EFFECT OF DAMPING AND THERMAL GRADIENT ON VIBRATIONS OF ORTHOTROPIC RECTANGULAR PLATE OF VARIABLE THICKNESS

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Abstract

In this present paper, damped vibrations of an orthotropic rectangular plate resting on elastic foundation with thermal gradient is modeled, considering variable thickness of plate. Following Le`vy approach, the governed equation of motion is solved numerically using quintic spline technique with clamped and simply supported edges. The effect of damping parameter and thermal gradient together with taper constant, density parameter and elastic foundation parameter on the natural frequencies of vibration for the first three modes of vibration are depicted through Tables and Figures, and mode shapes have been computed for fixed value of plate parameter. It has been observed that the rate of decrease of frequency parameter with damping parameter \( D_k \) for C-SS plate is higher than that for C-C plate keeping all other parameter fixed.

Keywords: Damping; Thermal gradient; Taperness; Elastic foundation; Orthotropy

MSC 2010 No.: 41A15, 81V99

1. Introduction

Structural damping is essential for design engineering as damping materials enhance the performance of system by increasing structural stiffness and thermal stability and the effect of temperature on mechanics of solid bodies has acquired great interest because of rapid development in space technology, high speed atmospheric flights and nuclear energy applications. Numerous studies on free vibration for isotropic/orthotropic plates of uniform/non-uniform thickness with or without temperature effect have been reported. Among these, Laura
and Gutierrez (1980) discussed vibration analysis of a rectangular plate subjected to a thermal gradient. Tomar and Gupta (1983, 1985) analyzed the effect of thermal gradient on frequencies of an orthotropic rectangular plate with variable thickness. In this series, Gupta et al. (2007) observed the thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Effect of non-homogeneity on thermally induced vibration of orthotropic visco-elastic rectangular plate of linearly varying thickness studied by Gupta and Singhal (2010). Gupta et al. (2013) had evaluated the effect of thermal gradient on the vibration of parallelogram plate with linearly varying thickness in both directions and thermal effect in linear form only. Effect of thermal gradient on vibration of non-homogeneous orthotropic trapezoidal plate of varying thickness has been studied by Gupta and Sharma (2013). Khanna and Singhal (2014) analyzed the thermal effect on vibration of tapered rectangular plate. Gupta and Jain (2014) analyzed exponential temperature effect on frequencies of a rectangular plate of non-linear varying thickness using quintic spline technique.

Plates resting on elastic foundation have applications in pressure vessels technology such as petrochemical, marine and aerospace industry, building activities in cold regions and aircraft landing in arctic operations discussed by Mcfadden and Bennett (1991), and Civalek and Acar (2007). In a series of papers, Lal et al., (2001) have studied the transverse vibrations of a rectangular plate of exponentially varying thickness resting on an elastic foundation. Transverse vibration of non-homogeneous orthotropic rectangular plate with variable thickness was discussed by Lal and Dhanpati (2007). In many applications of vibration and wave theory the magnitudes of the damping forces are small in comparison with the elastic and inertia forces but these small forces may have very great influence under some special situations. Recently, Robin and Rana (2013) analyzed the damped vibrations of isotropic/orthotropic rectangular plate with varying thickness resting on elastic foundation. The study of damped vibration of infinite plate with variable thickness resting on elastic foundation was carried out by Rana and Robin (2015).

In reality all the vibrations are damped vibration and foundations of underlying vibrational problems are elastic in nature, and thermal effects on the damped vibrations of orthotropic rectangular plates of variable thickness with elastic foundations is not considered by researchers but it plays a key role in vibrational problems. Keeping this in view thermally induced damped vibrations of orthotropic rectangular plates with exponentially thickness variation resting on Winkler foundation is presented here using quintic spline method on the basis of classical plate theory. Effect of damping parameter and thermal gradient together with thickness variation and foundation parameter on the frequencies has been illustrated for the first three modes of vibration for two different combinations of clamped, and simply supported correct to four decimal places.

