2 against M, under Pr for t=1, Re =1, Ec=0.1, $\omega = 0.5, \varepsilon = 0.1$

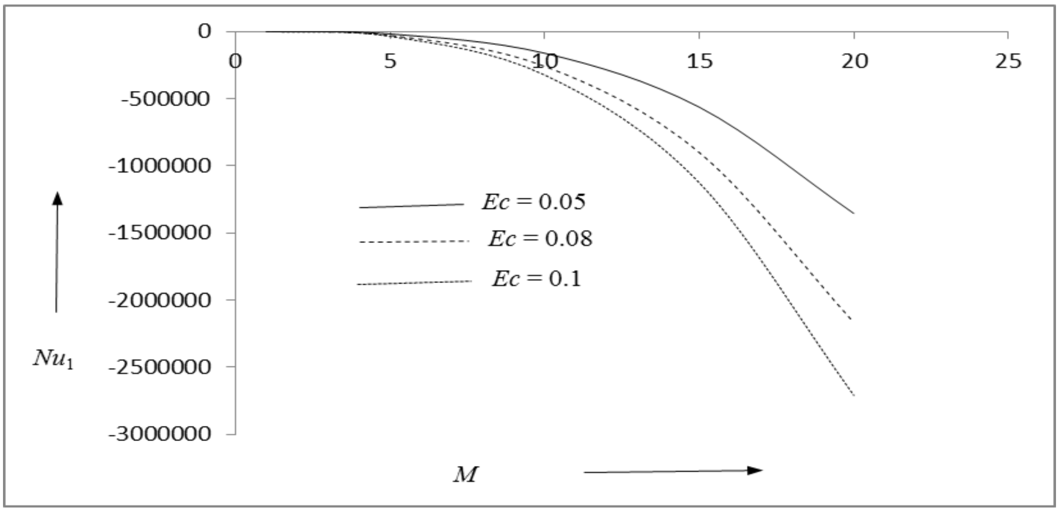

Figure 13. Nusselt number Nu_1 against M, under Ec for t=1, Re=1, Pr=7, $\omega = 0.5, \varepsilon = 0.1$

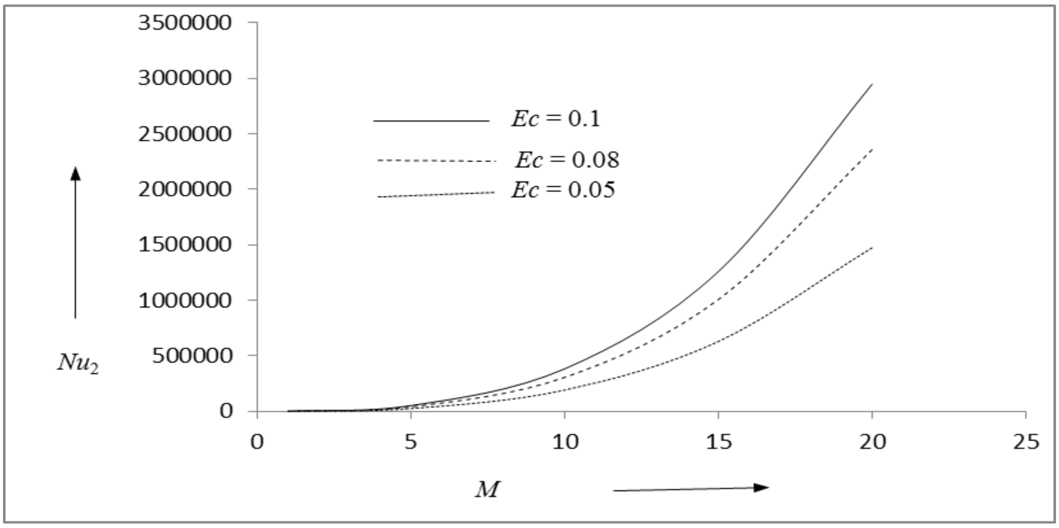

Figure 14. Nusselt number Nu_2 against M, under Ec for t=1, Re =1, Pr =7, $\omega = 0.5, \varepsilon = 0.1$

Acknowledgement:

The authors are thankful to the reviewer(s) for providing constructive suggestion to improve the quality of the paper. The authors are highly thankful to CSIR-HRDG for funding this research work under Research Grant-in-aid No. 25(0209)/12/EMR-II.

REFERENCES

Ahmed, N. and Barua, D.P. (2008). Magnetohydrodynamic three dimensional free convection Couette flow with heat and mass transfer in presence of a heat sink, Ultra Science, 20(1) M, pp. 41-58.

Ahmed, N. and Kalita, H. (2010). Oscillatory MHD Free and forced convection flow through a porous medium in presence of heat source with variable suction, International Journal of Heat and Technology, 28(1), pp. 141-147

Chang, C.C., and Yen, J.T. (1965). Magnetohydrodynamics channel flow under Time-dependent pressure gradient, Phys. fluids, 4, pp.1355.

