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A New Approach for Solving System of Local Fractional Partial Differential Equations

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Abstract

In this paper, we apply a new method for solving system of partial differential equations within local fractional derivative operators. The approximate analytical solutions are obtained by using the local fractional Laplace variational iteration method, which is the coupling method of local fractional variational iteration method and Laplace transform. Illustrative examples are included to demonstrate the high accuracy and fast convergence of this new algorithm. The obtained results show that the introduced approach is a promising tool for solving system of linear and nonlinear local fractional differential equations. Furthermore, we show that local fractional Laplace variational iteration method is able to solve a large class of nonlinear problems involving local fractional operators effectively, more easily and accurately; and thus it has been widely applicable in physics and engineering.

Keywords: System of local fractional differential equations; Local fractional Laplace variational iteration method; Local fractional derivative operators

MSC (2010) No.: 26A33; 34A12; 34A34; 35R11

1. Introduction

The local fractional variational iteration method was applied to solve the partial differential, integral and integro-differential equations arising in mathematical physics: Yang et al. (2013), Su et al. (2013), Yang et al. (2014) Wang et al. (2014), Baleanu et al. (2014) Chen, et al. (2014), Neamah (2014). Based on it, the local fractional Laplace variational iteration method had found successful applications in physics and applied mathematics as in the following examples: fractal heat conduction equation by Liu (2014), fractal vehicular traffic flow by Li, et al. (2014), heat equation by Xu et al. (2014), linear partial differential equations by Yang (2014), nonlinear partial differential equations by Jafari and Jassim (2014), Fokker Planck equation by Jassim (2015), diffusion-wave equations by Jassim et al. (2015).

The local fractional calculus theory has attracted a lot of interest for scientists and engineers because it is applied to model some problems for fractal mathematics and engineering. It plays a key role in many applications in several fields, such as physics Yang (2012), Kolwankar and Gangal (1998), Yang et al. (2013), Zhao et al. (2013), Li et al. (2014), heat conduction theory Yang (2012), Yang et al. (2013), Xu et al. (2014), fracture and elasticity mechanics Yang (2012), fluid mechanics Yang (2012), Yang et al. (2013) and so on. The local fractional partial differential equations arising in mathematical physics described the non-differentiable behaviors of physical laws. Finding the non-differentiable solutions is a hot topic. Useful techniques were successfully applied to deal with local fractional differential equations.

In this work, our aim is to use the local fractional Laplace variational iteration method to solve the system of linear and nonlinear local fractional partial differential equations. The structure of the paper is suggested as follows. In Section 2, we give analysis of the method used. In Section 3, some examples for systems of local fractional partial differential equations are given. Finally, the conclusions are considered in Section 4.

2. Local Fractional Laplace Variational Iteration Method.

In order to illustrate this method, we investigate system of local fractional partial differential equations as follows:

$$L_\alpha u_i(x,y) + R_i(U) + N_i(U) = 0, \quad i = 1,2,\ldots,n, \tag{2.1}$$

with the initial conditions

$$\frac{\partial^{r\alpha} u_i(0,y)}{\partial x^{r\alpha}} = f_i(y), \quad r = 0,1,2,\ldots,k-1, \tag{2.2}$$

where

$$U=\left[u_1(x,t),u_2(x,t),\ldots,u_n(x,t)\right],$$

$L_\alpha=\dfrac{\partial^{k\alpha}}{\partial x^{k\alpha}}$ denotes linear local fractional derivative operator of order $k\alpha$, R_i and N_i denotes linear and nonlinear terms.

According to the rule of local fractional variational iteration method Yang et al. (2013), Su et al. (2013), Jafari et al. (2015) the correction local fractional functional for (2.1) is constructed as

$$u_{i(m+1)}(x)=u_{im}(x)+{}_0I_x^{(\alpha)}\left(\frac{\lambda_i(\tau)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_{im}(\tau)+R_i\left(\tilde{U}_m\right)+N_i\left(\tilde{U}_m\right)\right]\right), \tag{2.3}$$

which leads up to

$$u_{i(m+1)}(x)=u_{im}(x)+{}_0I_x^{(\alpha)}\left(\frac{\lambda_i(x-\tau)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_{im}(\tau)+R_i\left(\tilde{U}_m\right)+N_i\left(\tilde{U}_m\right)\right]\right), \tag{2.4}$$

where

$$\tilde{U}_m=\left[\tilde{u}_{1m},\tilde{u}_{2m},\ldots,\tilde{u}_{nm}\right]$$

and $\dfrac{\lambda_i(\tau)^\alpha}{\Gamma(1+\alpha)}$ is a fractal Lagrange multipliers.

