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LRS Bianchi Type-I Cosmology with Gamma Law EoS in $f(R,T)$ Gravity

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Abstract

We have studied the locally rotationally symmetric (LRS) Bianchi type-I line element in $f(R,T)$ (R is the Ricci scalar and T is the trace of the stress energy tensor) theory of gravity in presence of EoS parameter. The simplest case of $f(R,T)$ gravity, i.e. first choice, is considered. The "gamma-law" equations of state are considered to explore the role of particle creation in the early universe. The exact solutions of the field equations are obtained using the scalar expansion proportional to the shear. The physical and kinematical properties of the model are studied.

Keywords: LRS Bianchi type-I spacetime; $f(R,T)$ gravity; EOS parameter; gamma law; cosmological model; scalar expansion; shear scalar

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1. Introduction

The eternal human quest to comprehend the origin and content of the Universe has seen significant and exciting progress in recent years. There is a dramatic convergence of observation and theory as high precision probes like the Cosmic Microwave Background, the two supernovae projects by Reiss (1998) and Perlmutter (1999), have further refined the observational abilities in consonance with the theoretical work. This direct knowledge reaffirms that the accelerated expansion as well as the existence of dark energy and dark matter as the major components of the Universe and the Baryonic matter (Einstein et al. 2005) as only a tiny fraction of the existence. A web of interlocking observations has established that the expansion of the Universe is speeding up and not slowing, revealing the presence of some form of repulsive gravity. Within the context of general relativity the cause of cosmic acceleration is a mysterious strong negative pressure termed as "dark energy" accounting for about 73% of the Universe. The simplest explanation for dark energy is that it is a cosmological constant or vacuum energy.

A cosmological model is a mathematical representation of the Universe describing the geometry of space and time and the distribution and nature of matter within the framework of the Einstein theory of gravitation. The three assumptions that the Universe is expanding, isotropic and spatially homogeneous, form the basis of the cosmological explorations to date. Cosmological models that obey these assumptions were first described by Friedmann (1924) as well as Lemaitre (1931) and later analyzed from a geometrical perspective by Robertson (1936) and Walker (1937). These models may or may not cater to the cosmological constant. The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric or the more commonly used Friedmann-Roberson-Walker (FRW) metric thus defines the geometry of a isotropic, homogeneous, expanding Universe. The dynamics are driven by the energy content of the Universe and the equation of state of the components that make up energy density. Of late, it has not been sufficient to merely study the standard universes but there is a need to focus on the universe which is homogeneous but not always isotropic. In such cases the allowed solutions of the equations of general relativity are called Bianchi models, after the Italian mathematician Luigi Bianchi (1898). The recent advances in the high precision cosmology raise the issue of exact isotropy. Bianchi models are being widely used as a deviation from the FRW models. Hence a Bianchi type I cosmological model in $f(R,T)$ gravity will be the basis of our hypothesis.

In 1905, Albert Einstein proposed that the laws of physics are the same for all non-accelerating observers, and that the speed of light in a vacuum was independent of the motion of all observers. This theory was termed as "the theory of special relativity." It laid a new structure for all of physics and suggested new concepts of space and time. Einstein then spent ten years trying to incorporate acceleration in the theory and published his theory of General Relativity (GR) in 1915.

Despite the great success of Einstein's theory of General Relativity, the limitation was that it could not explain the late time acceleration, as the universe is believed to have higher accelerated expansion in the latter half of its lifetime. This late time acceleration could be due to an exotic dark energy (Mishra and Sahoo 2014) component or a modification of Einstein's laws of gravity.

The once-discarded Einstein cosmological constant (Weinberg 1989; Peebles et al. 2003) regained its popularity due to the late time acceleration. This expansion is sought to be catered to by adding a term in the Einstein Field Equations or by considering scalar field contribution thus attempting to evolve consistent cosmological models. Besides this, since late time acceleration brings a fundamental challenge to gravitational theories, there are attempts to modify the GR theory also by modifying the underlying geometry. One of the simplest modifications has been the replacement of the standard Einstein-Hilbert action by an arbitrary function of the Ricci scalar

R. Bertolami et al. (2007) have introduced an explicit coupling between the arbitrary function of the Ricci scalar and the matter Lagrangian density. These are the $f(R)$ theories.

