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Study of Reliability with Mixed Standby Components

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Abstract

This paper deals with the reliability characteristics of two different series system configurations with mixed standby (include cold and warm standby) components. The failure rates of the primary and warm standby components are assumed to follow the Weibull distribution. The repair time distribution of each server is exponentially distributed. Moreover, we will derive the mean time-to-failure, and the steady-state availability for a special case of a serial system of two primary components, two warm standby components, and one cold standby component, when the failure and repair rate are constant.

Keywords: Reliability, Availability, Time varying failure, standby components, Markov Method

MSC 2010 No.: 60K10; 62P30; 60J28

1. Introduction

System reliability and system availability have widely been studied because of their prevalence in power plants, manufacturing systems, and industrial systems. Maintaining a high or required level of reliability and/or availability is often an essential request. In this paper, we consider the manufacturing system or the power plant to be series system with mixed standby (include cold and worm standby) components. A standby component is called a 'cold standby' if its failure rate is zero. The standby component is referred as 'warm standby' when the failure rate is nonzero and is less than the failure rate of a primary component. Primary, warm, and cold components can be considered to be repairable.

The present study is differs from past work in that it presents a novel methodology to design a system configuration involving series and mixed standby components. The reliability characteristics of a system with M operating machines, S warm standby spares and R repairmen with exponential failure and exponential repair time distributions was investigated by Wang and Sivazlian (1989). Srinivasan and Gopalan (1973) studied one on-line unit (operating machine) with general lifetime distribution, w Warm standbys with exponential failure and exponential repair time distributions based on only one assumption, namely, the system fails when no spares are available to replace the failed operating machine.

Studies have been focused on assuming that the time-to-repair follows an exponential distribution [see Yadavalli et al. (2002); Chien et al. (2006)]. The reliability and availability characteristics of two different series system configurations studied by [El-Said and El-Sherbeny (2007)]. El-Sherbeny et al. (2009) derived the reliability and availability characteristics of three different series system configurations with warm standby components and a repairable service station. El-Sherbeny (2010) discussed the optimal system for series systems with mixed standby components.

In this paper, we are going to study three different system configurations of series and mixed standby components. Configurations 1,2 are compared based on their reliability. In addition, for configuration 3, which is a special case, we are going to develop the explicit expressions for the mean time-to-failure $MTTF$ and the steady-state availability $A_T(\infty)$, and to calculate the cost/benefit ratio (C/B) based on assumed numerical values given to the system parameters, as well as to the costs components.

2. Estimation of the 2-parameter Weibull distribution

The hazard function of a component following a 2-parameter Weibull distribution can be described by:

$$h(t) = \frac{\eta t^{\eta-1}}{\theta^{\eta}} , \tag{1}$$

where θ is a scale parameter, and η is a shape parameter.

The likelihood function for m items begin test at the same time by Farnum and Booth (1997) is:

$$L = \prod_{i=1}^{\tau} f(t_i) * \prod_{i=\tau+1}^{m} R(t_i)$$

$$= \prod_{i=1}^{\tau} [\frac{\eta}{\theta^{\eta}} t_i^{\eta-1} e^{-\left(\frac{t_i}{\theta}\right)^{\eta}}] * \prod_{i=\tau+1}^{m} e^{-\left(\frac{t_i}{\theta}\right)^{\eta}} .$$

$$\ln L = \tau \ln \eta - \eta \tau \ln \theta + \sum_{i=1}^{\tau} (\eta - 1) \ln t_i - \sum_{i=1}^{m} \left(\frac{t_i}{\theta}\right)^{\eta}.$$

The partial derivatives of the natural log of the likelihood function are:

$$\frac{\partial \ln L}{\partial \eta} = \frac{\tau}{\eta} - \tau \ln \theta + \sum_{i=1}^{\tau} \ln t_i - \sum_{i=1}^{m} \left(\frac{t_i}{\theta}\right)^{\eta} [\ln t_i - \ln \theta] = 0, \tag{2}$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{-\eta \tau}{\theta} + \frac{\eta}{\theta^{\eta+1}} \sum_{i=1}^{m} t_i^{\eta} = 0,$$

$$\hat{\theta} = \left(\frac{\sum_{i=1}^{m} t_i^{\eta}}{\tau}\right)^{1/\eta}, \tag{3}$$

and

$$\ln \theta = \frac{1}{\eta}\left[\ln \sum_{i=1}^{m} t_i^{\eta} - \ln \tau\right]. \tag{4}$$