2. Mathematical Formulation

Consider a non-homogeneous orthotropic rectangular plate of length ‘a’, breath ‘b’, thickness ‘h(x,y)’ and density ‘ρ’, with resting on a winkler-type elastic foundation ‘k_f’ occupying the domain 0 ≤ x ≤ a, 0 ≤ y ≤ b in the xy plane. The x-and y axes are taken along the principal directions and z –axes is perpendicular to the xy plane. The middle surface being z = 0 and the origin is at one of the corners of the plate. The differential equation which governs the damped transverse vibration of such plates is given by
\[ D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2 \frac{\partial H}{\partial x} \frac{\partial^3 w}{\partial x \partial y^2} + 2 \frac{\partial H}{\partial y} \frac{\partial^3 w}{\partial y \partial x^2} + 2 \frac{\partial D_y}{\partial x} \frac{\partial^3 w}{\partial x^3} + 2 \frac{\partial D_y}{\partial y} \frac{\partial^3 w}{\partial y^3} \]
\[ + \frac{\partial^2 D_x}{\partial x^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2} \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_{xy}}{\partial x \partial y} \frac{\partial w}{\partial x} + \frac{\partial^2 D_{xy}}{\partial y \partial x} \frac{\partial w}{\partial y} \]
\[ + 4 \frac{\partial^2 D_{xy}}{\partial x \partial y} \frac{\partial^2 w}{\partial x^2 \partial y} + \rho h \frac{\partial^2 w}{\partial t^2} + K \frac{\partial w}{\partial t} + K_j w = 0, \]  \tag{1}

where

\[ D_x = E_x h^3 / (1 - \nu_x \nu_y), D_y = E_y h^3 / (1 - \nu_x \nu_y), D_1 = \nu_y E_x h^3 / 12(1 - \nu_x \nu_y), H = D_1 + 2D_{xy}, \]
\[ D_{xy} = G_{xy} h^3 / 12, \nu_x E_x = \nu_y E_y, \]

\( w(x, y, t) \) is the transverse deflection, \( t \) is the time, and \( E_x, E_y, \nu_x, \nu_y \) and \( G_{xy} \) are material constants in proper directions defined by an orthotropic stress-strain law.

Let the two opposite edges \( y = 0 \) and \( y = b \) of the plate be simply supported and thickness \( h = h(x, y) \) varies exponentially along the length i.e., in the direction of \( x \)-axis. Thus, ‘\( h \)’ is independent of \( y \) i.e., \( h = h(x) \).

**Figure 1.**  Boundary conditions and vertical cross-section of the plate

For a harmonic solution, the deflection function \( w \) satisfying the condition at \( y = 0 \) and \( y = b \), is assumed

\[ w(x, y, t) = W(x) \sin \frac{m \pi y}{b} e^{-\gamma t} \cos pt, \]  \tag{2}

where ‘\( p \)’ is the circular frequency of vibration and ‘\( m \)’ is a positive integer. Furthermore, for elastically non-homogeneous material, it is assumed that Young’s moduli \( E_x, E_y \) and density \( \rho \) are functions of space variable \( x \) only, and shear modulus is

\[ G_{xy} = \sqrt{E_x E_y / 2(1 + \nu_x \nu_y)}. \]
Thus, Equation (1) becomes
\[
E_x h^3 W^{(iv)} + \left\{ 6E_x h' h'' + 2h^3 E_x \right\} W'' + \left\{ -2 \frac{m^2 \pi^2}{b^2} (\nu_y E_x h^3 \right. \\
+ 2G_{xy} (1 + \nu_y \nu_x) h^3) \right\} W'
\]
\[
-2 \frac{m^2 \pi^2}{b^2} \left\{ \nu_y (3E_x h'' h' + h' E_x') + 2(1 - \nu_x \nu_y) (3h^2 G_{xy} h'' + h' E_x') \right\} W \\
+ \left[ \frac{m^4 \pi^4}{b^2} E_x h^3 - \nu_y \frac{m^2 \pi^2}{b^2} \left\{ 3E_x h'' h'' + 6E_x h'' + 6h'h' E_x' + h^3 E_x'' \right\} \right] W \cos pt
\]
\[+ 12(1 - \nu_x \nu_y) K_j W \cos pt + \left[ 12(1 - \nu_x \nu_y) \rho h \left\{ (\gamma^2 - p^2) \cos pt \right. \\
+ 2p \gamma \sin pt \right\} W = 0.
\]

