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Non-Newtonian Prandtl fluid over stretching permeable surface

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Abstract

An analysis is made of the velocity and temperature distribution in the flow of a viscous incompressible fluid caused by the stretching permeable surface which issues in the Prandtl fluid. Parandtl fluid is a pseudoplastic visco-inelastic non-Newtonian fluid. The governing partial differential equations are reduced to ordinary differential equations using deductive group transformation and similarity solution is derived. Numerical solutions to the reduced non-linear similarity equations are then obtained by adopting shooting method using the Nachtsheim-Swigert iteration technique. The results of the numerical solution are then presented graphically in the form of velocity and temperature profiles. The corresponding skin friction coefficient and the Nusselt number are also calculated.

Keywords: Deductive group-theoretic method; Prandtl fluids, MSABC; skin friction; Nusselt number

MSC 2010: 65L99, 76A05, 76D05, 76D10, 76W05

Nomenclature

u, v	velocity component in the boundary layer along x,y - axis resp.	α, β	flow parameter
τ	the non-vanishing shear tensor	η	similarity variable
x, y	Cartesian coordinates	F, H, G	similarity functions
ρ	density of the fluid	∞	free stream condition
α	thermal conductivity	o	constant condition
L, U_0	characteristic length and velocity of the mainstream respectively	Pr	Prandtl number $(\mathrm{LU_0}/\alpha\mathrm{Re})$
ψ	stream function	Re	Reynold number $(\mathrm{LU_0}/\nu)$
C_f, Nu	Skin friction coefficient and Nusselt number-physical quantities		

1. Introduction

The present study has many applications in coating and suspensions, cooling of metallic plate, paper production, heat exchangers technology and materials processing exploiting. Aerodynamic extrusion of plastic sheets, glass fiber production, paper production, heat treated materials traveling between a feed roll and a wind-up roll, cooling of an infinite metallic plate in a cooling bath, manufacturing of polymeric sheets are some examples of practical applications of non-Newtonian fluid flow over a stretching surface. The production of fiber sheet, plastic sheet, extrusion of molten polymers through a slit die is an important process in polymer industry. In almost every extrusion application the prime aim is to maintain the surface quality of the extrudate. To meet the requirements for best appearance and optimum service properties such as low friction, transparency and strength, all coating processes demand a smooth glossy surface.

During a manufacturing process, the material is in a molten phase, thrust through an extrusion die and then cools and solidifies some distance away from the die before arriving at the collection stage. In the region between the die and the cooling mechanism the material, while cooling, is found to stretch. Because of the solidification that eventually occurs one can reasonably expect that the stretching process varies with distance from the die.

The quality of the final product as well as the cost of production is affected by the speed of collection and the heat transfer rate. In this sense knowledge of the flow properties of the ambient fluid is clearly desirable. In fact, a detailed knowledge of the velocity and temperature distributions in this layer is therefore of utmost importance in controlling the rate of cooling so as to obtain final products of desired characteristics. In the present paper we consider the modeling of a two-dimensional Prandtl fluid for a stretching sheet. Note that this stretching may not necessarily be linear. It may be quadratic, power-law, exponential and so on.

Sakiadis (1961) was the first to investigate the boundary layer flow on moving solid surfaces. Due to entrainment of the ambient fluid, this boundary layer is different from that in the Blasius flow past a flat plate. The problem was extended to the case when there is suction or blowing at the moving surface by Erickson et al. (1966) who presented numerical solutions of the boundary layer equations for momentum, heat and mass transfer for various values of the parameters. Crane (1970) studied the laminar boundary layer flow caused by a stretching sheet with stretching velocity varying linearly with distance from a fixed point. Gupta and Gupta (1977) analyzed the heat and mass transfer corresponding to the similarity solution for the boundary layer over a stretching sheet. Grubka and Bobba (1985) investigated the heat transfer characteristics of the stretching sheet problem with variable temperature.

