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Hydromagnetic Instability of Streaming Viscoelastic Fluids Through Porous Media

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Abstract

The hydromagnetic instability of the plane interface between two uniform, superposed and streaming Rivlin-Ericksen viscoelastic fluids through porous medium is considered. The case of two uniform streaming fluids separated by a horizontal boundary is studied. It is observed, for the special case where perturbations in the direction and transverse direction of streaming are ignored, that the system is stable for stable configuration and unstable for unstable configuration. If the perturbations in the direction of streaming only one ignored, then the system is stable for stable configuration. However, the magnetic field succeeds in stabilizing certain wave-number range, which is otherwise potentially unstable. In all other directions, a minimum wave-number value has been found beyond which the system is unstable; the instability is found to be postponed by the presence of the magnetic field.

Keywords: Kelvin-Helmholtz instability, Rivlin-Ericksen viscoelastic fluid, uniform magnetic field, porous medium

MSC 2010: 76A10, 76E25, 76S05

1. Introduction

When two superposed fluids flow one over the other with a relative horizontal velocity, the instability of the plane interface between the two fluids is known as the “Kelvin-Helmholtz instability”. The discontinuity arising at the plane interface between two superposed streaming fluids is of prime importance in various astrophysical, geophysical and laboratory situations. The Kelvin-Helmholtz instability arise in situations such as air blowing over mercury, highly ionized hot plasma surrounded by a slightly colder gas, or a meteor entering the earth's atmosphere.

The instability of the plane interface between two superposed semi-infinite inviscid fluids flowing with different velocities has been considered by Helmholtz (1868) and Kelvin (1910) and a review of this Kelvin-Helmholtz instability, under varying assumptions of hydrodynamics and hydromagnetics, was given by Chandrasekhar (1981). These problems of instabilities in hydrodynamic and hydromagnetic configuration continue to attract the attention of researchers due to their importance in actual physical situations. Some experimental observations of the Kelvin-Helmholtz instability were made by Francis (1954). Gerwin (1968) studied the stability problem of non-conducting, streaming gas flowing over an incompressible conducting fluid. The effect of rotation in a general oblique magnetic field on the Kelvin-Helmholtz instability was also studied by Sharma and Srivastava (1968). Sengar (1984) analyzed the stability of two superposed gravitating streams in a uniform, vertical magnetic field in the presence of magnetic resistivity. He found that magnetic resistivity had a destabilizing effect on the system. Mehta and Bhatia (1988) studied the Kelvin- Helmholtz instability of two viscous, superposed plasmas in the presence of finite Larmor radius (FLR) effects and showed that both viscosity and FLR effects suppressed the instability. An excellent reappraisal of the Kelvin-Helmholtz problem was made by Benjamin and Bridges (1997), who gave the Hamiltonian formulation of the classic Kelvin-Helmholtz problem in hydrodynamics. Allah (1998) investigated the effects of magnetic field, heat and mass transfer on the Kelvin-Helmholtz instability of superposed fluids. Meignin et al. (2001) and Watson et al. (2004) studied the Kelvin-Helmholtz instability in a Hele-Shaw cell and a weakly ionized medium, respectively. The medium was assumed to be non-porous and fluids Newtonian in the above studies.

The flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the book by Phillips (1991). The gross effect of the fluid slowly percolating through the pores of the rock is given by Darcy's law. As a result, the usual viscous term in the equations of motion of Rivlin-Ericksen viscoelastic fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. Viscoelasticity is the property of materials exhibiting both viscous and elastic characteristics when undergoing deformation. Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. Viscoelastic materials have elements of both of these properties and, as such, exhibit time dependent strain/stress. Whereas elasticity is usually the result of bond stretching along crystallographic planes in an ordered solid, viscosity is the result of the diffusion of atoms or molecules inside an amorphous material Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which, in the process of their journey, change from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in an astrophysical context [McDonnel (1978)]. The instability of the plane interface between two uniform superposed and streaming fluids through porous medium has been investigated by Sharma and Spanos (1982). Khan et al. (2010) have studied the Kelvin-Helmholtz instability arising at the interface separating two superposed, viscous, electrically conducting fluids through a porous medium in the presence of a uniform two-dimensional horizontal magnetic field and it was found that both viscosity and porosity suppressed the stability, while streaming motion had a destabilizing influence.