Modified gravity models have been proposed in the works of Carroll et al. (2004), Sotiriou et al. (2010), and Nojiri et al. (2007) and references therein. In these proposed modifications, $f(R)$ theory of gravity is considered as appropriate, due to cosmologically important $f(R)$ models. Bertolami (2007) proposed a generalization of $f(R)$ modified theories of gravity has been described by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. As a result of the coupling the motion of the massive particles is non-geodesic, and an extra force orthogonal to the four velocity arises.

Nojiri and Odintsov (2011) have reviewed various modified gravity theories that are considered as gravitational alternatives for dark energy. In addition, Multamaki (2006, 2007), Clifton et al. (2012) and Mishra (2014) have investigated $f(R)$ gravity in a different context. Shamir (2010) has proposed a physically viable $f(R)$ gravity model, which show the unification of early time inflation and late time acceleration.

In another extension Harko et al. (2011) have proposed another extension of GR, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar and of the trace T of the stress-energy tensor. It has been suggested that due to the coupling between matter and geometry, the theory should depend on a source term, representing the variation of the matter-stress-energy tensor with respect to the metric.

The utility of late time acceleration in $f(R,T)$ gravity has been the focus of several recent investigations. Recently, Sahoo et al. (2016) has obtained Kaluza-Klein cosmological model in $f(R,T)$ gravity with $\Lambda(T)$. In the framework of $f(R,T)$ gravity, Nath et al. (2016a) have discussed Bianchi type III cosmological model while Mishra and Sahoo (2014) and Reddy and Shanthikumar (2013) studied Bianchi type III dark energy model and some anisotropic cosmological models, respectively. Ahmed and Pradhan (2014) have studied Bianchi type-V string and perfect fluid cosmological models respectively by consdering $f(R,T) = f_1(R)+f_2(T)$. Besides, Sharif (2012), Alvarenga (2013), Biswal (2015), and Myrzakulov (2012, 2012a) have investigated different aspects of $f(R,T)$ gravity. Recently, Singh et al. (2016) has studied the cosmological constant Λ in the frame work of $f(R,T)$ gravity.

Therefore, the cosmic acceleration in the results of $f(R,T)$ gravity, is from both geometric effects and matter contribution. The $f(R,T)$ has several interesting features promising to resolve issues of significant interest in cosmology.

2. A review of $f(R,T)$ gravity

The modified gravity with $f(R,T)$ action is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, T the trace of energy-momentum tensor T_{ij} of the matter, $\mathcal{L}_m$ corresponds to the matter Lagrangian. The energy momentum tensor

T_{ij} of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{ij}} \tag{2}$$

and its trace $T = g^{ij}T_{ij}$. Here, the matter Lagrangian $\mathcal{L}_m$ depends only on the metric tensor g_{ij} rather than its derivatives. Hence, we can write

$$T_{ij} = g_{ij}\mathcal{L}_m - \frac{\partial \mathcal{L}_m}{\partial g^{ij}}. \tag{3}$$

The equations of motion of $f(R,T)$ gravity are obtained by varying the action S with respect to g_{ij}.

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + &(g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) \\ &= 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \end{aligned} \tag{4}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}\mathcal{L}_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}. \tag{5}$$

Here

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R}, \quad f_T(R,T) = \frac{\partial f(R,T)}{\partial T},$$

the d'Alembert operator $\Box \equiv \nabla^i\nabla_i$, where ∇_i denotes the covariant derivative. Using a contraction of indices in equation (4),

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta, \tag{6}$$

where $\Theta = g^{ij}\Theta_{ij}$. The field equations of $f(R,T)$ gravity depends on the physical nature of Θ_{ij}. Hence depending on the nature of the matter source, one can obtain several theoretical models corresponding to matter source. Harko et al. (2011) explained three possible models as follows

