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Effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction

M. A. Seddeek and A. A. Almushigeh
Department of Mathematics
Faculty of Education for Scientifically Departments
Al-Qassim University
P.O. Box 6595
Saudi Arabia
seddeek_m@hotmail.com

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Abstract

A similarity solution is proposed for the analysis of steady free convection heat and mass transfer over a stretching sheet. The effect of radiation, chemical reaction and variable viscosity on hydromagnetic heat and mass transfer in the presence of magnetic field are investigated. The governing partial differential equations are transformed to the ordinary differential equations using similarity variables, and then solved numerically by means of the fourth-order Runge–Kutta method with shooting technique. A comparison with exact solution is performed and the results are found to be in excellent. Numerical results for the velocity, temperature and concentration as well as for the skin-friction, Nusselt number, and the Sherwood number are obtained and reported graphically for various parametric conditions to show interesting aspects of the solution.

Key words: Shooting Method, Radiation, Chemical Reaction, Variable viscosity, Heat and Mass Transfer, Free Convection

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1. Introduction

The use of magnetic field that influences heat generation/absorption process in electrically conducting fluid flows has important engineering applications. See for instance, Moalem (1976), Chakrabarti et al. (1979), Vajravelu et al. (1992), Chiam (1995), Chamkha et al. (2000), Chandran et al. (1996), and Seddeek (2005). This interest stems from the fact that hydromagnetic flows and heat transfer have been applied in many industries. For example, in many metallurgical processes such as drawing of continuous filaments through quiescent fluids, and annealing and tinning of copper wires, the properties of the end product depend greatly on the rate of cooling involved in these processes. Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature. To accurately predict the flow and heat transfer rates it is necessary to take into account this variation of viscosity. Seddeek et al. (2007) introduced the effect of an axial magnetic field on the flow and heat transfer. Also, Seddeek et al. (2006) have analyzed the effects of variable viscosity with magnetic field on the flow and heat transfer.

In the context of space technology and in processes involving high temperature, the effects of radiation are of vital importance. Little is currently known about the boundary layer flows of radiating fluids. The majority of studies concerned with the interaction of thermal radiation and natural convection were made by Sparrow and Cess (1962), Cess (1966), Arpaci (1972), Cheng and Ozisik (1972), Hasegawa et al. (1972) and Bankston et al. (1977) for the case of a vertical semi-infinite plate. Seddeek (2000) has analyzed the effects of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation. Also, Seddeek (2001) studied the thermal radiation and buoyancy effects on MHD free convection external heat generation flow over an accelerating permeable surface with temperature dependent viscosity. Recently, Seddeek et al. (2006) has studied the effects of radiation and thermal diffusivity on heat transfer over a stretching surface.

Previous results Kafoussias et al. (1995), Hossain et al. (2001), Seddeek (2005), Seddeek et al. (2005) and Sunil et al. (2005) have shown that when the effects of variable viscosity are taken into consideration, the critical Rayleigh number for the onset of convection is substantially reduced from the classical value, although the associated wave number is nearly the same. Seddeek et al. (2007) has studied the effects of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation. Pantokratoras (2004) has examined the variable viscosity on flow and heat transfer to a continuous moving flat plate.

Hence, the aim of the present work is to study the effects of radiation, chemical reaction, magnetic field and variable viscosity on hydromagnetic heat and mass transfer.

2. Governing equations

We consider the steady two-dimensional laminar free convective flow and mass transfer over a stretching sheet. The fluid is assumed to be viscous, incompressible and electrically conducting, we also assume that the fluid properties are isotropic and constant, except for the fluid viscosity which is assumed to be an exponential function of temperature. u, v, T and C are the fluid *x*-component of velocity, *y*-component of velocity, temperature and concentration respectively.

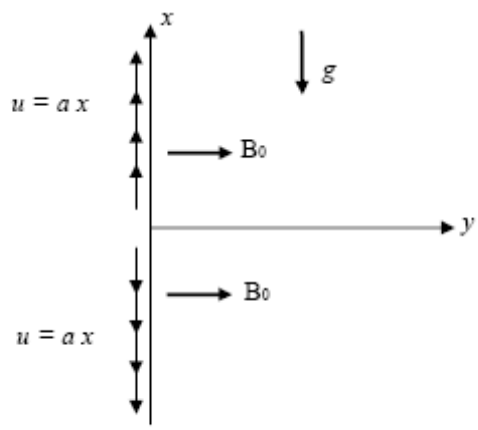

Fig. 1. The physical model and coordinate system

Under the above assumptions, the two dimensional boundary layer equations for the flow are given below, Ahmed (2004):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\beta_T\, g\, T+\beta_c\, g\, C-\frac{\sigma B_o^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_0 C^n. \tag{4}$$

Here, ρ is the fluid density, μ the viscosity coefficient, σ the electrical conductivity, B_o the magnetic field of constant strength, c_p the specific heat at constant pressure, β_T the coefficient of thermal expansion and β_C the coefficient of concentration expansion. g is the gravitational acceleration, k is the thermal conductivity, D is the mass diffusivity, n is order of reaction and

k_0 is the chemical reaction parameter. We consider the magnetic Reynolds number is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible.

The radiating gas is said to be a non-gray if the absorption coefficient K_λ is depending on the wave length λ Abd (2004). The equation which describes the conservation of radiative transfer in a unit volume (includes the radiative energy incident from all directions), for all wave length is

$$\nabla.\underline{q_r} = \int_0^\infty K_\lambda(T)(4e_{\lambda h}(T) - G_\lambda)\, d\lambda,$$

where q_r is the radiation heat flux, $e_{\lambda h}$ is the Plank's function and the incident radiation G_λ is defined as

$$G_\lambda = \frac{1}{\pi} \int_{\Omega=4\pi} e_{\lambda h}(\Omega) d\Omega,$$

where $\nabla.\underline{q_r}$ is the radiative flux divergence and Ω is the solid angle. Now, for an optically thin fluid exchanging radiation with a vertical cylinder at the average temperature value T_w and according to the above definition for the radiative flux divergence and Kirchhoff's law the incident radiation is given by

$$G_\lambda = 4e_{\lambda h}(T_w),$$

then

$$\nabla.\underline{q_r} = 4\int_0^\infty K_\lambda(T)(e_{\lambda h}(T) - e_{\lambda h}(T_w))d\lambda\ .$$