With the growing importance of viscoelastic fluids in modern technology and industries, investigations on such fluids are desirable. The Rivlin-Ericksen fluid is one such viscoelastic fluid. Rivlin and Ericksen (1955) studied the stress deformation relaxations for isotropic materials. Srivastava and Singh (1988) worked on the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of time-dependent pressure gradient. Sharma and Kumar (1997) studied the hydromagnetic stability of two Rivlin-Ericksen viscoelastic superposed conducting fluids and the analysis was carried out for two highly viscous fluids of equal kinematic viscosities and equal kinematic viscoelasticities. Thermal instability in Rivlin-Ericksen viscoelastic fluid in presence of magnetic field and rotation was also separately investigated by Sharma and Kumar (1996, 1997). In another study, Kumar (2000) discussed the Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluids in presence of suspended particles through a porous medium. The Kelvin-Helmholtz instability of Rivlin-Ericksen elastico-viscous fluid in a porous medium for the case of two uniform streaming fluid separated by a horizontal boundary is considered by Sharma et al. (2001) and it is found that for the special case when perturbations in the direction of streaming are ignored, perturbation transverse to the direction of streaming are found to be unaffected by the presence of streaming. El-Sayed (2002) has considered the electro hydrodynamic Kelvin-Helmholtz instability of the interface between two uniform superposed Rivlin-Ericksen dielectric fluid particle mixtures in a porous medium and the perturbations transverse to the direction of streaming are found to be unaffected by either streaming and applied electric fields for the potentially stable configuration as long as perturbations in the direction of streaming are ignored. In many geophysical fluid dynamical problems encountered, the fluid is electrically conducting and a uniform magnetic field of the Earth pervades the system. A study has, therefore, been made of the instability of electrical conducting streaming Rivlin-Ericksen viscoelastic fluids in a porous medium in the presence of a uniform magnetic field. It should be noted, however, that the linear stability analysis presented in this paper can be applied to the class of second order fluids as well. This is because of the fact that their deviatoric stress tensor reduces to the form $\mu A_1 + \alpha_1 \frac{\partial}{\partial t} A_1$, with $A_1 = 2D = \nabla u + (\nabla u)^T$ and $\alpha_1 < 0$, when linearized at a state of rest, see e.g. Joseph (1990). The problem is often encountered in chemical engineering, paper and pulp technology and several geophysical situations. These aspects form the motivation for the present study.

2. Formulation of the Problem and Perturbation Equations

The initial stationary state, whose stability we wish to examine is that of an incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid in which there is a horizontal streaming in the x-direction with a velocity $U(z)$ through a homogeneous, isotropic porous medium. A uniform magnetic field $\vec{H}(H_x, H_y, 0)$ pervades the system which is also acted on by a gravity force $\vec{g}(0,0,-g)$. The character of the equilibrium of this initial state is determined by supposing that the system is slightly disturbed and then by following its further evolution.

Let $\Gamma_{ij}, \tau_{ij}, e_{ij}, \delta_{ij}, u_i, x_i, p, \mu$ *and* μ' denote the stress tensor, shear stress tensor, rate-of-strain tensor, Kronecker delta, velocity vector, position vector, isotropic pressure, viscosity and viscoelasticity, respectively. The constitutive relations for the Rivlin-Ericksen viscoelastic fluid [Rivlin and Ericksen (1955), Sharma and Kumar (1997)] are

$$\left.\begin{aligned} \Gamma_{ij} &= -p\delta_{ij} + \tau_{ij}, \\ \tau_{ij} &= 2\left[\mu + \mu' \frac{\partial}{\partial t}\right] e_{ij}, \\ e_{ij} &= \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \end{aligned}\right\}. \tag{1}$$

Let $p, \rho, g, \nu, \nu', \vec{q}(U(z), 0, 0)$ denote, respectively, the pressure, density, magnitude of the acceleration due to gravity, kinematic viscosity, kinematic viscoelasticity and filter velocity of the Rivlin-Ericksen viscoelastic fluid. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1 and the interfacial tension is ignored. Then the equations of motion, continuity, incompressibility and Maxwell's equations for the Rivlin-Ericksen viscoelastic fluid through porous medium are given by