Chen and Char (1988) considered the continuous stretching surface with variable surface temperature. Abel and Veena (1995) examined unsteady boundary layer flow in a visco-elastic fluid past a stretching sheet. Tapanidis et al. (2003) discussed the application of scaling group of transformations to visco-elastic second-grade fluid flow. Mukhopadhyay et al. (2005) studied the MHD boundary layer flow and heat transfer over a stretching sheet with variable viscosity using the scaling group of transformations. Problems on stretching surface under other different physical situations were analyzed by Ishak et al. (2006), Wang (2009) and Patel and Timol (2011). Sajid and Hayat (2008) investigated the effect of thermal radiation on the boundary layer flow over an exponentially stretching sheet and found analytical solution. This thermal radiation and heat transfer of fluid flow applications occur in the modern system of electric power generation, plasma space vehicles, astrophysical flows and cooling of nuclear reactors. Recently, Nadeem et al. (2010) and Mukhopadhyay and Gorla (2012) studied various aspects of such problem either analytically or numerically.

The similarity solutions are quite popular because they result in the reduction of the independent variables of the problem. Most of the researchers in the field of fluid mechanics try to obtain the similarity solutions by introducing a general similarity transformation with unknown parameters into the differential equation and as a result obtain an algebraic system. Then, the solution of this system, if it exists, determines the values of the unknown parameters. We believe it is better to attack any problem of similarity solutions from the outset; that is, to find out the full list of the symmetries of the problem and then to study which of them are appropriate to provide group-invariant (or more specifically, similarity) solutions. To obtain symmetry of a differential equation is equivalent to the determination of the continuous groups of transformations associated with this symmetry.

Birkhoff (1960) initiated the applications of group theory to fluid mechanics which opened up the way for general similarity procedures. Building on this work, Morgan (1952) gave complete structure of the theory for reducing the number of independent variables. Ames (1966) applied the method for reducing more than one independent variable simultaneously by composing a multi-parameter group. Later, authors like Hansen (1965), Moran and Gaggioli (1968), Bluman and Cole (1974), Seshadri and Na (1985) and Bluman and Kumai (1989) contributed much to the development of the theory. The method has been applied intensively by Hansen and Na (1968), Timol and Kalthia (1986), Abd-el-Malek and Badran (1990), Pakdemirli (1994), Abd-el-Malek et al. (2002).

Further, most of the paints or protective coating applied on an extrudate is in general non-Newtonian fluids. In the present study we therefore examine the behavior of an incompressible non-Newtonian fluid obeying the Prandtl model due to stretching of the surface. For this purpose, we apply the deductive group transformations to investigate similarity and to analyze the steady boundary layer flow. By systematically searching the group of transformations, the set of governing partial differential equations for the flow and energy distribution transformed into a set of self-similar ordinary differential equations along with the appropriate boundary conditions. Numerical solution is derived to understand the fluid behavior. Velocity, temperature, the skin-friction and the Nusselt number for different values of thermo-physical parameters have been computed and the results are presented graphically and discussed qualitatively.

2. Mathematical formulation

Consider the case of a flat sheet issuing from a thin slit at $x = 0$, $y = 0$ and subsequently being stretched as in a polymer processing application shown in Figure1.

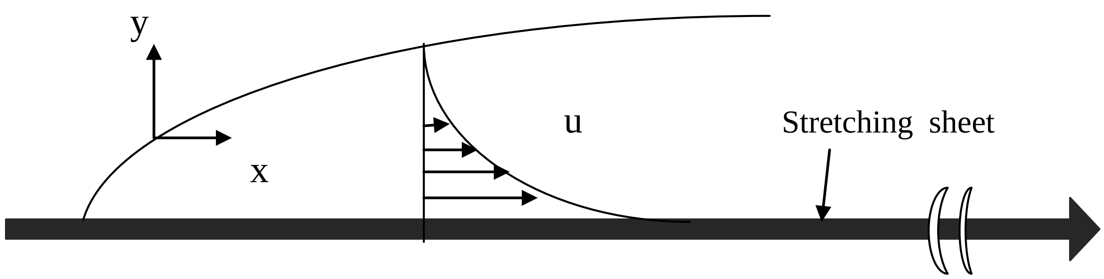

Figure 1. Schematic diagram

The flow caused by the stretching of this sheet is assumed to be laminar. Consider the boundary layer flow of incompressible, viscous-inelastic, two dimensional steady Prandtl non-Newtonian fluid over a stretching permeable surface. The governing differential equations of continuity, momentum and heat transfer following Pop and Na (1998) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho \left(u \, \frac{\partial u}{\partial x} + v \, \frac{\partial u}{\partial y} \right) = \frac{\partial \tau}{\partial y}, \tag{2}$$