Introducing the non-dimensional variables
\[
H = \frac{h}{a}; X = \frac{x}{a}; \overline{E} = \frac{E}{a}; \overline{W} = \frac{W}{a}; \overline{\rho} = \frac{\rho}{a}; \lambda^2 = m^2 \pi^2 \left( \frac{a}{b} \right)^2,
\]
and equating the coefficient of \( \sin(pt) \) and \( \cos(pt) \) independently to zero, Equation (3) reduces to
\[
\overline{E}_x H^3 \overline{W}^{(iv)} + 2 \left\{ 3\overline{E}_x H^2 H' + H^3 E_x \right\} \overline{W}'' + \left\{ 3\overline{E}_x H^2 H'' + 6\overline{E}_x H H' E_x + H^3 E_x'' \right\} \overline{W}'' \\
-2 \lambda^2 \left\{ \nu_y (3\overline{E}_x H^2 H' + H'' E_x) + 2(1 - \nu_x \nu_y) (3H^2 G_{xy} H' + H^3 G_{xy}') \right\} \overline{W}'
\]
\[
\left\{ \lambda^4 \overline{E}_x H^4 - \lambda^2 \nu_y \left\{ 3\overline{E}_x H^2 H'' + 6\overline{E}_x H H'' + 6H^2 H' E_x' + H^3 E_x'' \right\} \right. \\
+ \left[ 12(1 - \nu_x \nu_y) \left\{ \frac{-k^2}{4H \overline{\rho}} - a^2 p^2 \overline{\rho}^2 H + aK_f \right\} \right] \overline{W} = 0.
\]

The exponential temperature variation for the plate is assumed in the following form
\[ T = T_0 \left( \frac{e-x}{e-1} \right), \]  

where \( T \) is the temperature excess above the reference temperature at any point \( x \) and \( T_0 \) is the temperature excess above the reference temperature at the end \( x = 0 \). Furthermore, most of the engineering materials are found to have a linear relationship between the modulus of elasticity and temperature as discussed by Nowacki (1962).

Therefore, we get

\[ \bar{E} = \bar{E}_0 (1 - \xi T), \]  

where \( \bar{E}_0 \) is the modulus of elasticity of the material at the reference temperature and \( \xi \) is a constant. If the temperature at \( x = 1 \) is assumed as the reference temperature, the modulus variable becomes

\[ \bar{E}(x) = \bar{E}_0 \left( 1 - \eta \left( \frac{e-x}{e-1} \right) \right), \]

where

\[ \eta = \xi T_0, (0 \leq \eta \leq 1), \] is thermal gradient.

Substituting

\[ H = H_0 e^{\alpha x}; \bar{E}_x = E_1 \left( 1 - \eta \left( \frac{e-x}{e-1} \right) \right); \bar{E}_y = E_2 \left( 1 - \eta \left( \frac{e-x}{e-1} \right) \right); \rho = \rho_0 e^{\beta x}, \]

where

\[ H_0 = (H)_{x=0}, \bar{E}_1 = (\bar{E}_x)_{x=0}, \bar{E}_2 = (\bar{E}_y)_{y=0}, \rho_0 = (\rho)_{x=0}, \]

and ‘ \( \alpha \)’ is the taper constant due to exponentially varying thickness of plate, and equating the coefficients, the following equation is formed