Expanding $K_\lambda(T)$ and $e_{\lambda h}(T_w)$ in Taylor series around T_w for small $(T - T_w)$, then we can rewrite the radiative flux divergence as

$$\nabla.\underline{q_r} = 4(T - T_w)\int_0^\infty K_{\lambda w}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_w d\lambda,$$

where

$$K_{\lambda w} = K_\lambda(T_w).$$

Hence, an optical thin limit for a non-gray gas near equilibrium, the following relation holds

$$\nabla \underline{q_r} = -4(T - T_w)\Gamma \ , \tag{5}$$

and, hence,

$$\frac{\partial q_r}{\partial r} = 4(T - T_w)\Gamma , \tag{6}$$

where

$$\Gamma = \int_0^\infty K_{\lambda w}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_w d\lambda .$$

The boundary conditions are given by

$$\begin{aligned} &y = 0: \quad u = ax, \quad v = 0, \quad T = T_w, \quad C = C_w, \\ &y \to \infty: \quad u = 0, \quad T = 0, \quad C = 0, \end{aligned} \tag{7}$$

where a, T_w and C_w are constants.

The continuity equation (1) is satisfied by the stream function $\Psi(x, \mathrm{y})$ defined by

$$u = \frac{\partial \Psi}{\partial y}, \qquad v = -\frac{\partial \Psi}{\partial x} . \tag{8}$$

To transform equations (2), (3) and (4) into a set of ordinary differential equations, the following dimensionless variables are introduced:

$$\Psi = \sqrt{\frac{\mathrm{a}\ \mu_0}{\rho}} x\, f(\eta), \quad \eta = \sqrt{\frac{\mathrm{a}\ \mu_0}{\rho}} y, \quad \theta = \frac{T}{T_w}, \quad \phi = \frac{C}{C_w}. \tag{9}$$

The variations of viscosity are written in the form Elbashbeshy et al. (1993):

$$\frac{\mu}{\mu_o} = e^{-\beta_1 \theta}, \tag{10}$$

where μ_o is the viscosity at temperature T_w and β_1 is the viscosity parameter.

Using these new variables, the momentum, energy and concentration equations and their associated boundary conditions can then be written as

$$f''' + e^{\beta_1 \theta}\left[f f'' - f'^{\,2} + G_r\theta + G_c\phi - M^2 f' \right] - \beta_1 \theta' f'' = 0\,, \tag{11}$$

$$\theta'' + \Pr\left[f\,\theta' - F(\theta - 1)\right] = 0, \tag{12}$$

$$\varphi'' + Sc\left[f\,\varphi' - \gamma\,\varphi^n \right] = 0\,. \tag{13}$$

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \varphi(0) = 1, \\ &f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \varphi(\infty) = 0, \end{aligned} \tag{14}$$

where the prime indicates differentiation with respect to η. $M = \left(\frac{\sigma}{\rho\, a}\right)^{\frac{1}{2}} B_0$ (magnetic parameter), $\Pr = \frac{c_P \mu_0}{k}$ (Prandtl number), $Gr = \frac{g\beta_T\, T_w}{a^2\, x}$ (local modified Grashof's number), $Gc = \frac{g\beta_c\, c_w}{a^2\, x}$ (local Grashof's number), $\mathrm{S}c = \frac{\mu_0}{\rho \mathrm{C_w}}$ (Schmidt number), $\gamma = \frac{k_0}{\mathrm{a}} C_w^{n-1}$ (non-dimensional chemical reaction parameter), and $F = \frac{4}{a\,\rho\, c_p}$ (radiation parameter).

Fortunately, the boundary value problem (11) together with the boundary conditions (14) at $\mathrm{G}r = Gc = M = \beta_1 = 0$ has an exact solution in the form:

$$f(\eta) = 1 - e^{-\eta}\,. \tag{15}$$

The physical quantities of interest in this problem are the skin-friction parameter $\mathrm{c_f}$, local Nusselt number Nu, and the local Sherwood number which are defined by

$$\mathrm{c_f} = \frac{\tau_{\mathrm{w}}}{\mu\sqrt{\frac{\mathrm{a}\,\mu_0}{\rho}}\; a\,x} = e^{-\beta_1 \theta(\eta)} f''(0), \tag{16}$$

$$\mathrm{N}u = \frac{q_{\mathrm{w}}}{k\sqrt{\frac{\mathrm{a}\,\mu_0}{\rho}}\; T_w} = -\theta'(0), \tag{17}$$

$$\mathrm{N}u = \frac{m_{\mathrm{w}}}{D\sqrt{\frac{\mathrm{a}\ \mu_0}{\rho}}\ C_w} = -\varphi'(0), \tag{18}$$

Where m_w is the rate of mass flux at the wall and

$$m_{\mathrm{w}} = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} = -D\sqrt{\frac{\mathrm{a}\ \mu_0}{\rho}}\ C_w\varphi'(0).$$

The wall shear stress τ_w is given by

$$\tau_{\mathrm{w}} = \mu_0\left(\frac{au_0}{\rho}\right)^{\frac{3}{2}} x\ e^{-\beta_1\ \theta(\eta)} f''(0), \tag{19}$$

and the heat flux q_w at the wall is

$$q_{\mathrm{w}} = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\sqrt{\frac{\mathrm{a}\ \mu_0}{\rho}}\ T_w\ \theta'(0). \tag{20}$$

3. Numerical Method

The governing boundary layer equations (11), (12) and (13) subject to the boundary conditions (14) are solved numerically. The shooting method for linear equations is based on replacing the boundary value problem by two initial value problems, and the solution of the boundary value problem is a linear combination of the two initial value problems. The shooting method for non-linear second order boundary value problems is similar to the linear case, except that the solution of the non-linear problem can not be simply expressed as a linear combination between the solution of the two initial value problems. The maximum value of η_∞, to each group β_1, M, F, n, γ and Sc determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7}. Instead, we need to use a sequence of suitable initial values for the derivatives such that the tolerance at the end point of the range is very small. This sequence of initial values is given by the secant method and we use the fourth order Runge-Kutta method to solve the initial value problem.