$$u \, \frac{\partial T}{\partial x} + v \, \frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are components of velocity in x and y directions. ρ is the fluid density, τ is stress tensor and α is the thermal conductivity. Here, the shearing stress τ is related to the rate of stain by the arbitrary function

$$\mathcal{F} \left(\tau \, , \, \frac{\partial u}{\partial y} \right) = 0, \tag{4}$$

the form of which differs for different fluid models. It should be noted that the flow is caused solely by the stretching of the wall and there is no free stream velocity outside the boundary layer. The corresponding boundary conditions are

$$u = U_0 \, u_w(x), \quad v = - v_0, \quad T = T_w \quad for \quad y = 0, \tag{5a}$$

$$u = 0, \quad T = T_\infty \quad for \quad y \to \infty. \tag{5b}$$

where the uniform surface heat flux v_0 and the ambient temperature T_∞ are assumed constant. Note that $v_0 > 0$ corresponds to suction and $v_0 < 0$ to injection. Introducing the dimensionless quantities as

$$\bar{x} = \frac{x}{L}, \qquad \bar{y} = \frac{y}{L}\left(\frac{U_0 L}{v}\right)^{1/2}, \qquad \bar{u} = \frac{u}{U_0},$$

$$\bar{v} = \frac{v}{U_0}\left(\frac{U_0 L}{v}\right)^{1/2}, \qquad \bar{\tau} = \frac{\tau}{\rho U_0 \left(\frac{U_0 v}{L}\right)^{1/2}}, \qquad \theta = \frac{T - T_w}{T_w - T_\infty}.$$

and introducing the stream function ψ such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, continuity equation is satisfied identically and bars are dropped for simplicity. The above Equations (1) to (5a) and (5b) become

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \tau}{\partial y} = 0, \tag{6}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} - \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} = 0, \tag{7}$$

$$\mathcal{F}\left(\tau, \frac{\partial^2 \psi}{\partial y^2}\right) = 0, \tag{8}$$

and subject to the boundary conditions

$$\frac{\partial \psi}{\partial y} = u_w, \quad \frac{\partial \psi}{\partial x} = K, \quad \theta = 1 \quad for \ y = 0, \tag{9a}$$

$$\frac{\partial \psi}{\partial y} = 0, \quad \theta = 0 \quad for \quad y \to \infty, \tag{9b}$$

where Pr is the Prandtl number and $K = -v_0\left(\frac{L}{U_0 v}\right)^{1/2}$ is non-dimensional constant.

3. Methodology

Our method of solution depends on the application of a one-parameter deductive group of transformation to the partial differential Equations (6)-(8) along with auxiliary conditions (9a) and (9b). Under this transformation the two independent variables will be reduced by one and the differential equations will transform into ordinary differential equation. Recently, this method has been successfully applied to various two dimensional non-linear problems by Darji and Timol (2011), Hiral and Timol (2011), Adnan et al. (2013).

3.1. The group systematic formulation

Introducing the group theoretic method

$$G: \ ¥_a(@) = ₡^{@}(a)\, @ + ₭^{@}(a). \tag{10}$$

where @ stands for $x,\ y\ ,\psi\ ,\theta\ ,\tau,\ u_w$. $Ȼ's$ and ₭'s are real-valued and are at least differentiable in the real argument a.

3.2. The invariance analysis

For invariance, invoking the group (10) in (6)-(8), we get

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\ \frac{\partial^2\bar{\psi}}{\partial\bar{x}\partial\bar{y}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\ \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} - \frac{\partial\bar{\tau}}{\partial\bar{y}} = ₦_1(a)\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\tau}{\partial y}\right), \tag{11}$$

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\ \frac{\partial\bar{\theta}}{\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\ \frac{\partial\bar{\theta}}{\partial\bar{y}} - \frac{1}{Pr}\ \frac{\partial^2\bar{\theta}}{\partial\bar{y}^2} = ₦_2(a)\left(\frac{\partial\psi}{\partial y}\ \frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\ \frac{\partial\theta}{\partial y} - \frac{1}{Pr}\ \frac{\partial^2\theta}{\partial y^2}\right), \tag{12}$$

$$\mathcal{F}\left(\bar{\tau}, \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right) = ₦_3(a)\ \mathcal{F}\left(\bar{\tau}, \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right). \tag{13}$$