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g} - \frac{\rho}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{1}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H}, \tag{2}$$

$$\nabla.\vec{q} = 0, \tag{3}$$

$$\varepsilon\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0, \tag{4}$$

$$\nabla.\vec{H} = 0, \tag{5}$$

$$\varepsilon\frac{\partial \vec{H}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{H}\right). \tag{6}$$

Let $\delta p, \delta\rho, \vec{u}(u, v, w)$ and $\vec{h}(h_x, h_y, h_z)$ denote the perturbations in pressure p, density ρ, velocity $\vec{q}(U(z), 0, 0)$ and magnetic field $\vec{H}(H_x, H_y, 0)$. Then the linearized perturbation equations of the fluid layer become

$$\frac{\rho}{\in}\left[\frac{\partial \vec{u}}{\partial t} + \frac{1}{\in}(\vec{q}\cdot\nabla)\vec{u}\right] = -\nabla\delta p + \vec{g}\delta\rho - \frac{\rho}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\vec{u} + \frac{1}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\,, \tag{7}$$

$$\nabla\cdot\vec{u} = 0\,, \tag{8}$$

$$\left(\in\frac{\partial \rho}{\partial t} + (\vec{q}\cdot\nabla)\right)\delta\rho = -w\frac{d\rho}{dz}\,, \tag{9}$$

$$\nabla\cdot\vec{h} = 0\,, \tag{10}$$

$$\in\frac{\partial \vec{h}}{\partial t} + (\vec{q}\cdot\nabla)\vec{h} = \nabla\times\left(\vec{u}\times\vec{H}\right)\,. \tag{11}$$

Analyzing the disturbances into normal modes, we assume that the perturbed quantities have the space (x, y, z) and time (t) dependence of the form

$$exp\left[i\left(k_x x + k_y y + nt\right)\right]\,, \tag{12}$$

where k_x, k_y are horizontal wave numbers, $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave number and n is the rate at which the system departs from the equilibrium. For perturbation of the form (12), equations (7)–(11) give

$$\frac{\rho}{\in}\left[in + \frac{ik_x U}{\in}\right]u + \frac{\rho}{k_1}(\upsilon + i\upsilon' n)u = -ik_x\delta p + \frac{H_y}{4\pi}\left(ik_y h_x - ik_x h_y\right), \tag{13}$$

$$\frac{\rho}{\in}\left[in + \frac{ik_x U}{\in}\right]v + \frac{\rho}{k_1}(\upsilon + i\upsilon' n)v = -ik_y\delta p + \frac{H_x}{4\pi}\left(ik_x h_y - ik_y h_x\right), \tag{14}$$

$$\begin{aligned}\frac{\rho}{\in}\left[in + \frac{ik_x U}{\in}\right]w + \frac{\rho}{k_1}(\upsilon + i\upsilon' n)w = -D\delta p - g\delta\rho + \frac{H_x}{4\pi}\left(ik_x h_z - Dh_x\right)\\ + \frac{H_y}{4\pi}\left(ik_y h_z - Dh_y\right),\end{aligned} \tag{15}$$

$$\left(\in in + ik_x U\right)\delta\rho = -w(D\rho), \tag{16}$$

$$\left(in + ik_x U\right)\vec{h} = \left(ik_x H_x + ik_y H_y\right)\vec{q}\,, \tag{17}$$

$$ik_x u + ik_y v + Dw = 0, \tag{18}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \tag{19}$$

where

$$D = \frac{d}{dz}.$$

Eliminating δp between equations (13)-(15) with the help of equations (16)-(19), we obtain

$$D\left[\left\{\frac{i\rho}{\in^2}\left(\in n + k_x U\right) + \frac{\rho}{k_1}\left(\upsilon + in\upsilon'\right)\right\}Dw\right] - k^2\left[\frac{i\rho}{\in^2}\left(\in n + k_x U\right) + \frac{\rho}{k_1}\left(\upsilon + in\upsilon'\right)\right]w$$
$$-\frac{igk^2(D\rho)w}{\left(\in n + k_x U\right)} - \frac{i\left(k_x H_x + k_y H_y\right)^2}{4\pi\left(\in n + k_x U\right)}\left(D^2 - k^2\right)w = 0. \tag{20}$$

