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Stretching a Surface Having a Layer of Porous Medium in a Viscous Fluid

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Abstract

The present analysis deals with the steady, incompressible flow of a viscous fluid over a stretching sheet having a layer of porous medium of uniform thickness. The two-dimensional flow equations are derived in a Cartesian coordinate system. The semi-infinite region filled with a viscous fluid is divided into two regions namely, a clear fluid region and a region having a uniform pores. Darcy's law has been used for the flow of fluid in the porous medium region. An exact similar solution of the problem is obtained. The obtained solution is constrained by a relation between the porosity parameter and the parameter representing the viscosity ratios between the two regions. Our interest lies in determining the influence of porosity parameter, viscosities ratio parameter and thickness of the porous layer on the fluid velocity and the skin friction coefficient. The results for the Crane's problem in a complete clear and a complete porous region are retrieved as special cases of the present solution.

Keywords: Viscous fluid, stretching sheet, partially porous medium, Darcy law, exact solution, viscosity ratio

MSC 2010 No.: 76S05, 76D10

1. Introduction

The two-dimensional boundary layer flow induced by a stretching surface is the topic of interest of many researchers in the field from last four decades. This is due to the promising applications of such flows in several engineering processes. The examples may be seen in the extrusion process of a polymer sheet from a die or in the drawing of plastic films. Sakiadis (1961) initiated the study of boundary layer flows due to a moving plate. In studies (1961) it is assumed that sheet is moving with a constant velocity and the numerical results are obtained. The first study regarding the stretching flow of a viscous fluid is carried out by Crane (1970) and provided an exact closed form solution. The problem of two-dimensional stretching flow has been extended in many ways to incorporate various aspects of the flow. The uniqueness of the exact analytical solution provided by Crane (1970) is examined by McLeod and Rajagopal (1987).

The analysis for a porous stretching sheet is carried out by Gupta and Gupta (1977) subject to suction or injection. The flow inside a stretching channel or tube has been analyzed by Brady and Acrivos (1981) and the flow outside the stretching tube by Wang (1988). In another paper, Wang (1984) extended the flow analysis to the three-dimensional axisymmetric stretching surface. The unsteady flows induced by stretching film have been also discussed by Wang (1990) and Usha and Sridharan (1995). The boundary layer flow caused by the stretching of a flat surface in a rotating fluid has been studied by Wang (1988), Rajeswari and Nath (1992) and Nazar et al. (2004) for the Newtonian fluids. The flow and heat transfer of a viscous fluid over a stretching sheet and through a porous medium was discussed by Vajravelu (1994). He obtained an exact solution for the velocity and skin friction coefficient.

All the above mentioned studies for Newtonian fluids were carried out when the fluid is passing through a clear medium or the medium is porous as a whole. Due to the applications of the flows through partially filled porous media in chemical reactors, ceramic processing and electronic cooling it is important to discuss the flow when the medium is partially porous. In such a situation the flow domain is divided into two regions, a region of porous medium and a clear fluid region. The properties of fluid flow in such situation are discussed in a clear domain, a porous domain and an interface region between the two regions.

Some recent studies regarding the flow through partially porous media includes the work of Kuznetsov (1996, 1998 and 2000), Al-Nimr and Khadrawi (2003) and Chauhan and Agrawal (2010). In all these studies of partially filled porous medium the flow analysis is carried out in a finite domain. The results for the Blasius and Falkner-Skan flow problems for in the infinite domain with the surface having a layer of porous medium are discussed by Nield (2003) and Kuznetsov and Nield (2006). Motivated by these facts the Crane's problem is considered over a surface having a layer of porous medium of uniform thickness. To the best of the authors' knowledge this kind of study for a stretching flow is not carried out before.