\[ A_0 \frac{d^4 \bar{W}}{dX^4} + A_1 \frac{d^3 \bar{W}}{dX^3} + A_2 \frac{d^2 \bar{W}}{dX^2} + A_3 \frac{d \bar{W}}{dX} + A_4 \bar{W} = 0, \]  

where
\[ A_0 = 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right), \]
\[ A_1 = 2 \left[ \frac{\eta e^x}{e - 1} + 3\alpha \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right) \right], \]
\[ A_2 = 9\alpha^2 \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right) + (6\alpha + 1) \frac{\eta e^x}{e - 1} - 2\lambda^2 (\nu_y + \sqrt{E_2/E_1} (1 - \nu_y \nu_x)) \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right), \]
\[ A_3 = -2\lambda^2 \left[ \left( \nu_y + \sqrt{E_2/E_1} (1 - \nu_y \nu_x) \right) \left( \frac{\eta e^x}{e - 1} + 3\alpha \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right) \right) \right], \]
\[ A_4 = \lambda^4 E_2/E_1 \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right) - \lambda^2 \nu_y \left( 9\alpha^2 \left( 1 - \eta \left( \frac{e^{-e^x}}{e - 1} \right) \right) + (6\alpha + 1) \frac{\eta e^x}{e - 1} \right) \]
\[ - 3D_k I^2 e^{-(4\alpha - \beta)x} + \Omega^2 I^2 e^{(-2\alpha + \beta)x} - 12E_f e^{-3\alpha X} C^* \}, \]
\[ D_k = \frac{(1 - \nu_y \nu_x) K^2}{E_1 \rho_0^2}, I^* = \frac{1}{H_0^2}, C^* = \frac{1}{H_0^3}, E_f = \frac{a(1 - \nu_y \nu_x) K_L}{E_1}, \Omega^2 = \frac{12(1 - \nu_y \nu_x)a^2 \rho \rho_0^2}{E_1}, \]

and \( \Omega, D_k, E_f \) are frequency parameter, damping parameter and elastic foundation parameter, respectively. \( \rho_0 \) is the density of the plate and \( E_1, E_2 \) the Young’s moduli in proper directions at \( x = 0 \). The solution of Equation (8) together with boundary conditions at the edge \( x = 0 \) and \( x = 1 \) constitutes a two-point boundary value problem. As the PDE has several plate parameters, it becomes quite difficult to find its exact solution. Keeping this in mind which is complex for the purpose of computation, the quintic spline interpolation technique is used.

According to the spline technique, suppose \( W(x) \) be a function with continuous derivatives in \([0, 1]\) and interval \([0, 1]\) be divided into ‘n’ subintervals by means of points \( X_i \) such that

\[ 0 = X_0 < X_1 < X_2 < \ldots < X_n = 1, \]

where

\[ \Delta X = \frac{1}{n}, X_i = i\Delta X (i = 0,1,2,\ldots,n). \]

Let the approximating function \( \bar{W}(X) \) for the \( W(x) \) be a quintic spline with the following properties:

(i) \( \bar{W}(X) \) is a quintic polynomial in each interval \( (X_k, X_{k+1}) \).
(ii) \( \bar{W}(X) = W(X_k), k = 0,1,2,\ldots,n. \)
(iii) \( \frac{dW}{dX}, \frac{d^2W}{dX^2}, \frac{d^3W}{dX^3} \text{ and } \frac{d^4W}{dX^4} \) are continuous.

In view of the above axioms, the quintic spline takes the form

\[
\bar{W}(X) = a_0 + \sum_{i=0}^{4} a_i (X - X_0)^i + \sum_{j=0}^{n-1} b_j (X - X_j)^5,
\]

where

\[
(X - X_j)_+ = \begin{cases} 
0, & \text{if } X \leq X_j, \\
(X - X_j), & \text{if } X > X_j,
\end{cases}
\]

and \( a_i, b_j \) are constants.