Applying the local fractional Laplace transform on both sides of Equation (2.4), we get

$$Ł_\alpha\left\{u_{i(m+1)}(x)\right\}=Ł_\alpha\left\{u_{im}(x)\right\}+Ł_\alpha\left\{{}_0I_x^{(\alpha)}\left(\frac{\lambda_i(x-\tau)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_{im}(\tau)+R_i\left(\tilde{U}_m\right)+N_i\left(\tilde{U}_m\right)\right]\right)\right\}, \tag{2.5}$$

or

$$Ł_\alpha\left\{u_{i(m+1)}(x)\right\}=Ł_\alpha\left\{u_{im}(x)\right\}+Ł_\alpha\left\{\frac{\lambda_i(x)^\alpha}{\Gamma(1+\alpha)}\right\}Ł_\alpha\left\{L_\alpha u_n(x)+R_i\left(\tilde{U}_m\right)+N_i\left(\tilde{U}_m\right)\right\}. \tag{2.6}$$

Taking the local fractional variation of Equation (2.6) gives us

$$\delta^\alpha\left(Ł_\alpha\left\{u_{i(m+1)}(x)\right\}\right)=\delta^\alpha\left(Ł_\alpha\left\{u_{im}(x)\right\}\right)+\delta^\alpha\left(Ł_\alpha\left\{\frac{\lambda_i(x)^\alpha}{\Gamma(1+\alpha)}\right\}Ł_\alpha\left\{L_\alpha u_{im}(x)+R_i\left(\tilde{U}_m\right)+N_i\left(\tilde{U}_m\right)\right\}\right). \tag{2.7}$$

By using computation of Equation (2.7), we get:

$$\delta^\alpha\left(Ł_\alpha\left\{u_{i(m+1)}(x)\right\}\right)=\delta^\alpha\left(Ł_\alpha\left\{u_{im}(x)\right\}\right)+Ł_\alpha\left\{\frac{\lambda_i(x)^\alpha}{\Gamma(1+\alpha)}\right\}\delta^\alpha\left(Ł_\alpha\left\{L_\alpha u_{im}(x)\right\}\right). \tag{2.8}$$

Hence, from Equation (2.8) we get:

$$1+Ł_{\alpha}\left\{\frac{\lambda_i(x)^{\alpha}}{\Gamma(1+\alpha)}\right\}s^{k\alpha}=0, \tag{2.9}$$

where

$$\begin{aligned}\delta^{\alpha}\left(Ł_{\alpha}\{L_{\alpha}u_{im}(x)\}\right)&=\delta^{\alpha}\left(s^{k\alpha}Ł_{\alpha}\{u_{im}(x)\}-s^{(k-1)\alpha}u_{im}(0)-\cdots-u_{im}^{((k-1)\alpha)}(0)\right)\\&=s^{k\alpha}\delta^{\alpha}\left(Ł_{\alpha}\{u_{im}(x)\}\right).\end{aligned} \tag{2.10}$$

Therefore, we get

$$Ł_{\alpha}\left\{\frac{\lambda_i(x)^{\alpha}}{\Gamma(1+\alpha)}\right\}=-\frac{1}{s^{k\alpha}}. \tag{2.11}$$

Hence, we have the following iteration algorithm:

$$Ł_{\alpha}\left\{u_{i(m+1)}(x,y))\right\}=Ł_{\alpha}\{u_{im}(x,y)\}-\frac{1}{s^{k\alpha}}Ł_{\alpha}\{L_{\alpha}u_{im}(x,y)+R_i(U_m)+N_i(U_m)\}. \tag{2.12}$$

Therefore, the local fractional series solution of Equation (2.1) is

$$u_i(x,y)=\lim_{m\to\infty}Ł_{\alpha}^{-1}\left(Ł_{\alpha}\{u_{im}(x,y)\}\right). \tag{2.13}$$

3. Some Illustrative Examples

In this section, we give some illustrative examples for solving system of local fractional partial differential equations to demonstrate the efficiency of local fractional Laplace variational iteration method.

Example 1.

Let us consider the system of local fractional coupled partial differential equations with local fractional derivative Yang et al. (2014):

$$\begin{aligned}&\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}v(x,y)}{\partial y^{2\alpha}}-u(x,y)=0,\\&\frac{\partial^{2\alpha}v(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}-v(x,y)=0,\end{aligned} \tag{3.1}$$

subject to the initial conditions

$$\begin{aligned}&u(0,y)=0,\ \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=E_{\alpha}(y^{\alpha}),\\&v(0,y)=0,\ \frac{\partial^{\alpha}v(0,y)}{\partial x^{\alpha}}=-E_{\alpha}(y^{\alpha}).\end{aligned} \tag{3.2}$$