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T). \end{cases} \tag{7}$$

Here we have considered the first model, i.e., $f(R,T) = R + 2f(T)$, where $f(T)$ is arbitrary function of the trace of matter source and it represents the interaction between curvature and matter. Let us assume $f(T) = \lambda T$, where λ is a constant. In this case the field equations (4) can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)\lambda = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f_T(R,T) + f(T)g_{ij}. \tag{8}$$

Assuming $(g_{ij}\Box - \nabla_i\nabla_j)\lambda = 0$, we get

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f_T(R,T) + f(T)g_{ij}. \tag{9}$$

In this paper we consider the natural unit system with $G = c = 1$, where G is the Newtonian gravitational constant and c is the speed of light in vacuum.

3. Metric and Field Equations

We consider a homogeneous and anisotropic symmetric Bianchi type-I metric (Sahoo 2015) as,

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \tag{10}$$

where the scale factors A and B are functions of cosmic time t only. The metric has symmetry to $xy-$plane. The average scale factor, spatial volume, scalar expansion for the above metric are,

$$a = (A^2B)^{\frac{1}{3}}, \quad V = a^3 = A^2B, \quad \theta = u^i_{;i} = 2\frac{A'}{A} + \frac{B'}{B}, \tag{11}$$

where $'$ represents an ordinary derivative with respect to cosmic time t. The directional Hubble parameters are $H_1 = \frac{A'}{A}$ and $H_3 = \frac{B'}{B}$. The Hubble parameter H can be written as,

$$H = \frac{1}{3}(2H_1 + H_3). \tag{12}$$

In the presence of particle creation, the energy-momentum tensor is given by,

$$T_{ij} = (\rho_m + p_m + p_c)u_i u_j - (p_m + p_c)g_{ij}, \tag{13}$$

where ρ_m, p_m, p_c are the energy density, thermodynamical pressure and supplementary pressure respectively. The supplementary pressure is considered as a part of cosmological pressure which enters into the Einstein field equations. The supplementary pressure p_c is known as the creation pressure of particles and defined as (Calvao et al. 1992)

$$p_c = -\frac{(\rho_m + p_m)}{n}\frac{dN}{dV}, \tag{14}$$

where N is the particle number, V is volume, and $n = \frac{N}{V}$ is the particle number density. From literature it is observed that the value of p_c is either negative or zero depending on the presence of particle creation. $u^i = (0, 0, 0, 1)$ is the four velocity vector satisfying the condition $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$. The particle flux vector is of the form

$$N^\alpha = nu^\alpha \tag{15}$$

and this satisfies the balance equation (Pregogine et al. 1989)

$$N^\alpha_{;\alpha} = \psi, \tag{16}$$

where ψ is a particle source term which may be positive or negative depending whether there is production or annihilation of particles. In cosmology, ψ is usually considered to be zero. Here we have considered the relation between V and the particle number n as matter creation (decay) process. For the metric (10), equation (16) is

$$n' + 3nH = \psi. \tag{17}$$

Now equation (14) for adiabatic matter creation takes the form

$$p_c = -\frac{(\rho_m + p_m)}{3nH}\psi. \tag{18}$$

The stress energy tensor for perfect fluid in presence of particle creation is given by

$$T = \rho_m - 3(p_m + p_c). \tag{19}$$

From literature there is no unique definition of the matter Lagrangian, it can be assumed as $\mathcal{L}_m = -(p_m + p_c)$. By help of energy-momentum tensor and matter Lagrangian, equation (5) can be written as

$$\Theta_{ij} = -2T_{ij} - (p_m + p_c)g_{ij}. \tag{20}$$

Using this, the field equation (9) can be written as

$$G_{ij} = 8\pi T_{ij} + 2[T_{ij} + (p_m + p_c)g_{ij}]f_T(R, T) + f(T)g_{ij}. \tag{21}$$

Thus, the field equations for space time (10) and energy-momentum tensor (13) are,

$$H_1^2 + H_3^2 + H_1' + H_3' + H_1H_3 = -8\pi(p_m + p_c) + \lambda T \tag{22}$$

$$3H_1^2 + 2H_1' = -8\pi(p_m + p_c) + \lambda T \tag{23}$$

$$H_1^2 + 2H_1H_3 = 8\pi\rho_m + 2(\rho_m + p_m + p_c)\lambda + \lambda T. \tag{24}$$