Applying chain rule for transforming the derivatives under the group (10), we get

$$\left(\frac{Ȼ^{\psi}}{Ȼ^{y}}\right)\frac{\partial\psi}{\partial y}\left(\frac{Ȼ^{\psi}}{Ȼ^{x}Ȼ^{y}}\right)\frac{\partial^2\psi}{\partial x\partial y} - \left(\frac{Ȼ^{\psi}}{Ȼ^{x}}\right)\frac{\partial\psi}{\partial x}\left(\frac{Ȼ^{\psi}}{Ȼ^{y^2}}\right)\frac{\partial^2\psi}{\partial y^2} - \left(\frac{Ȼ^{\tau}}{Ȼ^{y}}\right)\frac{\partial\tau}{\partial y}$$
$$= ₦_1(a)\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\tau}{\partial y}\right), \tag{14}$$

$$\left(\frac{Ȼ^{\psi}}{Ȼ^{y}}\right)\frac{\partial\psi}{\partial y}\left(\frac{Ȼ^{\theta}}{Ȼ^{x}}\right)\frac{\partial\theta}{\partial x} - \left(\frac{Ȼ^{\psi}}{Ȼ^{x}}\right)\frac{\partial\psi}{\partial x}\left(\frac{Ȼ^{\theta}}{Ȼ^{y}}\right)\frac{\partial\theta}{\partial y} - \frac{1}{Pr}\left(\frac{Ȼ^{\theta}}{Ȼ^{y^2}}\right)\frac{\partial^2\theta}{\partial y^2}$$
$$= ₦_2(a)\left(\frac{\partial\psi}{\partial y}\ \frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} - \frac{1}{Pr}\ \frac{\partial^2\theta}{\partial y^2}\right), \tag{15}$$

$$\mathcal{F}\left(Ȼ^{\tau}\tau,\ \left(\frac{Ȼ^{\psi}}{Ȼ^{y^2}}\right)\frac{\partial^2\psi}{\partial y^2}\right) = ₦_3(a)\ \mathcal{F}\left(\tau,\ \frac{\partial^2\psi}{\partial y^2}\right). \tag{16}$$

For the invariance of above Equations (14)-(16),

$$\frac{Ȼ^{\psi^2}}{Ȼ^{x}Ȼ^{y^2}} = \frac{Ȼ^{\tau}}{Ȼ^{y}} = ₦_1(a), \tag{17a}$$

$$\frac{Ȼ^{\psi}Ȼ^{\theta}}{Ȼ^{x}Ȼ^{y}} = \frac{Ȼ^{\theta}}{Ȼ^{y^2}} = ₦_2(a), \tag{17b}$$

$$Ȼ^{\tau} = 1 = \frac{Ȼ^{\psi}}{Ȼ^{y^2}} = ₦_3(a)\ \ and\ \ ₭^{\tau} = 0, \tag{17c}$$

The invariance of boundary conditions (9a) and (9b) give,

$$\frac{Ȼ^{\psi}}{Ȼ^{y}} = Ȼ^{u_w},\quad Ȼ^{\theta} = 1\ \ \text{and}\quad ₭^{y} = ₭^{\theta} = ₭^{u_w} = 0, \tag{18}$$

On solving the Equations (17a), (17b), (17c) and (18) simultaneously, we obtained

$$Ȼ^{x} = Ȼ^{y^3},\quad Ȼ^{\psi} = Ȼ^{y^2},\quad Ȼ^{\tau} = 1,\ Ȼ^{u_w} = Ȼ^{y},$$

$$₭^{y} = ₭^{\tau} = ₭^{\theta} = ₭^{u_w} = 0.$$

Finally, we get the one-parameter group $\overline{G}$, which transforms invariantly the differential Equation (6)-(8) and the auxiliary conditions (9a) and (9b). This group $\overline{G}$ is of the form:

$$\overline{G}: \quad \bar{x} = \not\subset^{y^3}(a)\, x + \mathbb{K}^x(a), \qquad \bar{y} = \not\subset^{y}(a)\, y, \qquad \bar{\tau} = \tau,$$

$$\overline{\psi} = \not\subset^{y^2}(a)\, \psi + \mathbb{K}^{\psi}(a), \qquad \bar{\theta} = \theta, \qquad \overline{u_w} = \not\subset^{y}(a)\, u_w. \tag{19}$$