4. Results and Discussion

To study the behavior of the velocity, temperature and concentration profiles, curves are drawn for various values of the parameters that describe the flow. The results obtained for the steady flow are displayed in Figures 2-13. It should be mentioned that the results obtained here in

reduce to those reported by Afify when $\beta_1 = 0$ (constant viscosity) and $F = 0$ (no radiation) which gives validity of the present solution.

Figures 2, 3 and 4 illustrate the influence of the magnetic parameter M on the velocity, temperature and concentration profiles in the boundary layer, respectively. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase its temperature and concentration. Also, the effects on the flow and thermal fields become more so as the strength of the magnetic field increases. For positive values of β_1 the viscosity of the fluid decreases with an increase in the temperature and this is the case for fluids such as water, while for negative values of β_1 the viscosity of the fluid increases with an increase in the temperature.

The effects of the thermal radiation parameter F on the velocity temperature and concentration profiles in the boundary layer are illustrated in Figures 5, 6 and 7 respectively. Increasing the thermal radiation parameter F produces significant increases in the thermal condition of the fluid and its thermal boundary layer. Through the buoyancy effect, this increase in the fluid temperature induces more flow into the boundary layer thus causing the velocity of the fluid there to increase. In addition, the hydrodynamic boundary layer thickness increases as a result of increasing F.

Figs. 8- 10 display the variation of $f'(\eta)$, $\theta(\eta)$ and $\Phi(\eta)$ at different values of the chemical reaction parameter γ. It is seen that $f'(\eta)$, $\theta(\eta)$ and $\Phi(\eta)$ profiles decrease as the chemical reaction parameter increases.

Figures 11-13 display results for the velocity, temperature and concentration distribution, respectively. As shown, the velocity and concentration are increasing with decreasing the Schmidt number Sc. But, Figure 12 shows that the temperature increases as Sc increases. The analytical values of $f(\eta)$ in Equation (15) has been presented in Table 1. The agreement between analytical and numerical solutions is excellent.

Table 1. Values of $f(\eta)$ for different values of η at $Gr = Gc = M = \beta_1 = 0$

η	Exact solution in Eq. (15)	Present results	The error
0.00	0	-5.48586×10^{-21}	5.48586×10^{-21}
0.01	0.009950166250831893	0.009950166916633318	6.65801×0^{-10}
0.02	0.019801326693244747	0.0198013078386686	1.88546×10^{-8}
0.03	0.029554466451491845	0.029554447532648825	1.89188×10^{-8}
0.04	0.03921056084767682	0.03921054183033965	1.90173×10^{-8}
0.05	0.048770575499285984	0.048770556310743216	1.91885×10^{-8}
0.10	0.09516258196404048	0.09516256443452802	1.75295×10^{-8}

Table 2 Indicates that the skin-friction $f''(0)$ and the Sherwood number $-\phi'(0)$ increase as the radiation parameter F increase, while the Nusselt number $-\theta'(0)$ decreases as F increases. Also, $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ decrease when β_1 increase.

Table 2. Values of $f''(0), -\theta'(0)$ and $-\phi'(0)$ for different values of F and β_1 at $Gr = Gc = 0.5, n = 1, M = Sc = 0.1, F = 0$ and Pr=0.73

F	$f''(0)$			$-\theta'(0)$			$-\phi'(0)$		
	β_1=.7	β_1=-.3	$\beta_1 = 0$	β_1=-.7	β_1=-.3	$\beta_1 = 0$	$\beta_1 = -0.7$	$\beta_1 = -0.3$	$\beta_1 = 0$
0.00	-0.238357	-0.286959	-0.324971	0.613007	0.607422	0.603488	0.206665	0.205343	0.204439
0.01	-0.230968	-0.27618	-0.311263	0.5755	0.57034	0.566747	0.210404	0.209101	0.20822
0.02	-0.224763	-0.26715	-0.299814	0.543162	0.538345	0.535021	0.213267	0.211988	0.211131
0.03	-0.219327	-0.259266	-0.28985	0.514384	0.509855	0.506757	0.21561	0.214356	0.213523
0.04	-0.21445	-0.252221	-0.280979	0.488319	0.484041	0.48114	0.2176	0.216372	0.215562
0.05	-0.210008	-0.245832	-0.272962	0.464445	0.460391	0.457662	0.219331	0.218128	0.217341
0.10	-0.192151	-0.220465	-0.241437	0.368164	0.364967	0.362882	0.225659	0.224575	0.223884

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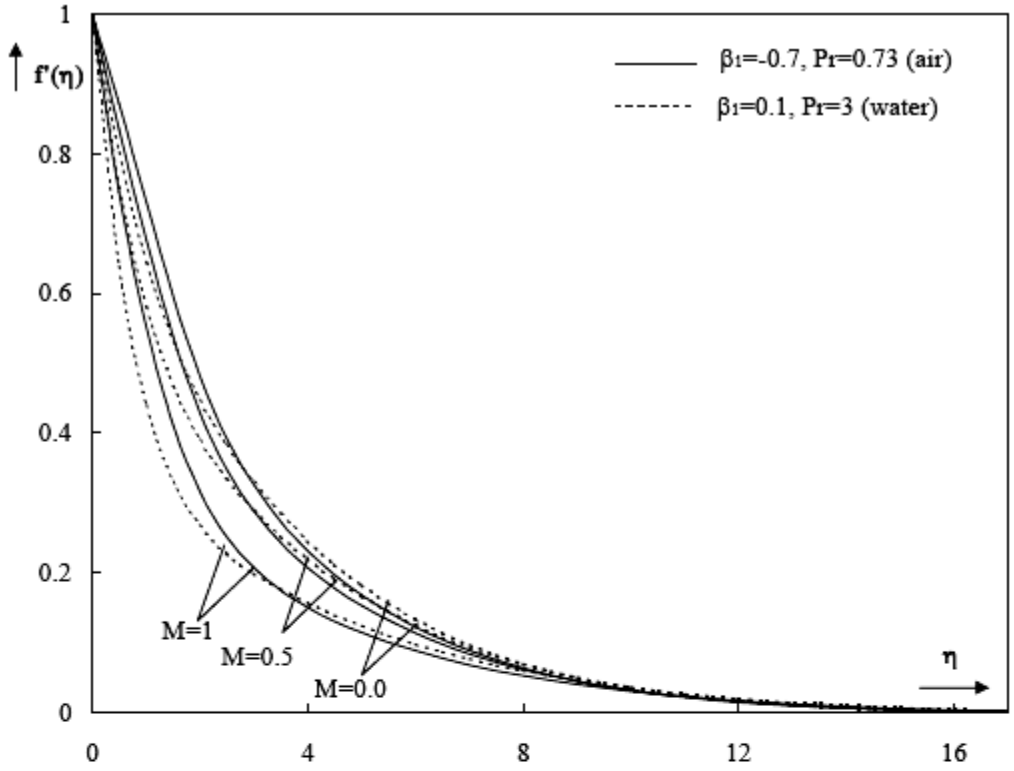