Substituting the results from equations (3), (4) into (2), then we have:

$$\frac{\partial \ln L}{\partial \eta} = \frac{\tau}{\eta} - \frac{\tau}{\eta}\left[\ln \sum_{i=1}^{m} t_i^{\eta} - \ln \tau\right] + \sum_{i=1}^{\tau} \ln t_i - \sum_{i=1}^{m} \left(\frac{t_i^{\eta} \tau}{\sum_{i=1}^{m} t_i^{\eta}}\right)\left[\ln t_i - \frac{1}{\eta}\left(\ln \sum_{i=1}^{m} t_i^{\eta} - \ln \tau\right)\right],$$

$$0 = \frac{\tau}{\eta} + \sum_{i=1}^{\tau} \ln t_i - \tau \sum_{i=1}^{m} \left(\frac{t_i^{\eta} \ln t_i}{\sum_{i=1}^{m} t_i^{\eta}}\right),$$

$$\frac{\tau}{\eta} = \tau \sum_{i=1}^{m} \left(\frac{t_i^{\eta} \ln t_i}{\sum_{i=1}^{m} t_i^{\eta}}\right) - \sum_{i=1}^{\tau} \ln t_i,$$

and

$$\eta = \left[\sum_{i=1}^{m} \left(\frac{t_i^{\eta} \ln t_i}{\sum_{i=1}^{m} t_i^{\eta}}\right) - \frac{1}{\tau} \sum_{i=1}^{\tau} \ln t_i\right]^{-1} = [h(\eta)]^{-1}. \tag{5}$$

For censoring, t_i is a recorded failure time for $i \le \tau$ and $t_i = t_s$ for $\tau + 1 \le i \le m$, where t_s is the maximum test time for censoring, τ is the number of items that fail before t_s. When all

t_i ($i = 1,2,3,..., m$) are available, the data are complete; complete data are a special case of right concerning for $r = m$.

Our empirical investigations suggest that choosing: $v = \lim_{\eta \to \infty} h(\eta)$.

Compute estimate of the parameters θ and η for number of failures times t_i. The proofs of the following results are presented in the appendix.

3. Description of the system

For the sake of discussion, we consider the requirements of a 10MW power plant. We also assume that generators are available in units of both 10 and 5 MW. Standby generators are always necessary in case of failure. We assume that the switch is perfect (Wang and Kuo, 2000). We also assume that the switchover time from warm standby component to primary component, from cold standby component to warm standby component, from failure to repair, or from repair to cold standby component (or primary component if the system is short) is instantaneous. Primary components and warm standby components can be considered to be repairable by Wang et al. (2006) and Xie et al. (2004). Each of the primary components fails independently of the state of the others and has time-dependent failure rate $\lambda_1(t)$ with parameters η_1, θ_1 .

Whenever one of the primary components fails, a warm standby moves into operation if any is available, and a cold standby is put on warm standby state if any is available, we now assume that when a warm standby moves into a primary component state, its failure characteristic will be that of the primary component, and when a cold standby moves into a warm standby state, its failure characteristic will be that of a warm standby. We assume that each of the available warm standby components fails independently of the state of all the others and has time-dependent failure rate $\lambda_2(t)$ with parameters η_2, θ_2 . Whenever a primary component or a warm standby component fails, it is immediately repaired in the order of breakdowns with a time-to-repair, which is exponentially distributed with parameter μ. Once a component is repaired, it is "as good as new", notice that a failed system is never repaired.

The following configurations are considered:

The first configuration is a serial system of one primary 10MW component, one warm standby 10MW component, and one cold standby 10MW component.

The second configuration is a serial system of two primary 5MW components, one warm standby 5MW component, and one cold standby 5MW component.

The third configuration (a special case): of two primary 5MW components, two warm stand by 5MW components, and one cold standby 5MW component, with constant failure rate λ_1, λ_2 ,and constant repair rate μ.