3.3. The complete set of absolute invariants

Now, we try to obtain a complete set of absolute invariants so that the original problem will be transformed into an ordinary differential equation in a similarity variable via group theoretic method. We have applied Hamad (2010) formulations for PDEs of 2-independent variables. By considering

$x_1 = x,\ x_2 = y,\ y_1 = \psi,\ y_2 = \theta, y_3 = \tau,\ y_4 = u_w$

and the definitions of $\propto_i, \beta_i$; $i = 1$ to 6, we get

$$\propto_i = \left.\frac{\partial \not\subset^i}{\partial a}\right|_{a = a_0} \text{ and } \beta_i = \left.\frac{\partial \mathbb{K}^i}{\partial a}\right|_{a = a_0}; \quad i = 1 \text{ to } 6, \tag{20}$$

where a_0 denotes the value of a which yield the identity element of the group. The generator is given by

$$X = (\propto_1 x_1 + \beta_1)\frac{\partial g}{\partial x_1} + (\propto_2 x_2 + \beta_2)\frac{\partial g}{\partial x_2} + (\propto_3 y_1 + \beta_3)\frac{\partial g}{\partial y_1}$$
$$+ (\propto_4 y_2 + \beta_4)\frac{\partial g}{\partial y_2} + (\propto_5 y_3 + \beta_5)\frac{\partial g}{\partial y_3} + (\propto_6 y_4 + \beta_6)\frac{\partial g}{\partial y_4}. \tag{21}$$

Hence, the characteristic equation becomes

$$\frac{dx}{\propto_1 x + \beta_1} = \frac{dy}{\propto_2 y} = \frac{d\psi}{\propto_3 \psi + \beta_3} = \frac{d\theta}{0} = \frac{d\tau}{0} = \frac{d\, u_w}{\propto_6 u_w}. \tag{22}$$

On solving this and using the relations between $\propto_i$'s & β_i's from Equations (19) and (20), we obtain similarity variables, Jain and Timol (2015), as follow

$$\begin{aligned} \eta &= y\, \text{л}^{-\frac{1}{3}}, \quad \text{where} \quad (x + \beta) = \text{л}, \ \beta = \frac{\beta_1}{\propto_1}, \\ \psi &= \text{л}^{\frac{2}{3}} F(\eta) - \frac{\beta_3}{\propto_3}, \\ \theta &= H(\eta), \\ \tau &= G(\eta), \\ u_w &= \text{л}^{\frac{1}{3}}. \end{aligned} \tag{23}$$

3.4. The reduction to an ordinary differential equation

The similarity transformations (23) maps Equations (6) to (9a) and (9b) into the following non-linear ordinary differential equations:

$$F'^2 - 2\,FF'' - 3G' = 0, \tag{24}$$

$$H'' + \frac{2\,Pr}{3}\; F\; H' = 0, \tag{25}$$

$$\mathcal{F}\,(H\,,\; F'') = 0, \tag{26}$$

$$F(0) = K, \quad F'\,(0) = 1\,, \quad H\,(0) = 1, \tag{27a}$$

$$F' = 0, \quad H = 0 \quad at \quad \eta \to \infty. \tag{27b}$$

4. Numerical solution and result discussion

To find the numerical solution, we have consider the non-Newtonian Prandtl fluid model. Mathematically, this model is given as

$$\tau = A\,sin^{-1}\left(\frac{1}{C}\;\frac{\partial u}{\partial y}\right). \tag{28}$$

Introducing the dimensionless quantities (defined in Section 3) and applying the similarity variables from (23)

$$G' = \frac{\alpha'\;F'''}{\sqrt{1-\beta\;F''^2}}, \tag{29}$$

where

$$\alpha' = \frac{A}{3\mu\,C} \text{ and } \beta = \frac{{U_0}^3}{C^2\,\nu\;L}$$

are dimensionless numbers and can be referred to as flow parameters. Substituting Equation (29) into Equation (24), we get

$$F''' = \frac{\left(1-\beta\;\;F''^2\right)^{1/2}}{\alpha'}\left(F'^2 - 2\,F\,F''\right). \tag{30}$$

Equations (30) and (25) are nonlinear ordinary differential equations and constitute the nonlinear boundary value problem along with the boundary conditions (27a) and (27b). As no specific method is available to solve this problem we opt for numerical method. We applied the Adams-Moulton procedure along with the shooting method due to Nachtsheim and Swigert (1965) based on the least square convergence criterion. The asymptotic boundary conditions are satisfied at the edge of the boundary layer by adjusting the initial conditions so that the mean square error between the computed variables and asymptotic values is minimized. Integration is carried out using the step size 0.5 starting from $\eta = 0$ until $\eta_{stop} = 3$. However, the success of this method depends greatly on the initial guesses made for $F''(0)$ and $H'(0)$ to begin the shooting method and such guesses are not made arbitrarily but vigilantly.