Fig. 2. The velocity profiles f'(η) with different M
(Gc = 0.5, Gr = 0.5, γ = 0.1, Sc = 0.1, n = 1, F = 0.001)

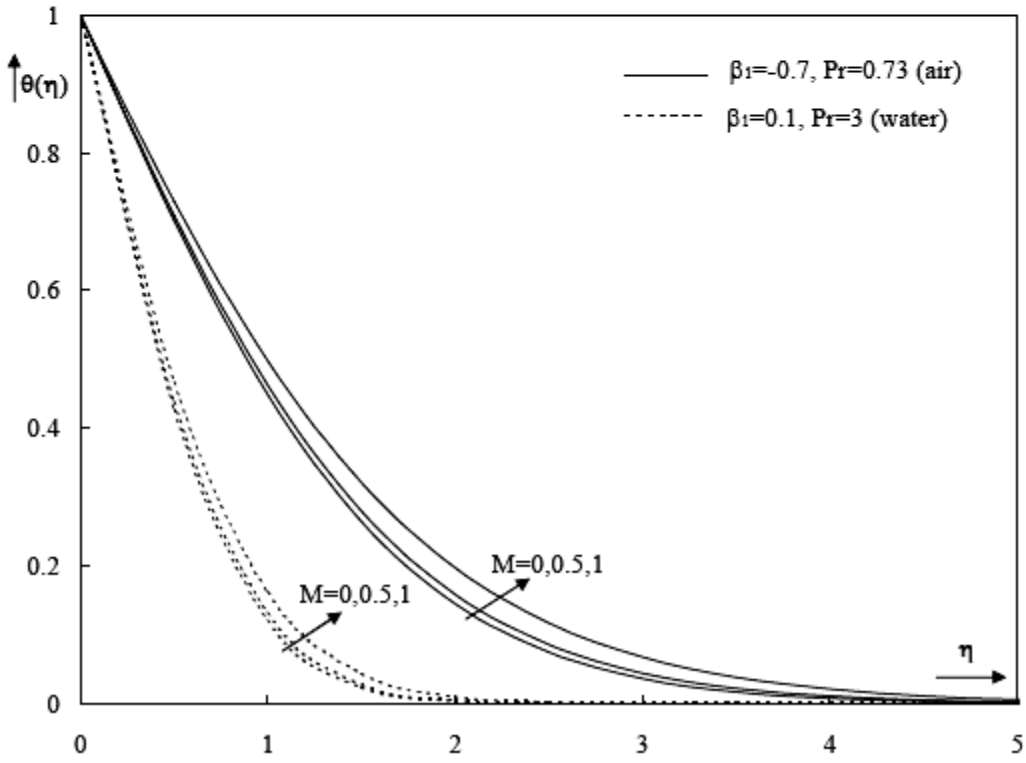

Fig. 3. The temperature profiles θ(η) with different M
(Gc = 0.5, Gr = 0.5, γ = 0.1, Sc = 0.1, n = 1, F = 0.001)

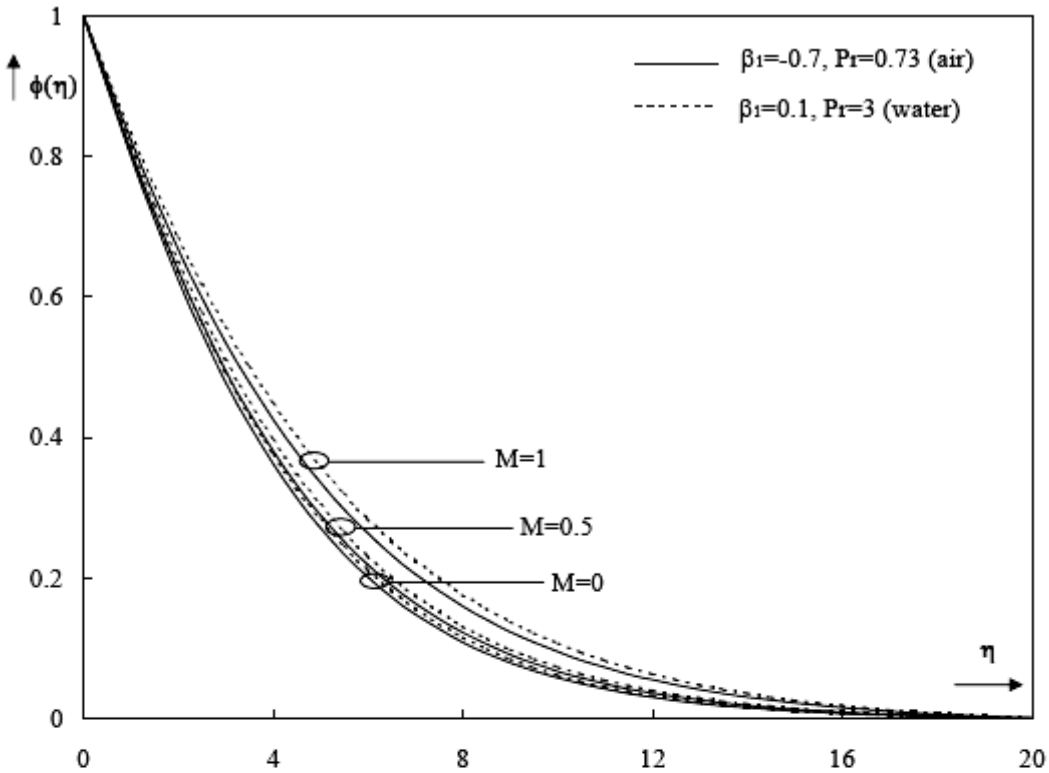

Fig. 4. The concentration profiles $\phi(\eta)$ with different M
(Gc = 0.5, Gr = 0.5, γ = 0.1, Sc = 0.1, n = 1, F = 0.001)