2. Mathematical Formulation

Consider a steady, incompressible flow of a viscous fluid over a stretching sheet having a layer of porous medium of uniform thickness d. For the mathematical formulation we have considered a Cartesian coordinate system such that the x-axis is taken along the flow and y-axis is perpendicular to it. The stretching surface lies at $y=-d$ and $y=0$ is the interface between the porous and a clear medium. We have divided our domain in two regions, a clear fluid region i.e. $\left(0 \le y < \infty\right)$, we call it as region I and a porous medium region i.e. $\left(-d \le y \le 0\right)$, we call it as region II.· The geometry of the problem is given in Figure 1.

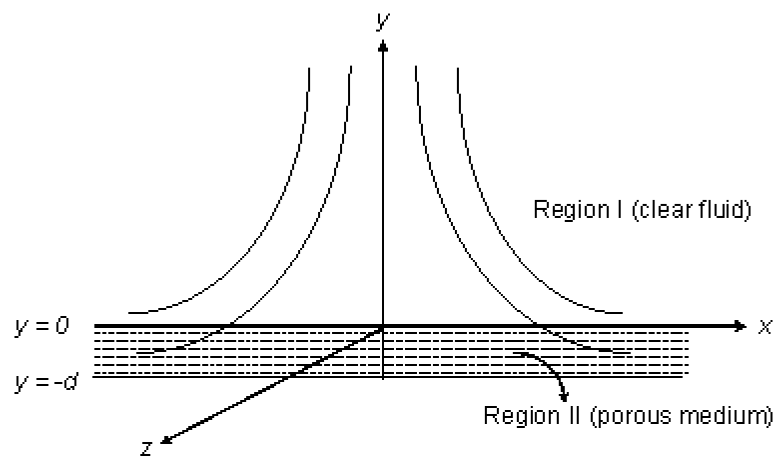

Figure 1. Geometry of the problem

The equations that govern the two-dimensional boundary layer flow of a viscous fluid in region I are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

with the condition far away from the plate

$$u \to 0 \quad \text{as} \quad y \to \infty. \tag{3}$$

Here u and v are the velocity components for region I in x and y directions respectively, ν is the kinematic viscosity in the clear fluid region and pressure gradient is neglected which is a consistent assumption with the condition (3). In region II the flow is governed by

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0, \tag{4}$$

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\overline{\mu}}{\rho}\frac{\partial^2 U}{\partial y^2}-\frac{\overline{\mu}\phi}{\rho k}U, \tag{5}$$

$$\frac{\partial p}{\partial y} = 0. \tag{6}$$

On elimination of pressure between Equations (5) and (6) one may get

$$U\frac{\partial^2 U}{\partial x \partial y} + V\frac{\partial^2 U}{\partial y^2} = \frac{\overline{\mu}}{\rho}\frac{\partial^3 U}{\partial y^3} - \frac{\overline{\mu}\phi}{\rho k}\frac{\partial U}{\partial y}, \tag{7}$$

in which U and V are components of velocity in region II, $\overline{\mu}$ is the dynamic viscosity in porous medium region, ρ is the fluid density, ϕ is the porosity and k is the permeability of the porous medium. We have considered the condition of matching velocity and shear stress at the fluid/solid interface so that no jump will occur in the velocity distribution. Therefore the appropriate boundary conditions at the surface and at the interface region are

$$U(x,y) = ax, \quad V(x,y) = 0 \quad \text{at} \quad y = -d, \tag{8}$$

$$u(x,y) = U(x,y), \quad v(x,y) = V(x,y), \quad \mu\frac{du}{dy} = \overline{\mu}\frac{dU}{dy} \quad \text{at} \quad y = 0. \tag{9}$$

Defining the similarity variables in the form

$$u = axf'(\eta), \quad v = -\sqrt{a\nu}\, f(\eta), \quad U = axF'(\eta), \quad V = -\sqrt{a\nu}\, F(\eta), \quad \eta = \sqrt{\frac{a}{\nu}}x. \tag{10}$$