Figure 2 depicts the velocity variation $F'(\eta)$ for different values of $K > 0$ (suction) for the fixed non-dimensional numbers $\alpha' = 10$, $\beta = 15 \times 10^{-3}$ and $Pr = 0.7$(air). Whereas by keeping the same α', β and Pr, for different $K > 0$(suction), temperature variation $H(\eta)$ is displayed in Figure 3. It is seen that both velocity and temperature decreases with the increase in suction. Figures 4 and 5 demonstrate velocity and temperature profiles for different values of $K < 0$ (injection), again for the fixed α', β and Pr. In this case velocity and temperature are increasing as K becomes more negative. It has been observed that α' and β have great influence on the velocity and temperature distributions of the Prandtl fluids. Again, keeping α' and β fixed, velocity and temperature variations are elucidated through Figures 6 to 9 for different Prandtl number for both suction and injection case. This time Prandtl number has no effect on velocity distribution but α' is affecting the solution for temperature profile. As the Prandtl number increases, the thickness of thermal boundary layer decreases. All graphs are plotted in terms of dimensionless parameters. The physical quantity of interest are the coefficient of local skin friction C_f and Nusslet number N_u which are plotted in Figures 10 and 11 and calculated as

$$C_f = \frac{2\,\tau_w}{\sqrt{R_e}}, \qquad \text{where} \quad \tau_w = \frac{\alpha}{3\sqrt{\beta}}\, sin^{-1}\left(\sqrt{\beta}\, F''(0)\right), \tag{31}$$

$$N_u = -\frac{x}{T_w - T_\infty} \left.\frac{\partial T}{\partial y}\right|_{y=0} = -\sqrt{R_e}\, H'(0). \tag{32}$$

From the graphs it is clear that skin friction coefficient and Nusselt number decrease when Reynolds number increases.

5. Conclusion

In the present work, the problem of non-Newtonian Prandtl flow over a nonlinear stretching permeable surface with variable temperature is studied. This Prandtl fluid is one of the pseudoplastic visco-inelastic fluid out of 26 (could be more) non-Newtonian fluids [Kapur et al. (1982)]. Solving each of non-Newtonian fluid flow problems is an intellectual challenge. We were motivated for the present study from the natural instinct to accept the challenges of solving unsolved problem and to get insight into the flow behaviors of non-Newtonian fluids. Further, Pop and Na (1998) considered linear velocity at the boundary edge while we have considered general form of this velocity as u_w.

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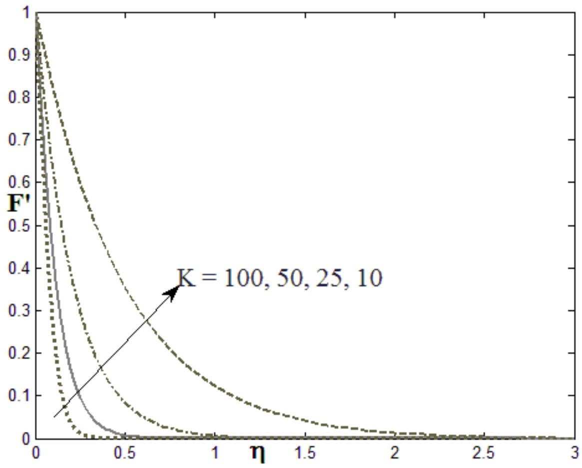