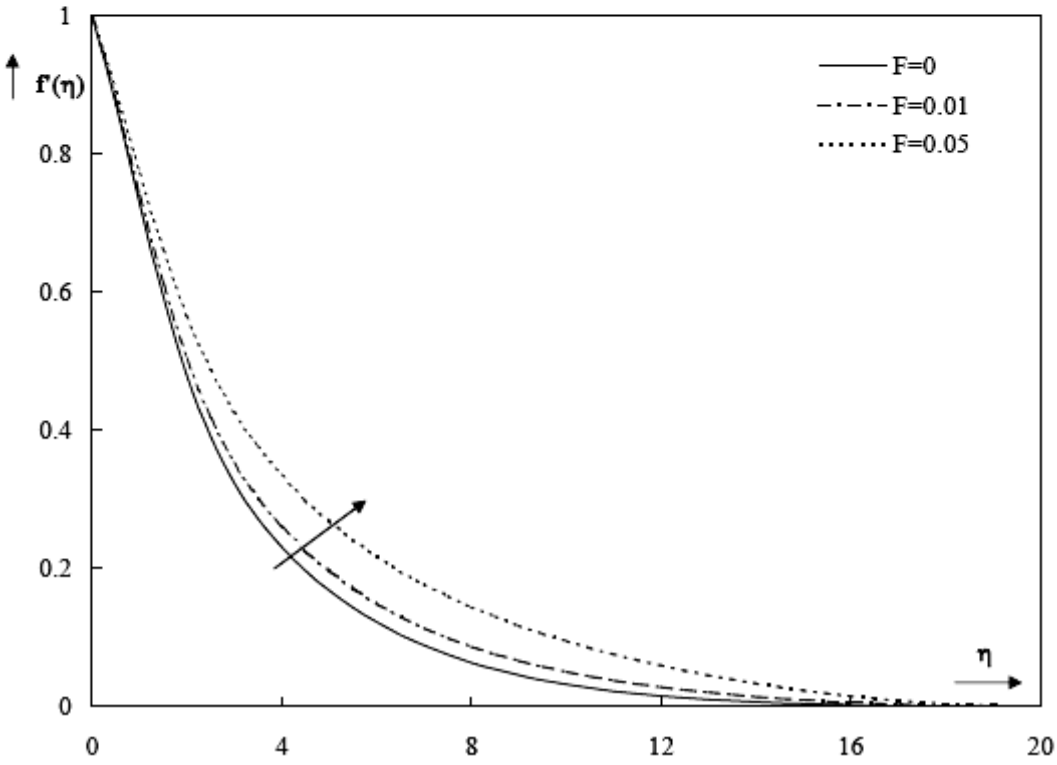

Fig. 5. The velocity profiles $f'(\eta)$ with different F
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1=-0.7, γ = 0.1, Sc = 0.1, n = 1, M = 0.1)

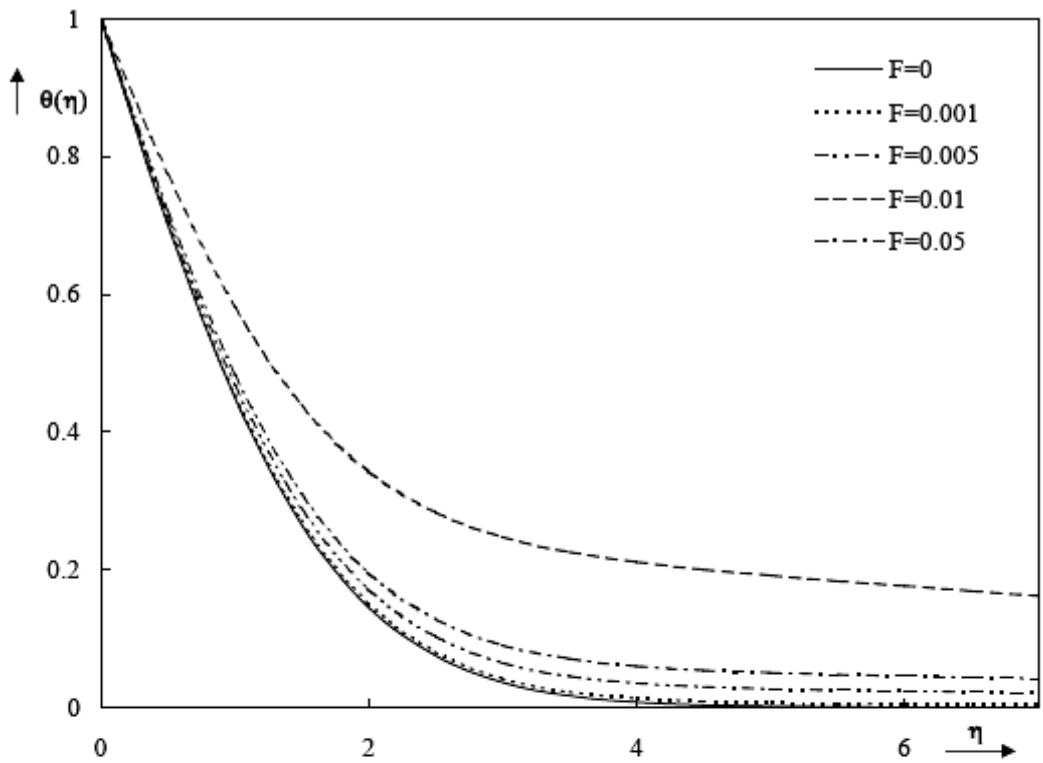

Fig. 6. The temperature profiles θ(η) with different F
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, γ = 0.1, Sc = 0.1, n = 1, M = 0.1)