Equations (1) and (4) are identically satisfied and Equations (2), (3) and (7)-(9) give

$$f''' - f'^2 + ff'' = 0, \tag{11}$$

$$\psi F^{iv} - \psi\phi_1 F'' + FF''' - F'F'' = 0, \tag{12}$$

$$F(-b) = 0, \quad F'(-b) = 1, \quad f(0) = F(0), \quad f'(0) = F'(0), \quad f''(0) = \psi F''(0), \quad f'(\infty) = 0, \tag{13}$$

where

$$\phi_1 = \frac{\phi\nu}{ak}, \quad \psi = \frac{\overline{\mu}}{\mu}, \quad b = \sqrt{\frac{a}{\nu}}d. \tag{14}$$

This choice of dimensionless parameters helps us to get the results in the limiting cases when there is no porous layer is present (1970) i.e. $b \to 0$ and when the whole medium is porous (1994) i.e. $b > 2$. The skin friction coefficient is given by

$$\mathrm{Re}_x^{1/2}\, C_f = \psi F''(-b), \tag{15}$$

in which $\mathrm{Re}_x = ax^2/\nu$ is the local Reynold number. It is important to point out that one can retrieve the results for Crane's problem when $\phi_1 \to 0$.

3. Exact Solution

The exact solution of Equation (12) satisfying the first two boundary conditions of Equation (13) is given by

$$F(\eta) = \sqrt{\frac{\psi}{1+\phi_1\psi}}\left\{1-e^{-(b+\eta)\sqrt{\frac{1+\phi_1\psi}{\psi}}}\right\}. \tag{16}$$

Using Equation (16) into Equation (13) for the interface conditions one obtains

$$f(0) = \sqrt{\frac{\psi}{1+\phi_1\psi}}\left\{1-e^{-b\sqrt{\frac{1+\phi_1\psi}{\psi}}}\right\}, \tag{17}$$

$$f'(0) = e^{-b\sqrt{\frac{1+\phi_1\psi}{\psi}}}, \tag{18}$$

$$f''(0) = -\sqrt{\psi\left(1+\phi_1\psi\right)}e^{-b\sqrt{\frac{1+\phi_1\psi}{\psi}}}. \tag{19}$$

A solution of Equation (11) subject to conditions (17)-(19) is given by

$$f(\eta) = A + Be^{-C\eta}, \tag{20}$$

where

$$A = \frac{\psi + (1-\psi)e^{-b\sqrt{\frac{1+\phi_1\psi}{\psi}}}}{\sqrt{\psi(1+\phi_1\psi)}}, \tag{21}$$

$$B = -\frac{e^{-b\sqrt{\frac{1+\phi_1\psi}{\psi}}}}{\sqrt{\psi\left(1+\phi_1\psi\right)}}, \tag{22}$$

$$C = \sqrt{\psi\left(1+\phi_1\psi\right)}, \tag{23}$$

provided that the following constraint must be satisfied between the parameters ϕ_1, ψ and b,

$$\psi - 1 + \phi_1\psi^2 e^{b\sqrt{\frac{1+\phi_1\psi}{\psi}}} = 0 \tag{24}$$

4. Results and Discussions

To see the valid range of the values of the parameters ψ and ϕ_1 the curves have been drawn in Figure 2 that satisfies Equation (24) for different values of the thickness of the porous medium layer. It is found that the provided exact solution is valid for the values $\psi \le 1$. The range of the porosity parameter depends upon the thickness of the porous layer. The range of admissible values of the parameters ψ and ϕ_1 decreases by increasing the layer thickness b.The curves in Figure 2 illustrate that if the value of the layer thickness b is greater or equal to the boundary layer thickness the present solution converges to the stretching flow through a porous medium (1994).

For a fixed value of b one cannot choose an independent value of parameter ψ because it depends upon the porosity parameter ϕ_1 through Equation (24). Moreover, Figure 2 shows that ψ is a decreasing function of ϕ_1. In Figure 3 the y-component of velocity is plotted for different values of the viscosity ratio ψ and porosity parameter ϕ_1. This Figure depict that the velocity and boundary layer thickness decreases by an increase in the porosity parameter which results in a decrease in the value of viscosity ratio ψ. The effects of these parameters on the x-component of velocity are shown in Figure 4. It is observed from this Figure that the velocity is a decreasing function of the parameters and the effect of the porosity parameter is strong inside the porous layer. Outside the porous layer this effect of porosity parameter damp very rapidly and there is no variation in velocity after a small distance beyond the interface region. The variation of the thickness of the porous layer on the velocity components is displayed in Figures 5 and 6 for a fixed value of the porosity parameter ϕ_1.