Figure 2. Velocity profile for $K > 0$

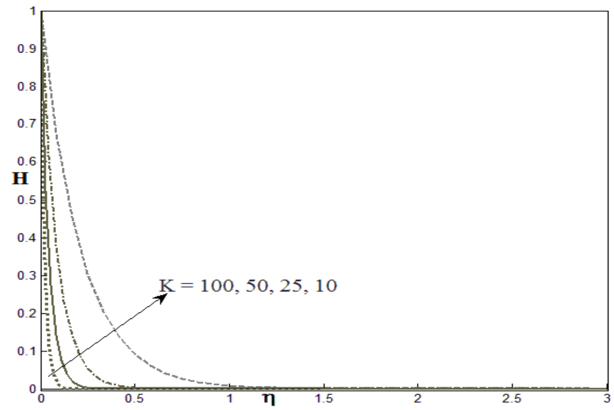

Figure 3. Temperature profile for $K > 0$

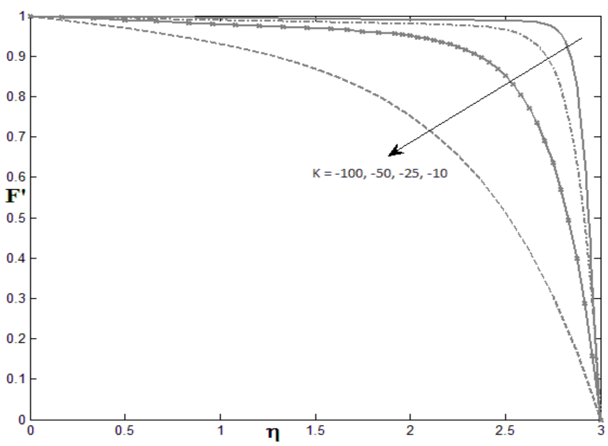

Figure 4. Velocity profile for $K < 0$

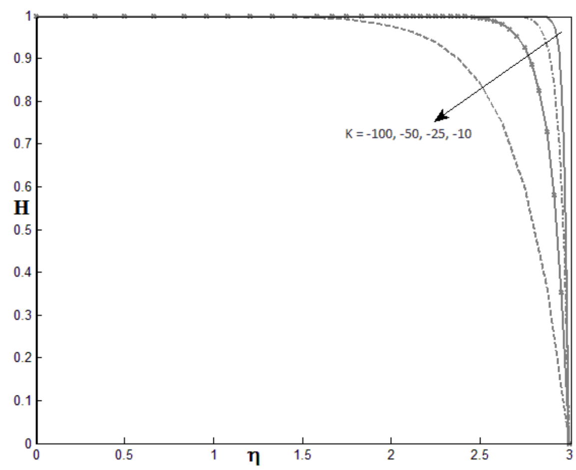

Figure 5. Temperature profile for $K < 0$

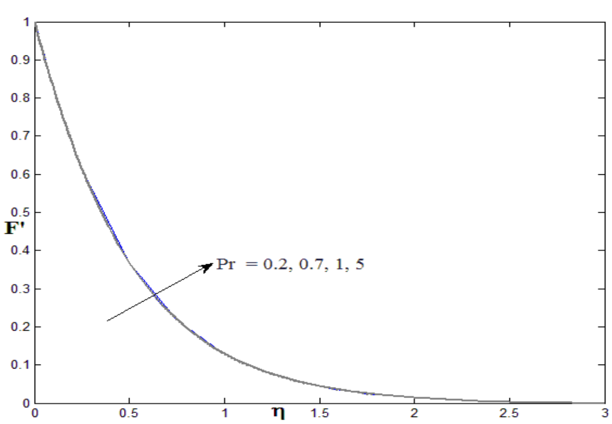

Figure 6. Velocity profile for $K > 0$

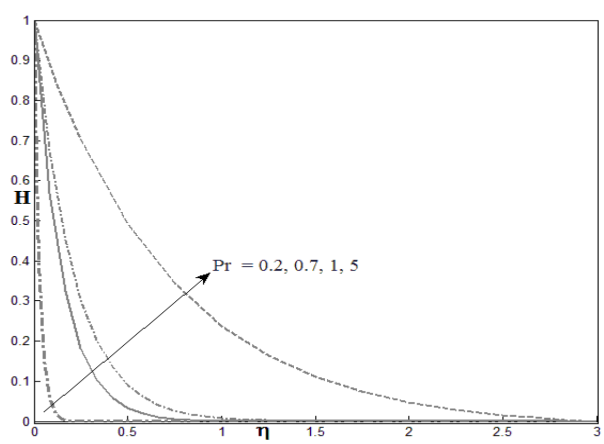