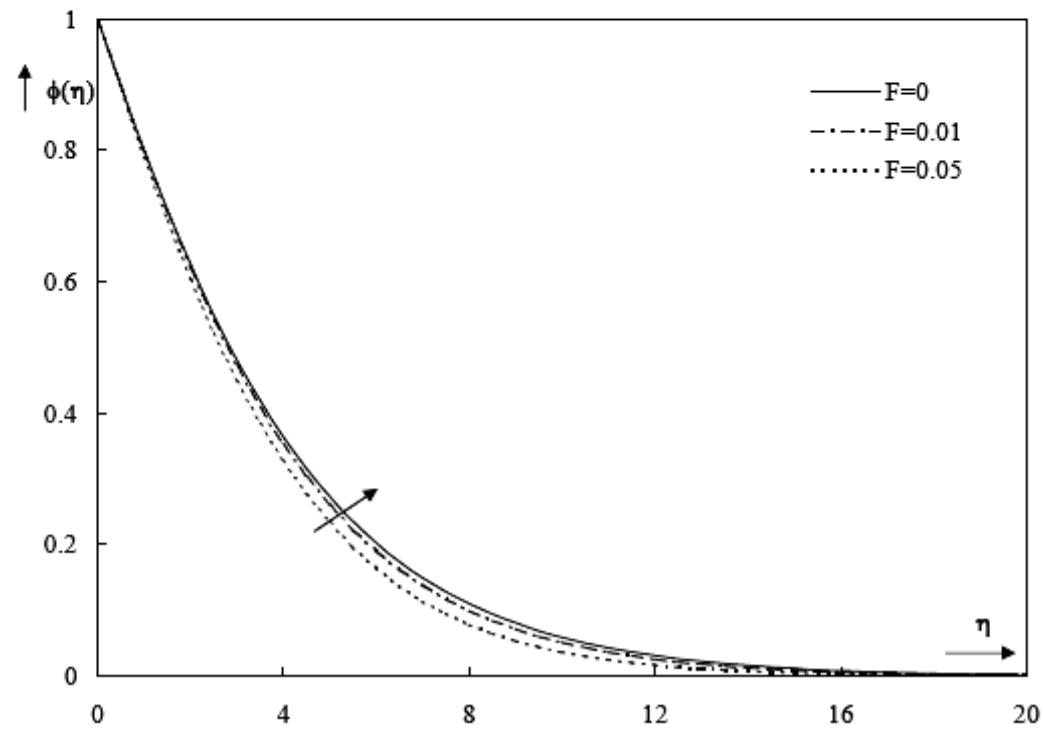

Fig. 7. The concentration profiles ϕ(η) with different F
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, γ = 0.1, Sc = 0.1, n = 1, M = 0.1)

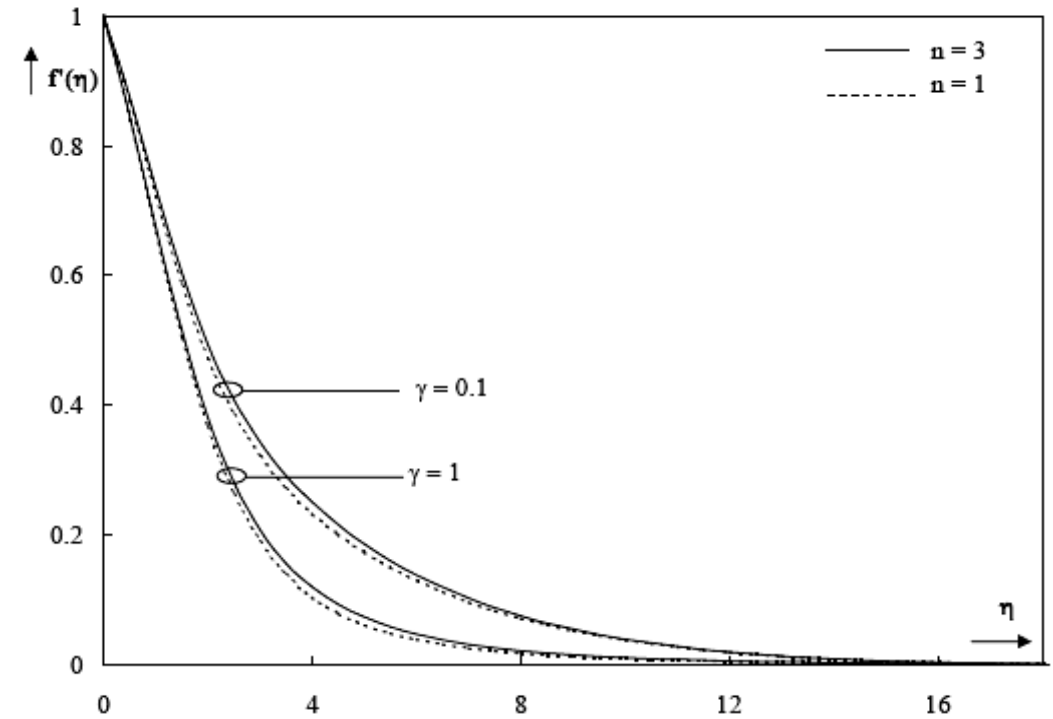

Fig. 8. The velocity profiles f'(η) with differentγ and n
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, F = 0.001, Sc = 0.1, M = 0.1)