3. Two Uniform Streaming Rivlin-Ericksen Viscoelastic Fluids Separated by a Horizontal Boundary

Consider the case when two superposed streaming Rivlin-Ericksen fluids of uniform densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 and uniform viscoelasticities μ_1' and μ_2' are separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and the upper fluids respectively. The density ρ_2 of the upper fluid is taken to be less than the density ρ_1 of the lower fluid so that, in the absence of streaming, the configuration is stable and the porous medium throughout is assumed to be isotropic and homogeneous. Let the two fluids be streaming with the constant velocities U_1 and U_2. Then in each of the two regions of constant ρ, μ, μ' and U, equation (20) reduces to

$$\left(D^2 - k^2\right)w = 0. \tag{21}$$

The boundary conditions to be satisfied are as follows:

(i) Since U is discontinuous at $z = 0$, the uniqueness of the normal displacement of any point on the interface implies that

$$\frac{w}{\left(\in n + k_x U\right)}, \tag{22}$$

must be continuous at an interface.

(ii) Integrating equation (20) between $z_0-\eta$ *and* $z_0+\eta$, and passing to the limit $\eta=0$, we obtain , in view of (22), the jump condition

$$\Delta_0\left[\left\{\frac{i\rho}{\in^2}\left(\in n+k_xU\right)+\frac{\rho}{k_1}\left(\upsilon+in\upsilon'\right)\right\}Dw\right]-\frac{ik_x^2H_x^2}{4\pi}\Delta_0\left(\frac{Dw}{\in n+k_xU}\right)$$
$$-\frac{ik_y^2H_y^2}{4\pi}\Delta_0\left(\frac{Dw}{\in n+k_xU}\right)-\frac{ik_xk_yH_xH_y}{2\pi}\Delta_0\left(\frac{Dw}{\in n+k_xU}\right)$$
$$-igk^2\Delta_0\left(\rho\right)\left(\frac{w}{\in n+k_xU}\right)_0=0. \tag{23}$$

Here $\Delta_0(f)=f(z_0+0)-f(z_0-0)$ is the jump which a quantity 'f' experiences at the interface $z=z_0$ and the subscript zero distinguishes the value, a quantity, known to be continuous at an interface, takes at $z=z_0$.

The general solution of equation (21) is a linear combination of the integrals e^{+kz} and e^{-kz}. Since $\frac{w}{(\in n+k_xU)}$ must be continuous on the surface z = 0 and w cannot increase exponentially on either side of the interface, the solutions appropriate for the two regions are

$$w_1=A\left(\in n+k_xU_1\right)e^{+kz},\qquad (z<0) \tag{24}$$

$$w_2=A\left(\in n+k_xU_2\right)e^{-kz},\qquad (z>0)\quad . \tag{25}$$

Applying the boundary condition (23) to the solutions (24) and (25), we obtain the dispersion relation

$$\left[1+\frac{\in}{k_1}\left(\alpha_1\upsilon_1'+\alpha_2\upsilon_2'\right)\right]n^2+\left[\frac{2k_x}{\in}\left(\alpha_1U_1+\alpha_2U_2\right)-\frac{i\in}{k_1}\left(\alpha_1\upsilon_1+\alpha_2\upsilon_2\right)+\frac{k_x}{k_1}\left(\alpha_1\upsilon_1'U_1+\alpha_2\upsilon_2'U_2\right)\right]n$$
$$+\left[\frac{k_x^2}{\in^2}\left(\alpha_1U_1^2+\alpha_2U_2^2\right)-\frac{ik_x}{k_1}\left(\alpha_1\upsilon_1U_1+\alpha_2\upsilon_2U_2\right)-2\left(k_xV_A+k_yV_B\right)^2-gk\left(\alpha_1-\alpha_2\right)\right]=0, \tag{26}$$

where $\upsilon_1\left(=\frac{\mu_1}{\rho_1}\right)$, $\upsilon_2\left(=\frac{\mu_2}{\rho_2}\right)$; and $\upsilon_1'\left(=\frac{\mu_1'}{\rho_1}\right)$, $\upsilon_2'\left(=\frac{\mu_2'}{\rho_2}\right)$ are the kinematic viscosities and the kinematic viscoelasticities of lower and upper fluids respectively,

$$\alpha_{1,2}=\frac{\rho_{1,2}}{\rho_1+\rho_2},\; V_A^2=\frac{H_x^2}{4\pi(\rho_1+\rho_2)}\text{ and }V_B^2=\frac{H_y^2}{4\pi(\rho_1+\rho_2)}.$$