4. The Reliability of the System

The state probability $P_j(t)$,for j=0,1,2,3 can be viewed as a result of solving a set of four first order linear differential equations given by the following identity:

$$\frac{dP_j(t)}{dt} = \dot{P}_j(t) = -P_j(t)\sum_{\substack{i=0\\ i\neq j}}^{3} a_{ji} + \sum_{\substack{i=0\\ i\neq j}}^{3} P_i(t)\, a_{ij} \tag{6}$$

where a_{ij} is the transition rate from state j to state i.

4.1. Calculations for Configuration 1

For configuration 1, let $P_3(t)$ be the probability that exactly 3 components are working at time t, $(t \geq 0)$. If we let $P(t)$ denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0) = \left[P_3(0), P_2(0), P_1(0), P_0(0)\right] = \left[1,0,0,0\right]. \tag{7}$$

The system-state equations for a Markov model which is the set of the first-order linear differential equations given by

$$\dot{P} = QP \quad .$$

The transition rate matrix Q for reliability according to configuration 1 is given by:

$$Q = \begin{pmatrix} -\lambda_1(t)-\lambda_2(t) & \mu & 0 & 0 \\ \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-\lambda_2(t)-\mu & 2\mu & 0 \\ 0 & \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-2\mu & 0 \\ 0 & 0 & \lambda_1(t) & 0 \end{pmatrix}$$

We will take the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called $\Lambda(t)$.

$$\Lambda(t) = \begin{pmatrix} -\lambda_1(t)-\lambda_2(t) & \mu & 0 \\ \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-\lambda_2(t)-\mu & 2\mu \\ 0 & \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-2\mu \end{pmatrix}.$$

We can write the system in the form:

$$\dot{P} = \Lambda(t).P(t), \tag{8}$$

where, $P(t) = \begin{bmatrix} P_3(t) \\ P_2(t) \\ P_1(t) \end{bmatrix}$.

To solve equation (8) with the initial condition

$$P(t=0) = \begin{bmatrix} P_3(0) \\ P_2(0) \\ P_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

Multiplying both sides of equation (8) by $e^{-\int_0^t \Lambda(s)ds}$ then we have:

$$\frac{d}{dt}\left[e^{-\int_0^t \Lambda(s)ds} \, . \, P(t) \right] = 0, \tag{10}$$

And, hence,

$$P(t) = e^{\int_0^t \Lambda(s)ds} \, . \, P(t=0). \tag{11}$$

Our problem now is how to determine the value of $e^{-\int_0^t \Lambda(s)ds}$.

So assume that

$$D = \int_0^t \Lambda(s)ds, \tag{12}$$

where

$$D=\int_0^t \Lambda(s)\, ds=\begin{pmatrix} -H_1(t)-H_2(t) & \mu t & 0 \\ H_1(t)+H_2(t) & -H_1(t)-H_2(t)-\mu t & 2\mu t \\ 0 & H_1(t)+H_2(t) & -H_1(t)-2\mu t \end{pmatrix},$$

and

$$H_1(t)=\int_0^t \lambda_1(x)dx \qquad , H_2(t)=\int_0^t \lambda_2(x)dx \cdot$$

The method of solving we follow gives us the value of e^D by the following relation

$$e^D=\alpha_0 I+\alpha_1 D+\alpha_2 D^2, \tag{13}$$

where I is the identity matrix of rank 3 and α_0 , α_1 , α_2 are the parameters obtained from the solution of the following system:

$$e^{S_1}=\alpha_0+\alpha_1 S_1+\alpha_2 S_1^2, \tag{14}$$
$$e^{S_2}=\alpha_0+\alpha_1 S_2+\alpha_2 S_2^2, \tag{15}$$
$$e^{S_3}=\alpha_0+\alpha_1 S_3+\alpha_2 S_3^2, \tag{16}$$

where S_1 , S_2 , S_3 are the characteristic roots of the matrix D. these roots are obtained from the characteristic equation g(s) of the matrix D given by:

$$g(s)=\begin{vmatrix} -H_1(t)-H_2(t)-s & \mu t & 0 \\ H_1(t)+H_2(t) & -H_1(t)-H_2(t)-\mu t-s & 2\mu t \\ 0 & H_1(t)+H_2(t) & -H_1(t)-2\mu t-s \end{vmatrix}. \tag{17}$$