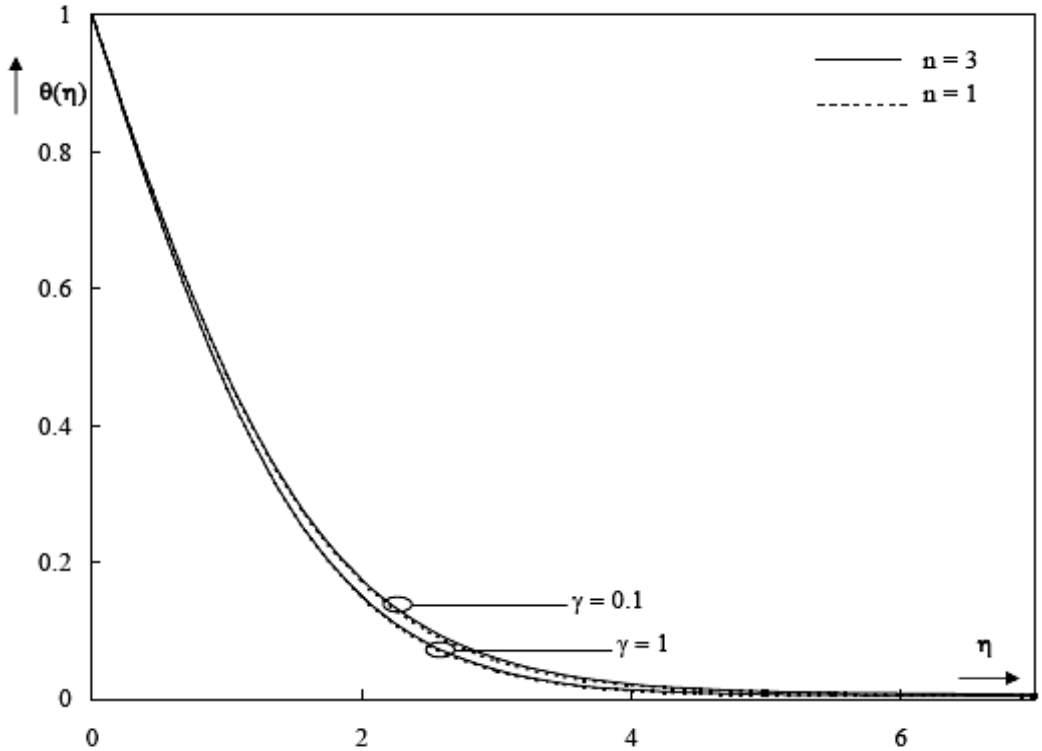

Fig. 9. The temperature profiles θ(η) with differentγ and n
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, F = 0.001, Sc = 0.1, M = 0.1)

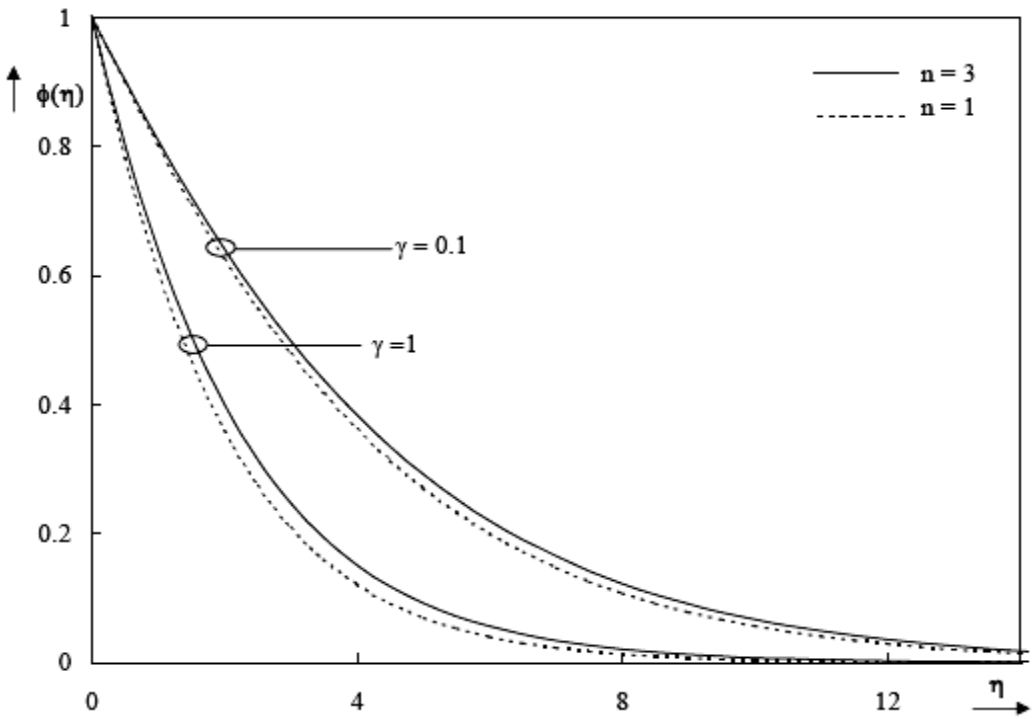

Fig. 10. The concentration profiles $\phi(\eta)$ with differentγ and n
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, F = 0.001, Sc = 0.1, M = 0.1)

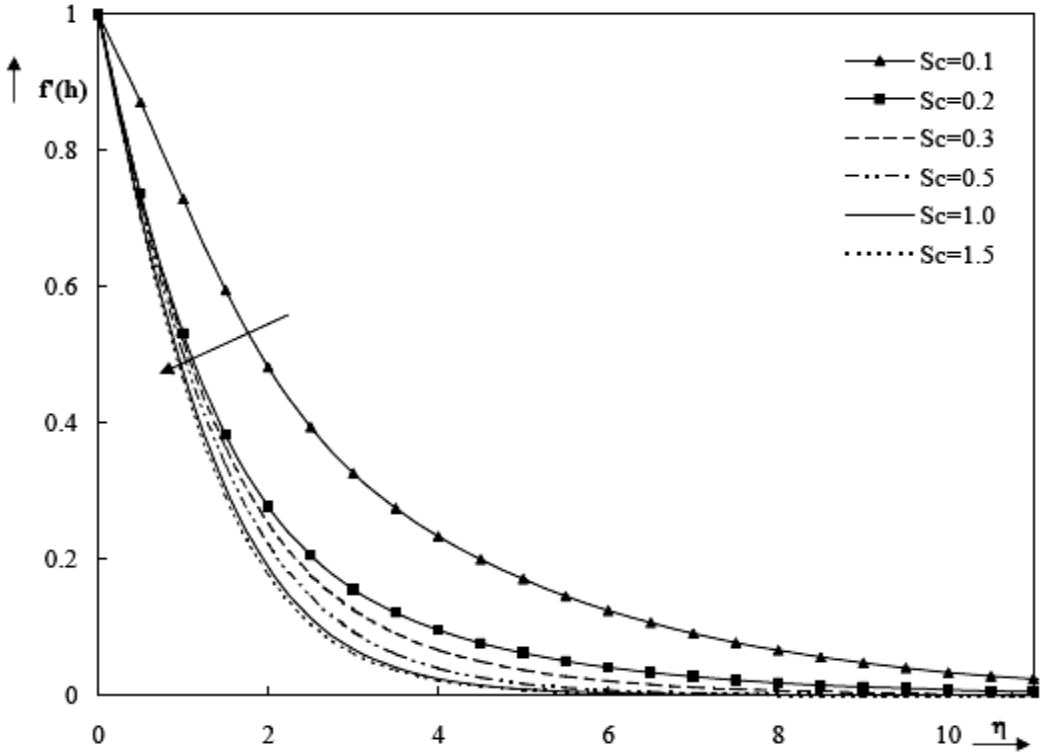

Fig. 11. The velocity profiles $f'(\eta)$ with different Sc
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, γ = 0.1, F = 0.001, n = 1, M = 0.1)