Thus, for the satisfaction at the \( n^{th} \) knot, Equation (8) reduces to

\[
A_4a_0 + [A_1(X_m - X_0) + A_2]a_1 + [A_4(X_m - X_0)^4 + 2A_3(X_m - X_0) + 2A_2]a_2 \\
+ [A_4(X_m - X_0)^3 + 3A_3(X_m - X_0)^2 + 6A_2(X_m - X_0) + 6A_1]a_3 \\
+ [A_4(X_m - X_0)^4 + 4A_3(X_m - X_0)^3 + 12A_2(X_m - X_0)^2 + 24A_1(X_m - X_0) + 24A_0]a_4 \\
+ \sum_{j=0}^{n-1} b_j [A_4(X_m - X_j)^5 + 5A_3(X_m - X_j)^4 + 20A_2(X_m - X_j)^3 + 60A_1(X_m - X_j)^2] \\
+ 120A_0(X_m - X_j)_+ = 0.
\]

(10)

For \( m = 0, 1, \ldots, n \), the system (10) contains \( (n + 1) \) homogeneous equation with \( (n + 5) \) unknowns, \( a_i, i = 0, 1, \ldots, 4 \), and \( b_j, j = 0, 1, 2, \ldots, (n - 1) \), can be represented in matrix form as

\[
[ A ] [ B ] = [ 0 ],
\]

where \([ A ]\) is a matrix of order \((n+1) \times (n+5)\), while \([ B ]\) and \([ 0 ]\) are column matrices of order \((n \times 5)\).

3. Boundary Conditions and Frequency Equation

The following two cases of boundary conditions have been considered:

(i) \( \text{(C-C): clamped at both the edge } X = 0 \text{ and } X = 1. \)

(ii) \( \text{(C-SS): clamped at } X = 0 \text{ and simply supported at } X = 1. \)

The relations that should be satisfied at clamped and simply supported, respectively, are
\[
W = \frac{dW}{dX} = 0, \quad W = \frac{d^2W}{dX^2} = 0.
\] 

(12)

Applying the boundary conditions C-C to the displacement function by Equation (9) one obtains a set of four homogeneous equations in terms of \((n+5)\) unknown constants which can be written as

\[
[B^c_c][B] = [0],
\]

(13)

where \(B^c_c\) is a matrix of order \(4 \times (n+5)\).

Therefore, Equation (11) together with the Equation (13) gives a complete set of \((n+5)\) homogeneous equations having \((n+5)\) unknowns which can be written as

\[
\begin{bmatrix}
A \\
B^c_c
\end{bmatrix} [B] = [0].
\]

(14)

For a non-trivial solution of Equation (11), the characteristic determinant must vanish, i.e.,

\[
\left| \frac{A}{B^c_c} \right| = 0.
\]

(15)

Similarly, for (C-SS) plate the frequency determinant can be written as

\[
\left| \frac{A}{B^{ss}} \right| = 0,
\]

(16)

where \(B^{ss}\) is a matrix of order \(4 \times (n+5)\).

4. Numerical Results and Discussions

The frequency Equations (15) and (16) have been solved to get the values of the frequency parameter \(\Omega\) for various values of plate parameters. By putting \(m = 1\) in frequency equations, the numerical results have been computed for first three modes of vibrations. The elastic constants for the plate material (‘ORTHOI’) are taken as

\[
E_1 = 1 \times 10^{10} \text{Mpa}, \quad E_2 = 5 \times 10^{10} \text{Mpa}, \quad \nu_x = 0.2, \quad \nu_y = 0.1.
\]