By solving equations (14) – (16), we have

$$\alpha_0=\frac{S_2S_3(S_2-S_3)e^{S_1}+S_1S_3(S_3-S_1)e^{S_2}+S_1S_2(S_1-S_2)e^{S_3}}{(S_1-S_2)(S_1-S_3)(S_2-S_3)}, \tag{18}$$

$$\alpha_1=\frac{(S_3^2-S_2^2)e^{S_1}+(S_1^2-S_3^2)e^{S_2}+(S_2^2-S_1^2)e^{S_3}}{(S_1-S_2)(S_1-S_3)(S_2-S_3)}, \tag{19}$$

$$\alpha_2 = \frac{(S_2 - S_3)e^{S_1} + (S_3 - S_1)e^{S_2} + (S_1 - S_2)e^{S_3}}{(S_1 - S_2)(S_1 - S_3)(S_2 - S_3)} . \tag{20}$$

Now we can obtain the value of e^{D} from equation (13) and obtain the values of required states probabilities from equation (11) which are:

$$P_3(t) = \alpha_0 - \alpha_1\left(H_1(t) + H_2(t)\right) + \alpha_2\left(\left(H_1(t) + H_2(t)\right)^2 + \mu\, t\left(H_1(t) + H_2(t)\right)\right)$$

$$P_2(t) = \alpha_1\left(H_1(t) + H_2(t)\right) - \alpha_2\left(\left(H_1(t) + H_2(t)\right)^2 + \left(H_1(t) + H_2(t)\right)\left(H_1(t) + H_2(t) + \mu\, t\right)\right) ,$$

$$P_1(t) = \alpha_2 (H_1(t) + H_2(t))$$

where

$$H_i(t) = \left(\frac{t}{\theta_i}\right)^{\eta_i} \quad , \ i = 1,2 .$$

The system reliability function of configuration 1 is:

$$R_1(t) = \sum_{i=1}^{3} P_i(t) = \alpha_0 .$$

Since from equation (18), one can obtain

$$R_1(t) = \frac{S_2 S_3 (S_2 - S_3)e^{S_1} + S_1 S_3 (S_3 - S_1)e^{S_2} + S_1 S_2 (S_1 - S_2)e^{S_3}}{(S_1 - S_2)(S_1 - S_3)(S_2 - S_3)} \quad , \tag{21}$$

where

$$s_1 = \frac{1/6(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{1/3} - 6\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{\left(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3}\right)^{1/3} - \frac{1}{3}a}$$

$$s_2, s_3 = \frac{-\frac{1}{12}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{1/3} + 3\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{\left(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3}\right)^{1/3} - \frac{1}{3}a}$$

$$\pm\frac{1}{2}i\sqrt{3}\left(\frac{\frac{1}{6}(36ab-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{1/3}+6\left(\frac{1}{3}b-\frac{1}{9}a^2\right)}{\left(36ba-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3}\right)^{1/3}}\right)$$

and

$$a=3H(t)_1+2H_2(t)+3\mu\ t$$

$$b=3[H_1(t)]^2+[H_2(t)]^2+4H_1(t).H_2(t)+3\mu\ t\ H_1(t)+2\mu\ t\ H_2(t)+2\mu^2t^2$$

$$c=[H_1(t)]^3+2\ [H_1(t)]^2.H_2(t)+H_1(t).[H_2(t)]^2$$

for

$$H_1(t)=\left(\frac{t}{97.22}\right)^{2.12}\ ,H_2(t)=\left(\frac{t}{111.66}\right)^{1.79}\ ,\mu=0.05.$$

4.2. Calculations for Configuration 2

Let $P_3(t)$ be the probability that exactly 4 components are working at time t, $(t\geq 0)$. If we let $P(t)$ denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0)=[P_3(0),P_2(0),P_1(0),P_0(0)]=[1,0,0,0]. \tag{22}$$

The transition rate matrix Q for reliability according to configuration 2 is given by:

$$Q=\begin{pmatrix} -2\lambda_1(t)-\lambda_2(t) & \mu & 0 & 0 \\ 2\lambda_1(t)+\lambda_2(t) & -2\lambda_1(t)-\lambda_2(t)-\mu & 2\mu & 0 \\ 0 & 2\lambda_1(t)+\lambda_2(t) & -2\lambda_1(t)-2\mu & 0 \\ 0 & 0 & 2\lambda_1(t) & 0 \end{pmatrix}.$$