Cowling, T.G. (1957). *Magnetohydrodynamics*, Interscience Publications, Inc. New York.

Dessie, H. and Kishan, N. (2014). Scaling group analysis on MHD free convective heat and mass transfer over a stretching surface with suction / injection, heat source / sink considering viscous dissipation and chemical reaction effects, Applications and Applied Mathematics, 9 (2), pp.553-572.

Govindarajulu, T. (1976). Couette flow in hydromagnetics with time- dependent suction, Indian J.Pure Appl. Math, 9(12), pp.1359-1364.

Hughes, W.F. and Young, Y.J. (1960). *The Electromagnetodynamics of Fluids*, J. Wiley, New York.

Israel Cookey, C., Ogulu, A. and Omubo Pepple, V.M. (2003). The influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction, Int. J. Heat Mass Transfer, 46 (13), pp. 2305 – 2311.

Jain, N.C. and Gupta, P. (2006). Three dimensional free convection Couette flow with transpiration cooling, Journal of Zhejiang University SCIENCE A, 7(3), pp. 340-346.

Kumar, H. (2009). Radiative Heat Transfer with Hydro-magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux, Thermal Science, 13(2), pp. 163 – 169.

Manjulatha, V., Varma, S.V.K. and Raju, V.C.C. (2014). Effects of radiation absorption and mass transfer on the free convection flow passed a vertical plate through a porous medium in an aligned magnetic field, Applications and Applied Mathematics, 9 (1), pp.75-93.

Masthanrao, S., Balamurugan, K.S., Barma, S.V.K. and Raju, V.C.C. (2013). Chemical reaction and hall effects on MHD convective flow along an infinite vertical porous plate with variable suction and heat absorption, Applications and Applied Mathematics, 8 (1), pp.268-288.

Muhuri, P.K. (1963). Flow formation in Couette motion in magnetohydrodynamics with suction, J. Phys. Soc. Japan, 18, pp.1671.

Oahimire, J.I. and Olajuwan, B.I. (2014). Effects of radiation absorption and thermo-diffusion on MHD heat and mass transfer flow of a micro-polar fluid in the presence of heat source, Applications and Applied Mathematics, 9 (2), pp.763-779.

Ramamoorty, P. (1962). A generalized porous wall Couette flow with uniform suction or blowing and uniform transverse magnetic field, J. Aerospace Sci., 29, pp.111.

Rathy, R.K. (1963). Hydromagnetic Couette's flow with suction and injection, ZAMM, 43, pp.370.

Sengupta, S. (2015). Free convective chemically absorption fluid past an impulsively accelerated plate with thermal radiation variable wall temperature and concentration, Applications and Applied Mathematics, 10 (1), pp.75-93.

Shukla, J. B. (1963). An exact solution for the Hydromagnetic flow of an electrically conducting fluid between two parallel porous plates, J. Phys. Soc. Japan, 18, pp.1214.

Soundalgekar, V.M. and Bhat, J.P. (1978). Oscillatory MHD channel flow and heat transfer, Indian J.Pure Appl. Math., 15(7), pp. 819-828.

Sutton, G.W. and Sherman, A. (1965). *Engineering Magnetohydrodynamics*, McGraw-Hill Book Co. Inc. New York.

Verma, P.D., and Bansal, J.L. (1966). Flow of a viscous incompressible fluid between two parallel plates, one in uniform motion and other at rest with uniform suction at the stationary plate, Proc. Indian Acad. Sci., 64, pp.385.

.

Appendix

$$m_2 = \frac{-Re + \sqrt{\left(Re^2 + 4M^2\right)}}{2}, \quad m_3 = \frac{-Re - \sqrt{\left(Re^2 + 4M^2\right)}}{2};$$

$$k' = \frac{1}{(m_3 - m_2)\sinh(m_2)\sinh(m_3)};$$

$$c_2 = k' m_3 \sinh(m_3); \quad c_3 = -k' m_2 \sinh(m_2);$$

$$c_1 = \frac{1}{M^2} - c_2 e^{m_2} - c_3 e^{m_3}; \quad c_4 = c_2 m_2 e^{m_2} + c_3 m_3 e^{m_3};$$

$$a = \frac{Re + \sqrt{\left(Re^2 + 4M^2\right)}}{2}; \quad b = \frac{-Re + \sqrt{\left(Re^2 + 4M^2\right)}}{2};$$

$$m_5 = \frac{-a + \sqrt{\left(a^2 + 4i\omega\right)}}{2}; \quad m_6 = \frac{-a - \sqrt{\left(a^2 + 4i\omega\right)}}{2};$$

$$m_7 = \frac{b + \sqrt{\left(b^2 + 4i\omega\right)}}{2}; \quad m_8 = \frac{b - \sqrt{\left(b^2 + 4i\omega\right)}}{2};$$