Equation (26) yields

$$
\begin{aligned}
in = \frac{1}{2}\Bigg[& \left\{ -\frac{\in}{k_1}\left(\alpha_1\upsilon_1 + \alpha_2\upsilon_2\right) - \frac{2ik_x}{\in}\left(\alpha_1 U_1 + \alpha_2 U_2\right) - \frac{ik_x}{k_1}\left(\alpha_1\upsilon_1' U_1 + \alpha_2\upsilon_2' U_2\right) \right\} \\
& \pm \Bigg\{ \left[\frac{\in}{k_1}\left(\alpha_1\upsilon_1 + \alpha_2\upsilon_2\right) \right]^2 - \frac{4ik_x\alpha_1\alpha_2}{k_1}\left(\upsilon_1 - \upsilon_2\right)\left(U_1 - U_2\right) + \frac{4k_x^2\alpha_1\alpha_2}{\in k_1}\left(\upsilon_2' U_1 - \upsilon_1' U_2\right)\left(U_1 - U_2\right) \\
& - \frac{2\in ik_x}{k_1^2}\Big[\left(\alpha_1^2\,\upsilon_1\upsilon_1' U_1 + \alpha_2^2\,\upsilon_2\upsilon_2' U_2\right) + \alpha_1\alpha_2\left(\upsilon_1\upsilon_2' U_1 + \upsilon_1'\upsilon_2 U_2\right) \\
& + \alpha_1\alpha_2\left(U_1 - U_2\right)\left(\upsilon_1\upsilon_2' - \upsilon_1'\upsilon_2\right) \Big] - \left[\frac{k_x}{k_1}\left(\alpha_1\upsilon_1' U_1 + \alpha_2\upsilon_2' U_2\right) \right]^2 \\
& + \frac{4\alpha_1\alpha_2 k_x^2}{\in^2}\left(U_1 - U_2\right)^2 - 4gk\left(\alpha_1 - \alpha_2\right)\left[1 + \frac{\in}{k_1}\left(\alpha_1\upsilon_1' + \alpha_2\upsilon_2'\right) \right] \\
& - 8\left(k_x V_A + k_y V_B\right)^2 \left[1 + \frac{\in}{k_1}\left(\alpha_1\upsilon_1' + \alpha_2\upsilon_2'\right) \right] \Bigg\}^{1/2} \Bigg] \left[1 + \frac{\in}{k_1}\left(\alpha_1\upsilon_1' + \alpha_2\upsilon_2'\right) \right]^{-1}. \qquad (27)
\end{aligned}
$$

Some Cases of Interest are now considered:

(a) When $k_x = 0$, $k_y = 0$, equation (27) yields

$$
\begin{aligned}
in = \frac{1}{2}\left[\left\{ -\frac{\in}{k_1}\left(\alpha_1\upsilon_1 + \alpha_2\upsilon_2\right) \right\} \pm \left\{ \frac{\left[\frac{\varepsilon}{k_1}\left(\alpha_1\nu_1 + \alpha_2\nu_2\right) \right]^2 - 4gk\left(\alpha_1 - \alpha_2\right)}{\left[1 + \frac{\varepsilon}{k_1}\left(\alpha_1\nu_1' + \alpha_2\nu_2'\right) \right]} \right\}^{1/2} \right] \\
\times \left[1 + \frac{\in}{k_1}\left(\alpha_1\upsilon_1' + \alpha_2\upsilon_2'\right) \right]^{-1}. \qquad (28)
\end{aligned}
$$