We will take the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called $\Lambda(t)$.

$$\Lambda(t)=\begin{pmatrix} -2\lambda_1(t)-\lambda_2(t) & \mu & 0 \\ 2\lambda_1(t)+\lambda_2(t) & -2\lambda_1(t)-\lambda_2(t)-\mu & 2\mu \\ 0 & 2\lambda_1(t)+\lambda_2(t) & -2\lambda_1(t)-2\mu \end{pmatrix}.$$

We can write the system in the form:

$$\dot{P} = \Lambda(t).P(t), \tag{23}$$

where,

$$P(t)=\begin{bmatrix} P_3(t) \\ P_2(t) \\ P_1(t) \end{bmatrix}.$$

To solve equation (23) with the initial condition

$$P(t=0)=\begin{bmatrix} P_3(0) \\ P_2(0) \\ P_1(0) \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \tag{24}$$

We will solve equation (23) with the aid of the method used in the previous section and hence:

$$D=\int_0^t \Lambda(s)\, ds=\begin{pmatrix} -2H_1(t)-H_2(t) & \mu t & 0 \\ 2H_1(t)+H_2(t) & -2H_1(t)-H_2(t)-\mu t & 2\mu t \\ 0 & 2H_1(t)+H_2(t) & -2H_1(t)-2\mu t \end{pmatrix}$$

and

$$H_1(t)=\int_0^t \lambda_1(x)dx \qquad , H_2(t)=\int_0^t \lambda_2(x)dx .$$

Here, the value of e^D will be given by the same relation, which is:

$$e^D=\alpha_0 I+\alpha_1 D+\alpha_2 D^2, \tag{25}$$

where I is the identity matrix of rank 3 and α_0 , α_1 , α_2 are the parameters obtained from the solution of the following system:

$$e^{r_1} = \alpha_0 + \alpha_1 r_1 + \alpha_2 r_1^2 , \tag{26}$$

$$e^{r_2} = \alpha_0 + \alpha_1 r_2 + \alpha_2 r_2^2 , \tag{27}$$

$$e^{r_3} = \alpha_0 + \alpha_1 r_3 + \alpha_2 r_3^2 , \tag{28}$$

where r_1 , r_2 , r_3 are the characteristic roots of the matrix D. these roots are obtained from the characteristic equation g(r) of the matrix D given by:

$$g(r) = \begin{pmatrix} -2H_1(t) - H_2(t) - r & \mu\, t & 0 \\ 2H_1(t) + H_2(t) & -2H_1(t) - H_2(t) - \mu\, t - r & 2\mu\, t \\ 0 & 2H_1(t) + H_2(t) & -2H_1(t) - 2\mu\, t - r \end{pmatrix}. \tag{29}$$

By solving equations (26) – (28), we have

$$\alpha_0 = \frac{r_2 r_3 (r_2 - r_3) e^{r_1} + r_1 r_3 (r_3 - r_1) e^{r_2} + r_1 r_2 (r_1 - r_2) e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)}, \tag{30}$$

$$\alpha_1 = \frac{(r_3^2 - r_2^2) e^{r_1} + (r_1^2 - r_3^2) e^{r_2} + (r_2^2 - r_1^2) e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)}, \tag{31}$$

and

$$\alpha_2 = \frac{(r_2 - r_3) e^{r_1} + (r_3 - r_1) e^{r_2} + (r_1 - r_2) e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)}. \tag{32}$$

Now we can obtain the value of e^D from equation (25) and obtain the values of required states probabilities from equation (23) which are:

$$P_3(t) = \alpha_0 - \alpha_1 \big(2H_1(t) + H_2(t)\big) + \alpha_2 \Big(\big(2H_1(t) + H_2(t)\big)^2 + \mu\, t \big(2H_1(t) + H_2(t)\big)\Big)$$

$$P_2(t) = \alpha_1 \big(2H_1(t) + H_2(t)\big) - \alpha_2 \Big(\big(2H_1(t) + H_2(t)\big)^2 + \big(2H_1(t) + H_2(t)\big)\big(2H_1(t) + H_2(t) + \mu\, t\big)\Big)$$