Figure 7. Temperature profile for $K > 0$

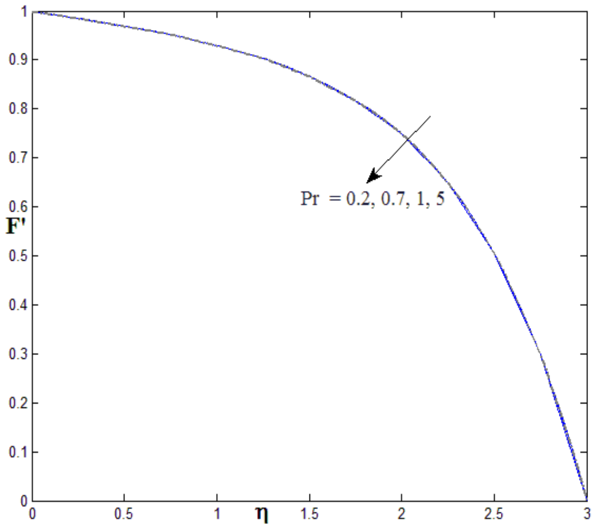

Figure 8. Velocity profile for $K < 0$

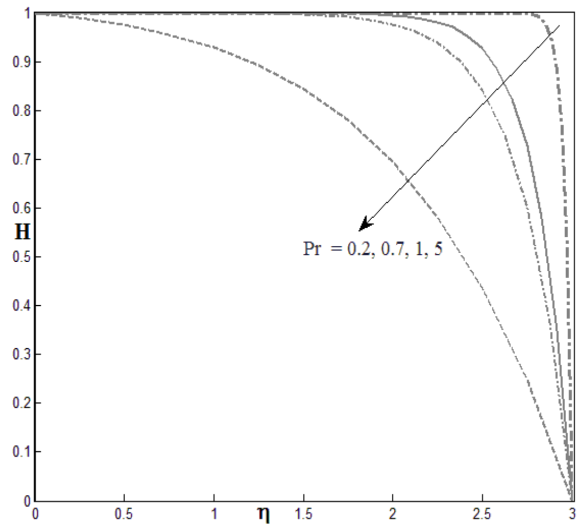

Figure 9. Temperature profile for $K < 0$

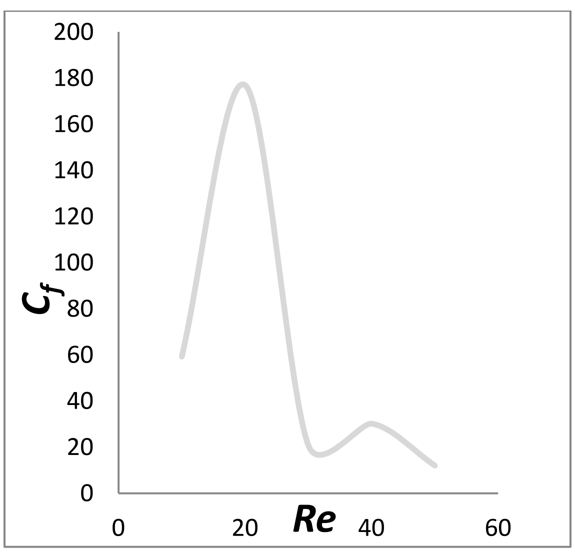

Figure 10. Skin Friction coefficient C_f for suctionn- injection together

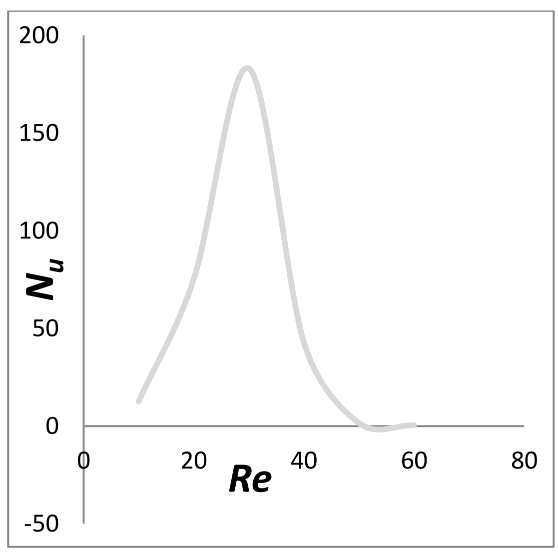

Figure 11. Nusselt number *Nu* for suction- injection together