Here we assume kinematic viscosities υ_1, υ_2 and kinematic viscoelasticities υ_1', υ_2' of the two fluids to be equal, i.e., $\upsilon_1 = \upsilon_2 = \upsilon$, $\upsilon_1' = \upsilon_2' = \upsilon'$. However, any of the essential features of the problem are not obscured by this simplifying assumption. Equation (28), then, becomes

$$in = \frac{1}{2}\left[\left\{-\frac{\varepsilon\nu}{k_1}\right\} \pm \left\{\left[\frac{\varepsilon\nu}{k_1}\right]^2 - 4gk(\alpha_1 - \alpha_2)\left[1+\frac{\varepsilon\nu'}{k_1}\right]\right\}^{1/2}\right]\left[1+\frac{\varepsilon\nu'}{k_1}\right]^{-1}. \tag{29}$$

(ii) Unstable Case

For the potentially unstable configuration i.e. heavier fluid overlying lighter fluid $(\alpha_2 > \alpha_1)$, it is evident from equation (29) that one of the values of '*in*' is positive which means that the perturbations grow with time and so the system is unstable.

(iii) Stable Case

For the potentially stable configuration $(\alpha_2 < \alpha_1)$, equation (29) yields that both the values of '*in*' are either real, negative or complex conjugates with negative real parts implying stability of the system.

(b) When $k_x = 0,\ k_y \neq 0$, equation (27) yields

$$in = \frac{1}{2}\left[\left\{-\frac{\varepsilon\nu}{k_1}\right\} \pm \left\{\left[\frac{\varepsilon\nu}{k_1}\right]^2 + 4\left[1+\frac{\varepsilon\nu'}{k_1}\right]\left[gk(\alpha_2 - \alpha_1) - 2k_y^2 V_B^2\right]\right\}^{1/2}\right]\left[1+\frac{\varepsilon\nu'}{k_1}\right]^{-1}, \tag{30}$$

where we have assumed kinematic viscosities υ_1, υ_2 and kinematic viscoelasticities υ_1', υ_2' of the two fluids to be equal (let $\upsilon_1 = \upsilon_2 = \upsilon,\ \upsilon_1' = \upsilon_2' = \upsilon'$), but this simplifying assumption does not obscure any of the essential features of the problem.

(i) Unstable Case

For the potentially unstable configuration $(\alpha_2 > \alpha_1)$, it is evident from equation (30) that one of the values of '*in*' is positive implying that the system is unstable if

$$gk(\alpha_2 - \alpha_1) > 2k_y^2 V_B^2, \tag{31}$$

whereas both the values of '*in*' are either real, negative or complex conjugates with negative real parts implying stability of the system if

$$gk(\alpha_2 - \alpha_1) < 2k_y^2 V_B^2 \ . \tag{32}$$

(ii) Stable Case

For the potentially stable configuration $(\alpha_2 < \alpha_1)$, equation (30) yields that both the values of '*in*' are either real, negative or complex conjugates with negative real parts implying stability of the system.

(c) In every other direction, instability occurs when

$$\frac{\alpha_1\alpha_2 k_x^2}{\in^2}(U_1-U_2)^2\left\{1+\frac{\in\upsilon'}{k_1}\right\} > \left[\frac{k_x\upsilon'}{2k_1}(\alpha_1U_1+\alpha_2U_2)\right]^2$$

$$+\left[gk(\alpha_1-\alpha_2)+2(k_xV_A+k_yV_B)^2\right]\left\{1+\frac{\in\upsilon'}{k_1}\right\}. \tag{33}$$

The kinematic viscosities υ_1 and υ_2 and the kinematic viscoelasticities υ_1' and υ_2' of two fluids here are assumed to be equal i.e. $\upsilon_1=\upsilon_2=\upsilon$, $\upsilon_1'=\upsilon_2'=\upsilon'$, but this simplifying assumption does not obscure any of the essential features of the problem.