4. Solutions of the field equations

In order to get the exact solution to the field equations (22)-(24), i.e., three equations with five unknowns, namely A, B, ρ_m, p_m, p_c, two more assumptions are needed. Equation (18) gives the value of p_c from ρ_m and p_m, which implies we required only one additional assumption. To solve the set of field equations, we have considered the shear scalar is proportional to the scalar expansion. This yields

$$H_3 = lH_1, \tag{25}$$

where $l \neq 1$ is a constant. For $l = 1$, the model is isotropic and anisotropic otherwise. From equations (22) and (23) we obtain

$$H_1 = \frac{1}{(2+l)t + k_1}, \tag{26}$$

where $k_1$is integration constant. Using equation (26) in equation (25), we get

$$H_3 = \frac{l}{(2+l)t + k_1}. \tag{27}$$

The metric potentials $A(t)$ and $B(t)$ are

$$A(t) = k_2[(2+l)t + k_1]^{\frac{1}{2+l}} \tag{28}$$

and

$$B(t) = k_3[(2+l)t + k_1]^{\frac{l}{2+l}}, \tag{29}$$

where $k_2, k_3$are integrating constants. The metric (10) becomes

$$ds^2 = dt^2 - k_2^2[(2+l)t + k_1]^{\frac{2}{2+l}}(dx^2 + dy^2) - k_3^2[(2+l)t + k_1]^{\frac{2l}{2+l}}dz^2, \tag{30}$$

which is the anisotropic model of the universe in $f(R,T)$ gravity. Here we have considered the EoS parameter of perfect fluid which is known as gamma-law in cosmological domain as

$$p_m = (\gamma - 1)\rho_m, \tag{31}$$

where γ is a constant lies in $[0,2]$. Using (26), (27) and (31) in the set (22)-(24), the energy density of matter, particle creation pressure and the pressure of the matter are given by

$$\rho_m = \frac{1+2l}{(2\lambda + 8\pi)[(2+l)t + k_1]^2}, \tag{32}$$

$$p_c = \frac{(1+2l)(2-\gamma)}{(2\lambda + 8\pi)[(2+l)t + k_1]^2} \tag{33}$$

and

$$p_m = \frac{(1+2l)(\gamma-1)}{(2\lambda + 8\pi)[(2+l)t + k_1]^2}. \tag{34}$$

From (12), the Hubble parameter H can be written as

$$H = \frac{2+l}{3[(2+l)t + k_1]}. \tag{35}$$

The volume V is obtained as

$$V = k_2^2 k_3[(2+l)t + k_1]. \tag{36}$$

It is observed that the Hubble parameter decreases while the spatial volume increase with increase in time. The values scale factor, scalar expansion and the shear scalar are obtained as

$$a = k_4[(2+l)t + k_1]^{\frac{1}{3}}, \qquad k_4 = (k_2^2 k_3)^{\frac{1}{3}}, \tag{37}$$

$$\theta = \frac{2+l}{(2+l)t + k_1} \tag{38}$$

and

$$\sigma^2 = \frac{(1-l)^2}{3[(2+l)t + k_1]^2}. \tag{39}$$

The anisotropy parameter $\Delta = 6\left(\frac{\sigma}{\theta}\right)^2$ is

$$\Delta = \frac{1-l^2}{3(l+2)}. \tag{40}$$

4. Conclusion

We have constructed LRS Bianchi type I cosmological model in $f(R,T)$ gravity with the equation of state (EoS) parameter. We have restricted γ between [0,2], so that the energy density behavior for both the positive and negative pressure can be studied. We have observed that the scalar expansion of the model decreases with increase in time for $l > 0$. As the mean anisotropy parameter is constant, which is a measure of deviation from isotropic expansion, the universe does not represent isotropic. However, for $l = 1$, one can obtain the isotropic behavior of the model.

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