$$P_1(t) = \alpha_2 (2H_1(t) + H_2(t))$$

where

$$H_i(t) = \left(\frac{t}{\theta_i}\right)^{\eta_i}, \ i = 1,2 \ .$$

The system reliability function of configuration 2 is:

$$R_2(t) = \sum_{i=1}^{3} P_i(t) = \alpha_0 \ .$$

Since from equation (30), one can obtain

$$R_2(t) = \frac{r_2 r_3 (r_2 - r_3) e^{r_1} + r_1 r_3 (r_3 - r_1) e^{r_2} + r_1 r_2 (r_1 - r_2) e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)} \ , \tag{33}$$

where

$$r_1 = \frac{1/6(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{1/3} - 6\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{\left(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3}\right)^{1/3} - \frac{1}{3}a}$$

$$r_2, r_3 = \frac{-\frac{1}{12}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{1/3} + 3\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{\left(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3}\right)^{1/3} - \frac{1}{3}a}$$

$$\pm \frac{1}{2} i\sqrt{3}\left(\frac{\frac{1}{6}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{1/3} + 6\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{\left(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3}\right)^{1/3}}\right)$$

and

$$a = 6H(t)_1 + 2H_2(t) + 3\mu \ t$$

$$b = 12[H_1(t)]^2 + [H_2(t)]^2 + 8H_1(t).H_2(t) + 6\mu \ t \ H_1(t) + 2\mu \ t \ H_2(t) + 2\mu^2 t^2$$

$$c = 8[H_1(t)]^3 + 8 \ [H_1(t)]^2.H_2(t) + 2H_1(t).[H_2(t)]^2$$

for

$$H_1(t)=\left(\frac{t}{97.22}\right)^{2.12},H_2(t)=\left(\frac{t}{111.66}\right)^{1.79},\mu=0.05.$$

4.3. Calculations for Configuration 3:

Mean time to failure of the system:

For configuration 3, let $P_4(t)$ be the probability that exactly 5 components are working at time t, $(t \ge 0)$. If we let $P(t)$ denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0)=\left[P_4(0),P_3(0),P_2(0),P_1(0),P_0(0)\right]=\left[1,0,0,0,0\right], \tag{34}$$

where the transition rate matrix Q for reliability according to configuration 3 is given by:

$$Q=\begin{pmatrix} -2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0 \\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0 \\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0 \\ 0 & 0 & 2\lambda_1+\lambda_2 & -2\lambda_1-3\mu & 4\mu \\ 0 & 0 & 0 & 2\lambda_1 & -4\mu \end{pmatrix}.$$

To evaluate the transient solution is too complex. Therefore, we will restrict ourselves in calculating the MTTF. Therefore, we will take the transpose matrix of Q and delete the rows and columns for the absorbing state. The new matrix is called A.

$$A=\begin{pmatrix} -2\lambda_1-2\lambda_2 & 2\lambda_1+2\lambda_2 & 0 & 0 \\ \mu & -2\lambda_1-2\lambda_2-\mu & 2\lambda_1+2\lambda_2 & 0 \\ 0 & 2\mu & -2\lambda_1-\lambda_2-2\mu & 2\lambda_1+\lambda_2 \\ 0 & 0 & 3\mu & -2\lambda_1-3\mu \end{pmatrix}. \tag{35}$$

The expected time to reach an absorbing state is calculated from:

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)(-A^{-1})\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \tag{36}$$

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTTF_3 = \tag{37}$$

$$\frac{\lambda_1\left(3\mu^3 + 6\mu^2\lambda_2 + 8\mu^2\lambda_1 + (6\mu + 4\lambda_2)\lambda_2^2 + (18\mu + 24\lambda_2)\lambda_2\lambda_1 + (12\mu + 36\lambda_2 + 16\lambda_1)\lambda_1^2\right)}{8(\lambda_1 + \lambda_2)^3}.$$

Availability Analysis of the System:

For the availability case of configuration 3, we will use the initial condition initial conditions for this problem from equation (34):