At the edge \(X = 0\), the thickness \(h_0\) have been considered as 0.1. To choose the appropriate interval \(\Delta X\), a computer program was developed for the evaluation of frequency parameter \(\Omega\), and run for \(n = 10, 20, 30, \ldots, 150\), and for different sets of the values of parameters for both the boundary conditions. Figure 2 represents the percentage error in the numerical values of frequency parameter \(\Omega\) up to fourth decimal places with the increase in the number of nodes. In all the computation \(n = 140\) has been fixed to achieve the decimal accuracy.
Figure 2. Percentage error in frequency parameter $\Omega$: (a) for c-c plate (b) c-ss plate for $a/b = 0.25$, $\alpha = 0.04$, $\beta = 0.4$, $E_f = 0.02$, $D_k = 0.01$; $\diamond$, First mode; $\circ$, Second mode; $\Box$, Third mode; Percentage error $= [(\Omega_n - \Omega_{140})/\Omega_{140}] \times 100$; $n = 10, 20, 30, \ldots, 140$

The results are presented in Tables 1-3 and Figures 3-5, for different values of damping parameter $D_k = 0.0, 0.001, 0.002, 0.003, 0.004$, and 0.005, thermal gradient $\eta = 0.0, 0.2, 0.4, 0.6$, foundation parameter $E_f = 0.0, 0.2$, density parameter $\beta = \pm 0.5$ and taper parameter $\alpha = \pm 0.5$ for both the boundary conditions C-C and C-SS, by taking the fixed values of aspect ratio $a/b = 1.0$. It is found that the values of frequency parameter $\Omega$ for a C-C plate are greater than that for C-SS plate for the same set of values of plate parameters.

Table 1 shows the values of frequency parameter $\Omega$ with the increasing value of damping parameter $D_k$ for the fixed value of density parameter $\beta = 0.5$ and foundation parameter $E_f = 0.02$, for first three modes of vibration of C-C and C-SS plates. Figure 3a shows the behavior of frequency parameter $\Omega$. It decreases with the increasing values of damping parameter $D_k$ for two different values of taper parameter $\alpha = \pm 0.5$, thermal gradient $\eta = 0.0, 0.5$ and density parameter $\beta = 0.5$ for both the plates. The rate of decrease of $\Omega$ with damping parameter $D_k$ for C-SS is higher than that for C-C plate keeping all other plate parameters fixed. This rate increases with the increase in the value of non taper parameter $\alpha$ and thermal gradient $\eta$. A similar inference can be drawn from Figures 3b and 3c, when the plate is vibrating in the second mode as well as in the third mode of vibration except that the rate of decrease of $\Omega$ with $D_k$ is lesser as compared to the first mode.

Tables 2 and 3, provide the inference of thermal gradient $\eta$ on frequency parameter $\Omega$ for two values of damping parameter $D_k = 0.0$, and 0.001, taper parameter $\alpha = \pm 0.5$ density parameter $\beta = \pm 0.5$ and foundation parameter $E_f = 0.0, 0.02$ for the fixed value of aspect ratio $a/b = 1.0$. It is noticed that the frequency parameter $\Omega$ decreases continuously with the increasing value of thermal gradient $\eta$ for C-C and C-SS plates, whatever be the value of other plate parameters. It is
found that the rate of decrease of frequency parameter $\Omega$ for C-C plate is higher than C-SS plate for three modes. Figure 4a gives the inference of thermal gradient $\eta$ on frequency parameter $\Omega$ for the first mode of vibration for fixed values of $\beta = 0.5$ and $\alpha = 0.5$. This rate decreases with the increase in the value of foundation parameter $E_f$, and also increases with the increase in the value of damping parameter $D_k$ and the number of modes, as is clear from Tables 3b and 3c when the plate is vibrating in the second and third mode of vibration.