It is evident from these Figures that when $b \to 0$ the solution of the Crane's problem (1970) is recovered. Also the solution for a complete porous region (1994) is obtained for $b > 2$. Hence the present analysis contains the solution of the stretching flows with and without porous medium as special cases.

The numerical values of the skin friction coefficient are given in Table 1. It is clear from Table 1 that the magnitude of skin friction coefficient decreases by an increase in the porosity parameter. Similarly, for the large values of the porous medium layer thickness the magnitude of skin friction coefficient is small. Moreover, the values converge to Crane's solution for $\phi_1 = 0$.

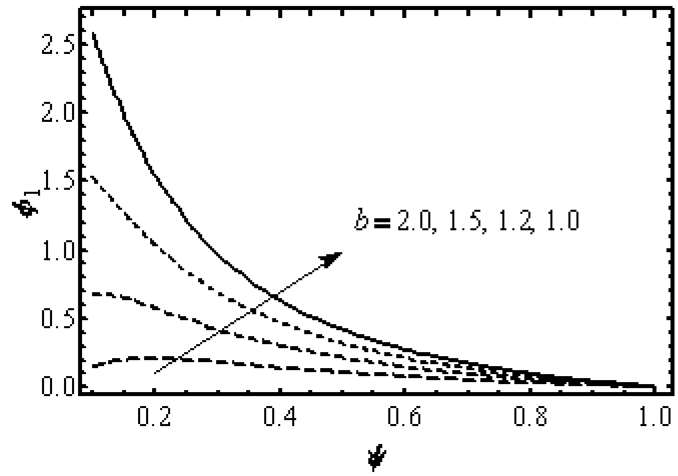

Figure 2. Contours of the valid range of the parameter values.

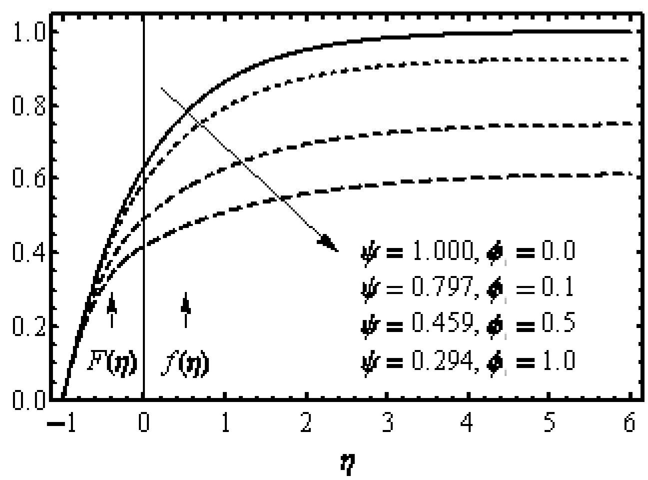

Figure 3. Influence of porosity parameter ϕ_1 and viscosity ratio parameter ψ on the vertical component of velocity.

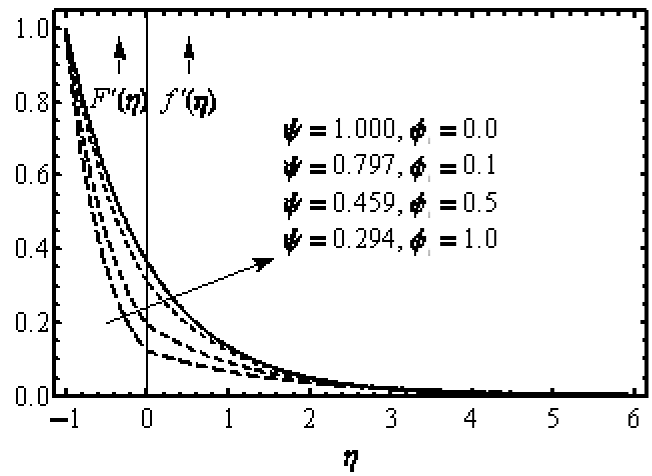

Figure 4. Influence of porosity parameter ϕ_1 and viscosity ratio parameter ψ on the horizontal component of velocity.