Thus, for a given difference in velocity $U_1 - U_2$ and for a given direction of the wave-vector $\vec{k}$, instability occurs for all wave numbers

$$k > \left[\frac{g\in^2(\alpha_1-\alpha_2)\left(1+\frac{\in\upsilon'}{k_1}\right)}{\left[\alpha_1\alpha_2\cos^2\theta(U_1-U_2)^2\left(1+\frac{\in\upsilon'}{k_1}\right)-\frac{\upsilon'^2(\alpha_1U_1+\alpha_2U_2)^2\in^2\cos^2\theta}{4k_1^2}-2\in^2(V_A\cos\theta+V_B\sin\theta)^2\left(1+\frac{\in\upsilon'}{k_1}\right)\right]}\right], \tag{34}$$

where θ is the angle between the directions of $\vec{k}(k_x, k_y, 0)$ and $\vec{U}(U,0,0)$ i.e. $k_x = k\cos\theta$, $k_y = k\sin\theta$. Hence, for a given velocity difference $U_1 - U_2$, instability occurs for the least wave number when $\vec{k}$ is in the direction of $\vec{U}$ and this minimum wave number k_{min}, is given by

$$k_{\min} = \left[\frac{g\in^2(\alpha_1-\alpha_2)\left(1+\frac{\in\upsilon'}{k_1}\right)}{\left[\alpha_1\alpha_2(U_1-U_2)^2\left(1+\frac{\in\upsilon'}{k_1}\right)-\frac{\upsilon'^2(\alpha_1U_1+\alpha_2U_2)^2\in^2}{4k_1^2}-2\in^2V_A^2\left(1+\frac{\in\upsilon'}{k_1}\right)\right]}\right]. \tag{35}$$

For $k > k_{min}$, the system is unstable. It is clear from equation (35) that the presence of magnetic field increases the value of k_{min} for which the system is unstable. Thus, the instability of the system is postponed. We thus obtain the stabilizing effect of magnetic field.

(d) Since the perturbations most sensitive to the Kelvin-Helmholtz instability are in the direction of streaming, we put $k_x = k_y = k$.

In the presence of magnetic field, equation (27) yields that stability occurs when

$$\frac{\alpha_1\alpha_2}{\in^2}(U_1-U_2)^2\left\{1+\frac{\in\upsilon'}{k_1}\right\}<\left[\frac{\upsilon'}{2k_1}(\alpha_1U_1+\alpha_2U_2)\right]^2$$

$$+\left[\frac{g(\alpha_1-\alpha_2)}{k}+2(V_A+V_B)^2\right]\left\{1+\frac{\in\upsilon'}{k_1}\right\}. \tag{36}$$

The right hand side (RHS) of the above inequality has a minimum when $\frac{d\,(RHS)}{dk}=0$, i.e., when $k\to\infty$. Therefore, we shall have stability if

$$(U_1-U_2)^2<\frac{\varepsilon^2}{\alpha_1\alpha_2}\left[\frac{\upsilon'}{2k_1}(\alpha_1U_1+\alpha_2U_2)\right]^2\left\{1+\frac{\in\upsilon'}{k_1}\right\}^{-1}+\left[2(V_A+V_B)^2\frac{\varepsilon^2}{\alpha_1\alpha_2}\right]. \tag{37}$$

The magnetic field, therefore, has stabilizing effect and completely suppresses the Kelvin-Helmholtz instability for small wavelengths. The medium porosity reduces the stability range given in terms of a difference in streaming velocities and the Alfv′en velocities.

4. Conclusions

The foregoing analysis has shown that the stability of an interface between two Rivlin-Ericksen viscoelastic fluids through porous medium in the presence of the magnetic field is affected by the streaming of the fluids in a direction parallel to the interface. In the special case where perturbations in the direction and transverse direction of streaming are ignored, the system is stable for stable configuration and unstable for unstable configuration. Further, for the special case where the perturbations in only the direction of streaming are ignored; the system is found to be stable for a stable configuration. However, in this case, the magnetic field succeeds in stabilizing certain wave-number range, which could not be done in the absence of amagnetic field for the potentially unstable configuration. When the flow parallel to the interface was included in the analysis, the interface was found to be unstable for sufficiently small wavelengths and the instability of the system is found to be postponed due to the presence of the magnetic field.

Similar results can also be obtained for a three layer flow (multilayer flow; plane Poiseuille flow) where the stability criteria is similar to that for the two layer flow as the reduction in the effective viscosity ratio does not affect the stability criteria of the flow. Furthermore, the reduction in the effective viscosity ratio enables us to explain the diminish growth rates which are observed for flows of compatible fluids but not the reduction in the size of the unstable region [see e.g. Rousset et al. 2005, 2006].

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