The differential equations form can be expressed as:

$$\begin{pmatrix}\dot{P}_4(t)\\ \dot{P}_3(t)\\ \dot{P}_2(t)\\ \dot{P}_1(t)\\ \dot{P}_0(t)\end{pmatrix} = \begin{pmatrix}-2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0\\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0\\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0\\ 0 & 0 & 2\lambda_1+\lambda_2 & -2\lambda_1-3\mu & 4\mu\\ 0 & 0 & 0 & 2\lambda_1 & -4\mu\end{pmatrix} \cdot \begin{pmatrix}P_4(t)\\ P_3(t)\\ P_2(t)\\ P_1(t)\\ P_0(t)\end{pmatrix}.$$

The steady state availability can be obtained using the following procedure. In the steady state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state probabilities with:

$$A_T(\infty) = 1 - P_0(\infty), \tag{38}$$

and,

$$QP(\infty) = 0,$$

or, in the matrix form:

$$\begin{pmatrix} -2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0 \\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0 \\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0 \\ 0 & 0 & 2\lambda_1+\lambda_2 & -2\lambda_1-3\mu & 4\mu \\ 0 & 0 & 0 & 2\lambda_1 & -4\mu \end{pmatrix} \cdot \begin{pmatrix} P_4(\infty) \\ P_3(\infty) \\ P_2(\infty) \\ P_1(\infty) \\ P_0(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{39}$$

Using the following normalization condition:

$$\sum_{i=0}^{4} P_i(\infty) = 1. \tag{40}$$

We substitute (40) in any one of the redundant rows in (39) to yield

$$\begin{pmatrix} -2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0 \\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0 \\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0 \\ 0 & 0 & -2\lambda_1-\lambda_2 & -2\lambda_1-3\mu & 4\mu \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} P_4(\infty) \\ P_3(\infty) \\ P_2(\infty) \\ P_1(\infty) \\ P_0(\infty) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{41}$$

Solution of (41) provides the steady-state probabilities in the availability case. The explicit expression for $A_T(\infty)$ is given by:

$$\begin{aligned} A_T(\infty) = {} & [2\mu\lambda_1(3\mu^3 + (6\mu^2\lambda_2 + (6\mu + 2\lambda_2)\lambda_2^2 + (6\mu^2 + (12\mu + 8\lambda_2)\lambda_2 \\ & + (6\mu + 10\lambda_2 + 4\lambda_1)\lambda_1)\lambda_1)]/[6\mu^4 + 12\mu^3\lambda_2 + (12\mu^2 + \mu\lambda_2)\lambda_2^2 + 12\mu^3\lambda_1 + 24\mu^2\lambda_2\lambda_1 \\ & + (10\mu^2 + 2\lambda_2)\lambda_2^2\lambda_1 + 12\mu^2\lambda_1^2 + (17\mu + 8\lambda_2)\lambda_2\lambda_1^2 + (8\mu + 10\lambda_2 + 4\lambda_1)\lambda_1^3] \end{aligned} \tag{42}$$

Cost/Benefit Ratio:

The notion of cost-benefit analysis is simple in principle. We assume the size-proportional cost for the primary components, warm standby components, and cold standby components, respectively, shown in table (2) with this we calculate the costs for configuration 3. It utilizes the cost/benefit ratio (C/B) as a means to rank alternatives, let:

C = the cost for the configuration3,
B_1 = the *MTTF* of configuration 3,

or

B_2 = the $A_T(\infty)$ of configuration 3.

Table 1. The size-proportional cost for the components

Component	**Cost (in $)**
Primary 5MW	5E+6
Warm standby 5MW	3E+6
Cold standby 5MW	2E+6

The cost for configuration 3 (where there is two primary components, two warm standby components, and one cold standby component= $18E+6).

A numerical illustration is provided by considering the following parameters:

$$\lambda_1 = 0.6, \qquad \lambda_2 = 0.05, \qquad \mu = 1.0 .$$

Given these values, we can calculate for configuration 3:

1) cost/MTTF=1.39E+6
2) $\cos t / A_T(\infty)$ =18.5E+6

5. Conclusion

We have provided in this paper, the reliability of two configurations, when the components have time-dependent failure rate and a constant repair rate. By comparing the R(t) in both configurations, we can see that in the first configuration the reliability is higher than reliability in second configuration as shown in Figure 1. Moreover, from numerical results for the cost/benefit measure have been obtained for configuration 3 (special case), we have provided a systematic methodology to develop the mean time to system failure and the steady-state availability of series system with mixed standby components. By comparing the MTTF and the $A_T(\infty)$, we can draw a conclusion that the mean time to system failure and the steady-state availability are significantly improved by adding cold standby components.