Table 1. Values of frequency parameter $\Omega$ for C-C and C-SS plate for $a/b= 1.0$ $E_f= 0.02$, $\beta= 0.5$

<table>
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<th>MODE</th>
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<td>C-C</td>
<td>C-SS</td>
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<td>C-SS</td>
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<tr>
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Table 2. Values of frequency parameter $\Omega$ for C-C and C-SS plate for $a/b=1.0$, $\alpha=0.5$, $\beta=0.5$

<table>
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<th>$Dk=0.0, Ef=0.02$</th>
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<tr>
<td>$\eta=0.4$</td>
<td></td>
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<tr>
<td>II</td>
<td>66.5659</td>
<td>54.4427</td>
<td>65.9166</td>
<td>53.5925</td>
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<tr>
<td>III</td>
<td>125.2135</td>
<td>107.9092</td>
<td>124.8594</td>
<td>107.4836</td>
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<tr>
<td>$\eta=0.6$</td>
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<td></td>
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</tr>
<tr>
<td>I</td>
<td>24.5758</td>
<td>18.6252</td>
<td>22.9292</td>
<td>15.9702</td>
</tr>
<tr>
<td>II</td>
<td>60.4844</td>
<td>49.3811</td>
<td>59.7835</td>
<td>48.4588</td>
</tr>
<tr>
<td>III</td>
<td>113.6045</td>
<td>97.7699</td>
<td>113.2213</td>
<td>97.308</td>
</tr>
</tbody>
</table>

Table 3. Values of frequency parameter $\Omega$ for C-C and C-SS plate for $a/b=1.0$, $Ef=0.02$, $Dk=0.001$

<table>
<thead>
<tr>
<th>MODE</th>
<th>$\alpha=0.5, \beta=0.5$</th>
<th>$\alpha=0.5, \beta=0.5$</th>
<th>$\alpha=0.5, \beta=0.5$</th>
<th>$\alpha=0.5, \beta=0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C-C</td>
<td>C-SS</td>
<td>C-C</td>
<td>C-SS</td>
</tr>
<tr>
<td>$\eta=0.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>62.7187</td>
<td>54.5053</td>
<td>48.23</td>
<td>41.4371</td>
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<tr>
<td>III</td>
<td>113.8324</td>
<td>100.8062</td>
<td>87.6308</td>
<td>77.1639</td>
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<tr>
<td>$\eta=0.2$</td>
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<td></td>
</tr>
<tr>
<td>I</td>
<td>29.917</td>
<td>27.3433</td>
<td>23.1179</td>
<td>20.5718</td>
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<tr>
<td>II</td>
<td>59.2707</td>
<td>51.5817</td>
<td>45.7001</td>
<td>39.3095</td>
</tr>
<tr>
<td>III</td>
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<td>94.8687</td>
<td>82.6838</td>
<td>72.7941</td>
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<tr>
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<tr>
<td>II</td>
<td>55.456</td>
<td>48.327</td>
<td>42.8678</td>
<td>36.9442</td>
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<td>$\eta=0.6$</td>
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<tr>
<td>II</td>
<td>50.9022</td>
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<td>III</td>
<td>90.803</td>
<td>80.452</td>
<td>70.6227</td>
<td>62.1942</td>
</tr>
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</table>
From Figure 4b, the effect of thermal gradient \( \eta \) is found to decrease the frequency parameter \( \Omega \); however, the rate of decrease gets increased to more than twice of the first mode for both the boundary conditions. In case of third mode, this rate of decrease further increases and becomes nearly twice of the second mode as is evident from Figure 4c.