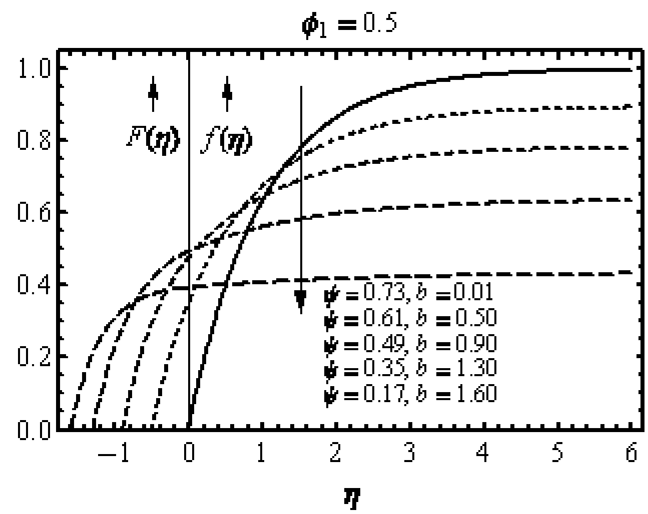

Figure 5. Influence of layer thickness b and viscosity ratio parameter ψ on the vertical component of velocity.

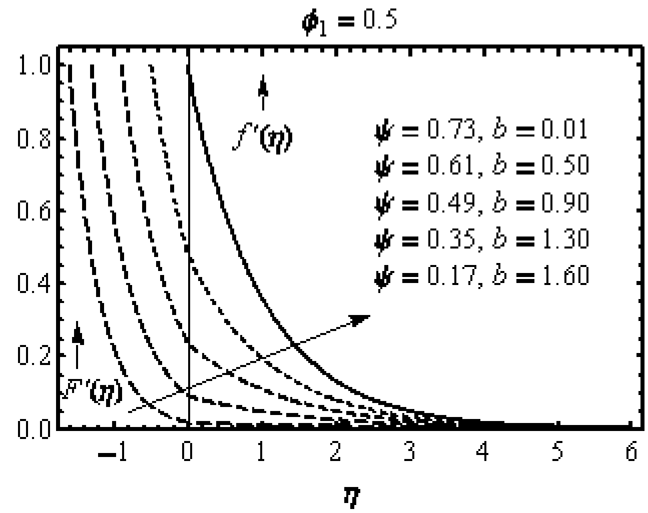

Figure 6. Influence of layer thickness b and viscosity ratio parameter ψ on the horizontal component of velocity.

Table 1. Influence of porosity parameter and layer thickness on skin friction coefficient $-C_f \mathrm{Re}_x^{1/2}$.

ϕ_1	b=0.01	b=0.5	b=1.0
0.0	1	1	1
0.1	0.999540	0.971339	0.927450
0.5	0.998164	0.893691	0.751137
1.0	0.996903	0.829276	0.617330
2.0	0.994996	0.740683	0.440196

5. Conclusion

In this paper, the viscous flow due to stretching of a sheet having a layer of porous medium is considered. The exact solution of the problem is evaluated by dividing the domain into two regions one representing a porous medium and the other representing a clear fluid. This problem is infecting another generalization of the Crane's problem and still possesses an exact solution. The graphical results are presented and discussed under the influence of the pertinent parameters of interest. It is also important to point out that however the lateral velocity far away from the plate is zero but pressure variation cannot be neglected inside the porous medium region.

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