Numerical results for the cost /benefit measure have been obtained for the configuration 3 gives smallest cost/MTTF than the cost/MTTF by configuration (two primary 5MW components, one warm standby 5MW component, and one cold standby 5MW component), and the configuration 3 gives smallest $\cos t / A_T(\infty)$ than $\cos t / A_T(\infty)$ by configurations (one primary 10MW component, one warm standby 10MW component, and one cold standby 10MW component), (one primary 10MW component, one warm standby 10MW component, and two cold standby

10MW components), and (one primary 10MW component, two warm standby 10MW components, and one cold standby 10MW component).

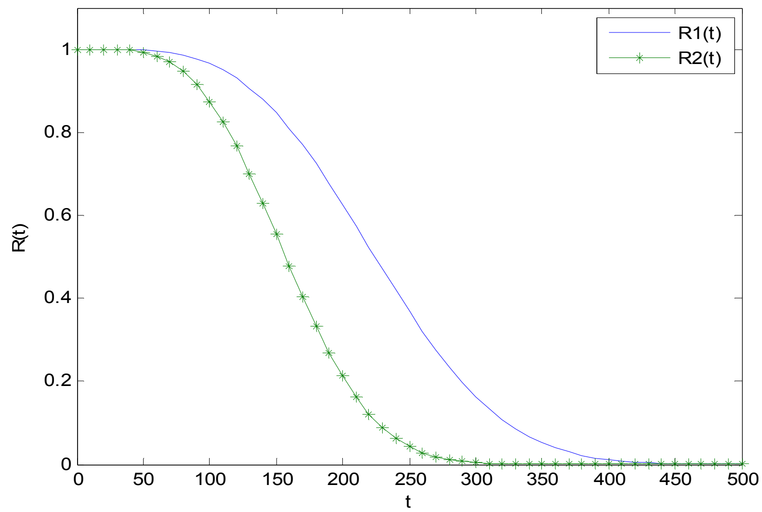

Figure 1: Relationships between R(t) and Time (hr.) in Configuration 1, 2

Appendix

Then from equation (4), we have:

$$v = \ln t_s - \frac{1}{\tau}\sum_{i=1}^{\tau} \ln t_i \tag{A.1}$$

and

$$\begin{aligned}\lim_{\eta\to 0} h(\eta) &= \frac{\sum_{i=1}^{\tau} \ln t_i + (m-\tau)\ln t_s}{m} - \frac{1}{\tau}\sum_{i=1}^{m} \ln t_i \\ &= \left[\ln t_s - \frac{1}{\tau}\sum_{i=1}^{m} t_i\right] - \frac{\tau}{m}\left[\ln t_s - \frac{1}{\tau}\sum_{i=1}^{\tau} t_i\right] \\ &= v - \frac{\tau}{m} v = \left(1 - \frac{\tau}{m}\right) v .\end{aligned} \tag{A.2}$$

Using equations (A.1) and (A.2) to obtain $\hat{\eta}$:

$$\hat{\eta} = \left[\frac{v + (1-\tau/m)v}{2}\right]^{-1} = \frac{2}{v(2-\tau/m)} . \tag{A.3}$$

This approximation simplifies to $\hat{\eta} = \frac{2}{v}$. Equation (A.3) provides a quick approximation to $\hat{\eta}$ and can be used as an initial estimate of $\hat{\eta}$ for iterative MLE routines.

Table 2. Computing estimate of the parameters α and β for number of failures times t_i.

Order failures times t_i For i=1,2,…,10										V	$\hat{\eta}=2/V$	$\hat{\theta}=\left(\sum_{i=1}^{10}\frac{t_i^{\eta}}{10}\right)^{1/\eta}$
1	2	3	4	5	6	7	8	9	10			
37	58	72	88	115	136	152	165	185	213	0.682	2.933	138.07
31	43	56	65	73	82	96	101	111	135	0.948	2.12	97.22
27	35	66	83	96	101	131	145	199	222	0.884	2.26	128.41
24	32	41	66	79	89	98	120	180	235	1.117	1.79	111.66
18	26	39	53	77	93	108	135	220	253	1.216	1.64	118.84

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