Figure 5a provides graphs of frequency parameter \( \Omega \) verses thermal gradient \( \eta \) for the first mode of vibration. There is a continuous decrease in the value of frequency parameter \( \Omega \) for fixed values of \( E_f = 0.02 \) and damping parameter \( D_k = 0.001 \) for both the boundary conditions. This rate decreases with the increase in the value of density parameter \( \beta \); it increases with the increase in the value of taper parameter \( \alpha \) and in increases the number of modes, as is clear from Tables 3b and 3c, when the plate is vibrating in the second and third mode of vibration. From Figure 5b, the effect of thermal gradient \( \eta \) is found to decrease the frequency parameter \( \Omega \); however, the rate of decrease gets increased to more than twice of the first mode for both the boundary conditions. In case of third mode, this rate of decrease further increases and becomes nearly twice of the second mode as is evident from Figure 5c. The normalized displacements for the two boundary conditions C-C and C-SS are plotted in Figures 6 and 7, respectively. The plate thickness varies parabolically in \( X \)-direction and the plate is considered resting on elastic foundations \( E_f = 0.02 \) with damping parameter \( D_k = 0.01 \). Mode shapes for a rectangular plate i.e., \( a/b = 0.25 \) have been computed and observed that the nodal lines are seen to shift towards the edge, i.e., \( X = 1 \) as the edge \( X = 0 \) increases in thickness for both the plates. No special change was seen in the pattern of nodal lines by taking different values of \( \beta \) and \( E_f \). The normalized displacements were differing only at the third or fourth place after decimal for both the boundary conditions.

![Graphs of frequency parameter \( \Omega \) verses thermal gradient \( \eta \) for the first mode of vibration.](image)

**Figure 3.** Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode , for \( a/b=1.0, \beta=0.5, \ E_f=0.02 \), ..., C-C; --,C-SS; \( \Delta, \alpha=-0.5, \eta=0.0 \); \( \bullet, \alpha=-0.5, \eta=0.5 \); \( \square, \alpha=0.5, \eta=0.0 \); \( \blacktriangle, \alpha=0.5, \eta=0.5 \)
Figure 4. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode, for $a/b=1.0, \beta=0.5, \alpha=0.5$, $\Delta, C-C$; $\Delta, C-SS$; $\Delta, D_k=0.0, E_f=0.0$, •, $D_k=0.001, E_f=0.0$, □, $D_k=0.0, E_f=0.02$; ▲, $D_k=0.001, E_f=0.02$

Figure 5. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode, for $a/b=1.0, E_f=0.02, D_k=0.001$, $\Delta, C-C$; $\Delta, C-SS$; $\Delta, \alpha=-0.5, \beta=-0.5$; •, $\alpha=-0.5, \beta=0.5$; □, $\alpha=0.5, \beta=-0.5$; ▲, $\alpha=0.5, \beta=0.5$
Figure 6. Normalized displacements for c-ss-c-ss plate, for $a/b=1; \ h=0.1, \ \beta=0.5, D_k=0.005, \ E_f=0.02$; -, First mode; ——, Second mode; ......Third mode, □, $\alpha=-0.5, \eta=0.5$; ▲, $\alpha=0.5, \eta=0.5$; o, $\alpha=0.5, \eta=0.0$

Figure 7. Normalized displacements for c-ss-ss-ss plate, for $a/b=1; \ h=0.1, \ \beta=0.5, D_k=0.005, \ E_f=0.02$; -, First mode; ——, Second mode; ......Third mode, □, $\alpha=-0.5, \eta=0.5$; ▲, $\alpha=0.5, \eta=0.5$; o, $\alpha=0.5, \eta=0.0$
5. Conclusion

In the present study results are computed using MATLAB within the permissible range of parameters up to the desired accuracy ($10^{-8}$), which validates the actual phenomenon of vibrational problem. The rate of decrease of frequency parameter $\Omega$ with thermal gradient $\eta$, decreases with the increase in the value of foundation parameter and non-homogeneity parameter and this rate increases with the increase in the value of taper parameter and damping parameter. Temperature effect, variation in thickness, elastic foundation, damping parameter and non-homogeneity parameter are of great interest because engineers often try to know natural frequency and modes of vibration before finalizing the design of a structure or machine. Our endeavor is to provide a mathematical model for analyzing the vibrational behavior of orthotropic rectangular plate resting on elastic foundation for different values of thermal gradient, taperness and damping parameter. Thus, the present study may be helpful in the determination of natural frequencies and mode shapes by proper choice of plate parameters so that robustness of structure can be determined for improved structural design.

REFERENCES