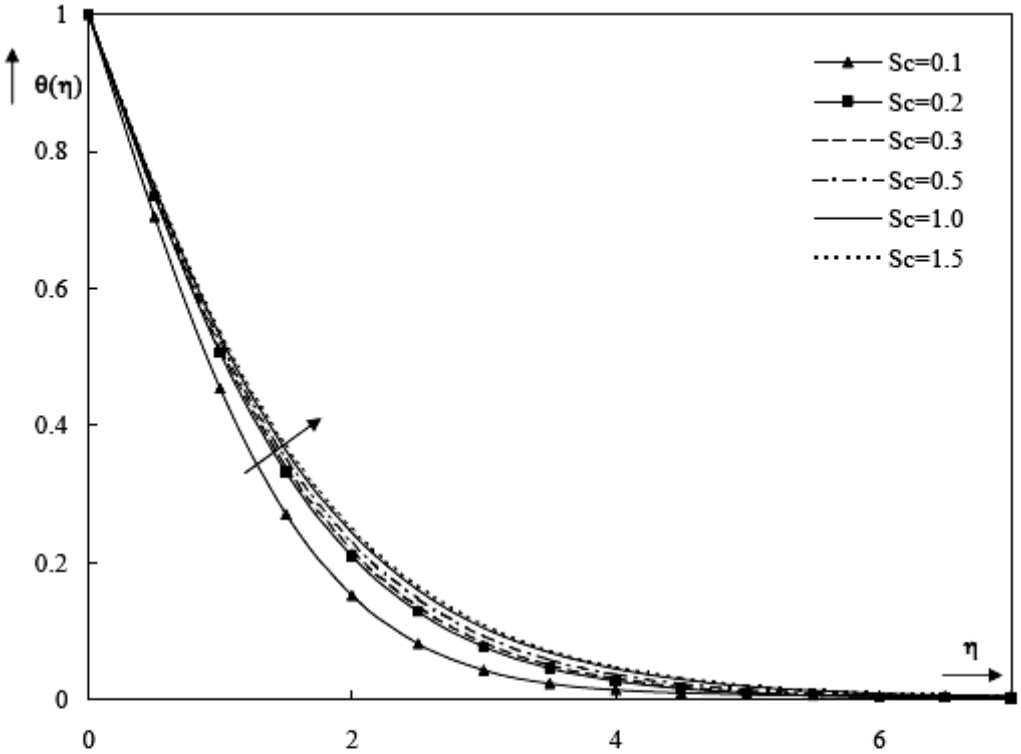

Fig. 12. The temperature profiles θ(η) with different Sc
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, γ = 0.1, F = 0.001, n = 1, M = 0.1)

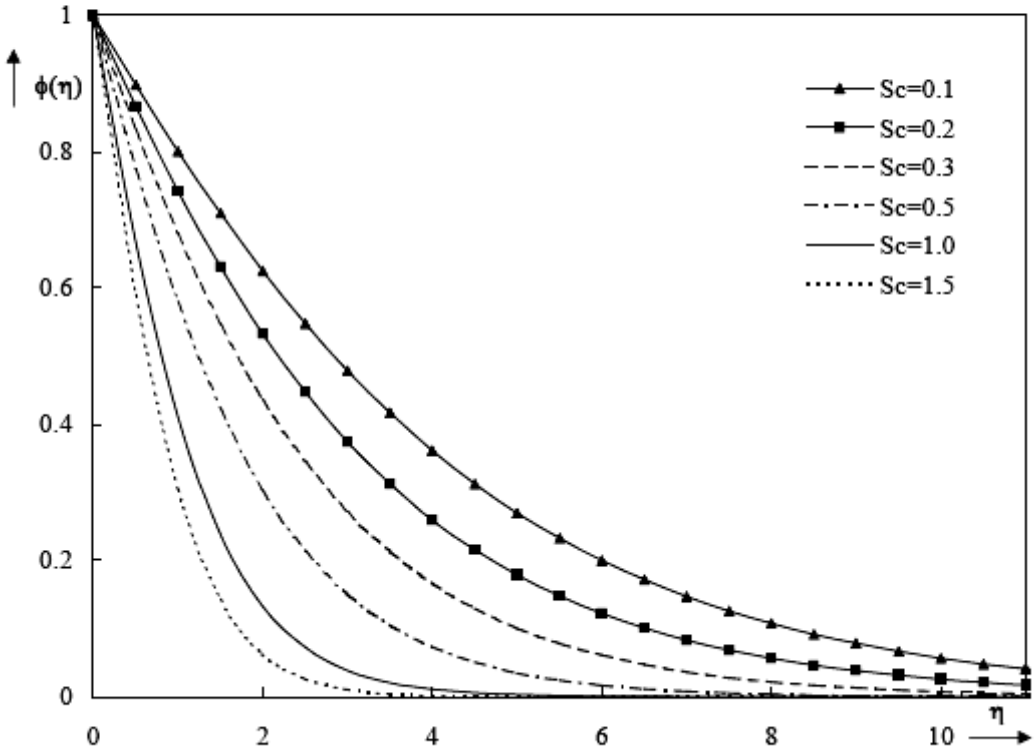

Fig. 13. The concentration profiles ϕ(η) with different Sc
(Gc = 0.5, Gr = 0.5, Pr =0.73, β_1 =-0.7, γ = 0.1, F = 0.001, n = 1, M = 0.1)