$$k'' = \frac{ia}{\omega Re\{\sinh(m_5)\cosh(m_6) - \cosh(m_5)\sinh(m_6)\}};$$

$$c_5 = k''\sinh(m_6); \quad c_6 = -k''\sinh(m_5);$$

$$k''' = \frac{-ib}{\omega Re\{\sinh(m_7)\cosh(m_8) - \cosh(m_7)\sinh(m_8)\}};$$

$$c_7 = k'''\sinh(m_8); \quad c_8 = -k'''\sinh(m_5);$$

$$P_{11} = \frac{m_2 c_2^2 \left(m_2^2 + M^2\right)}{(4m_2 + 2PrRe)}; \;\; P_{12} = \frac{m_3 c_3^2 \left(m_3^2 + M^2\right)}{(4m_3 + 2PrRe)};$$

$$P_{13} = \frac{2c_2}{\left(m_2 + PrRe\right)}; \quad P_{14} = \frac{2c_3}{\left(m_3 + PrRe\right)}; \quad P_{15} = \frac{1}{PrReM^2};$$

$$c_{10} = \frac{-PrEc}{\sinh(PrRe)}\left\{P_{11}\sinh(2m_2) + P_{12}\sinh(2m_3) - P_{13}\sinh(m_2) - P_{14}\sinh(m_3) + P_{15}\right\};$$

$$c_9 = PrEc\left\{P_{11}\cosh(2m_2) + P_{12}\cosh(2m_3) - P_{13}\cosh(m_2) - P_{14}\cosh(m_3)\right\} - c_{10}\cosh(PrRe);$$

$$m_{11} = \frac{-PrRe + \sqrt{\left(Pr^2 Re^2 + 4i\omega Pr\right)}}{2};$$

$$m_{12} = \frac{-PrRe - \sqrt{\left(Pr^2 Re^2 + 4i\omega Pr\right)}}{2};$$

$$P_{16} = \frac{2bc_2 m_2 c_5 m_5}{\left(m_2 + m_5\right)^2 + PrRe\left(m_2 + m_5\right) - i\omega Pr};$$

$$P_{17} = \frac{2bc_2 m_2 c_6 m_6}{\left(m_2 + m_6\right)^2 + PrRe\left(m_2 + m_6\right) - i\omega Pr};$$

$$P_{18} = \frac{\left(a + b\right)c_2 m_2 c_7 m_7}{\left(m_2 + m_7\right)^2 + PrRe\left(m_2 + m_7\right) - i\omega Pr};$$

$$P_{19} = \frac{\left(a + b\right)c_2 m_2 c_8 m_8}{\left(m_2 + m_8\right)^2 + PrRe\left(m_2 + m_8\right) - i\omega Pr};$$

$$P_{20} = \frac{\left(b - a\right)c_3 m_3 c_5 m_5}{\left(m_3 + m_5\right)^2 + PrRe\left(m_3 + m_5\right) - i\omega Pr};$$

$$P_{21} = \frac{\left(b - a\right)c_3 m_3 c_6 m_6}{\left(m_3 + m_6\right)^2 + PrRe\left(m_3 + m_6\right) - i\omega Pr};$$

$$P_{22} = \frac{bc_5 m_5}{M^2\left(m_5^2 + PrRem_5 - i\omega Pr\right)};$$

$$P_{23} = \frac{bc_6 m_6}{M^2\left(m_6^2 + PrRem_6 - i\omega Pr\right)};$$

$$P_{24} = \frac{ac_7 m_7}{M^2\left(m_7^2 + PrRem_7 - i\omega Pr\right)};$$

$$P_{25} = \frac{ac_8 m_8}{M^2\left(m_8^2 + PrRem_8 - i\omega Pr\right)};$$

$$A_{11} = 2PrEc(P_{16}\cosh(m_2 + m_5) + P_{17}\cosh(m_2 + m_6) + P_{18}\cosh(m_2 + m_7) + P_{19}\cosh(m_2 + m_8) + P_{20}\cosh(m_3 + m_5) + P_{21}\cosh(m_3 + m_6) - P_{22}\cosh(m_5) - P_{23}\cosh(m_6) - P_{24}\cosh(m_7) - P_{25}\cosh(m_8));$$

$$A_{12} = 2PrEc(P_{16}\sinh(m_2 + m_5) + P_{17}\sinh(m_2 + m_6) + P_{18}\sinh(m_2 + m_7) + P_{19}\sinh(m_2 + m_8) + P_{20}\sinh(m_3 + m_5) + P_{21}\sinh(m_3 + m_6) - P_{22}\sinh(m_5) - P_{23}\sinh(m_6) - P_{24}\sinh(m_7) - P_{25}\sinh(m_8));$$

$$c_{11} = \frac{A_{11}\sinh(m_{12}) - A_{12}\cosh(m_{12})}{\sinh(m_{11})\cosh(m_{12}) - \cosh(m_{11})\sinh(m_{12})};$$

$$c_{12} = \frac{-A_{11}\sinh(m_{11}) + A_{12}\cosh(m_{11})}{\sinh(m_{11})\cosh(m_{12}) - \cosh(m_{11})\sinh(m_{12})}.$$