In view Equation (2.12) and Equation (3.1) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} &Ł_\alpha\{u_{m+1}(x,y)\} = Ł_\alpha\{u_m(x,y)\} - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}u_m(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}v_m(x,y)}{\partial y^{2\alpha}} - u_m(x,y)\right\}, \\ &Ł_\alpha\{v_{m+1}(x,y)\} = Ł_\alpha\{v_m(x,y)\} - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}v_m(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}u_m(x,y)}{\partial y^{2\alpha}} - v_m(x,y)\right\}, \end{aligned} \tag{3.3}$$

which leads to

$$\begin{aligned} &Ł_\alpha\{u_{m+1}(x,y)\} = \frac{1}{s^\alpha}u_m(0,y) + \frac{1}{s^{2\alpha}}u_m^{(\alpha)}(0,y) - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}v_m(x,y)}{\partial y^{2\alpha}} - u_m(x,y)\right\}, \\ &Ł_\alpha\{v_{m+1}(x,y)\} = \frac{1}{s^\alpha}v_m(0,y) + \frac{1}{s^{2\alpha}}v_m^{(\alpha)}(0,y) - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}u_m(x,y)}{\partial y^{2\alpha}} - v_m(x,y)\right\}, \end{aligned} \tag{3.4}$$

where the initial value reads:

$$\begin{aligned} &Ł_\alpha\{u_0(x,y)\} = Ł_\alpha\left\{\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)\right\} = \frac{E_\alpha(y^\alpha)}{s^{2\alpha}}, \\ &Ł_\alpha\{v_0(x,y)\} = Ł_\alpha\left\{-\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)\right\} = -\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}. \end{aligned} \tag{3.5}$$

Making use of Equation (3.4) and Equation (3.5), we get the first approximation, namely:

$$\begin{aligned} &Ł_\alpha\{u_1(x,y)\} = \frac{1}{s^\alpha}u_0(0,y) + \frac{1}{s^{2\alpha}}u_0^{(\alpha)}(0,y) - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}v_0(x,y)}{\partial y^{2\alpha}} - u_0(x,y)\right\}, \\ &Ł_\alpha\{v_1(x,y)\} = \frac{1}{s^\alpha}v_0(0,y) + \frac{1}{s^{2\alpha}}v_0^{(\alpha)}(0,y) - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{\partial^{2\alpha}u_0(x,y)}{\partial y^{2\alpha}} - v_0(x,y)\right\}. \end{aligned} \tag{3.6}$$

Hence, we have

$$\begin{aligned} &Ł_\alpha\{u_1(x,y)\} = \frac{E_\alpha(y^\alpha)}{s^{2\alpha}} - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{-\frac{2x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)\right\} \\ &Ł_\alpha\{v_1(x,y)\} = -\frac{E_\alpha(y^\alpha)}{s^{2\alpha}} - \frac{1}{s^{2\alpha}} Ł_\alpha\left\{\frac{2x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)\right\} \\ &\qquad = \frac{E_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{2E_\alpha(y^\alpha)}{s^{4\alpha}}, \\ &\qquad = -\frac{E_\alpha(y^\alpha)}{s^{2\alpha}} - \frac{2E_\alpha(y^\alpha)}{s^{2\alpha}}. \end{aligned} \tag{3.7}$$

From Equation (3.4) and Equation (3.7), we arrive the second approximation:

$$Ł_\alpha\{u_2(x,y)\}=\frac{1}{s^\alpha}u_1(0,y)+\frac{1}{s^{2\alpha}}u_1^{(\alpha)}(0,y)-\frac{1}{s^{2\alpha}}Ł_\alpha\left\{\frac{\partial^{2\alpha}v_1(x,y)}{\partial y^{2\alpha}}-u_1(x,y)\right\},$$
$$Ł_\alpha\{v_2(x,y)\}=\frac{1}{s^\alpha}v_1(0,y)+\frac{1}{s^{2\alpha}}v_1^{(\alpha)}(0,y)-\frac{1}{s^{2\alpha}}Ł_\alpha\left\{\frac{\partial^{2\alpha}u_1(x,y)}{\partial y^{2\alpha}}-v_1(x,y)\right\}. \tag{3.8}$$

Therefore, we get

$$Ł_\alpha\{u_2(x,y)\}=\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}-\frac{1}{s^{2\alpha}}Ł_\alpha\left\{-\frac{2x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)-\frac{4x^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(y^\alpha)\right\}$$
$$Ł_\alpha\{v_2(x,y)\}=-\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}-\frac{1}{s^{2\alpha}}Ł_\alpha\left\{\frac{2x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)+\frac{4x^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(y^\alpha)\right\}$$
$$=\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}+\frac{2E_\alpha(y^\alpha)}{s^{4\alpha}}+\frac{4E_\alpha(y^\alpha)}{s^{6\alpha}},$$
$$=-\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}-\frac{2E_\alpha(y^\alpha)}{s^{4\alpha}}-\frac{4E_\alpha(y^\alpha)}{s^{6\alpha}}, \tag{3.9}$$
$$\vdots$$
$$Ł_\alpha\{u_m(x,y)\}=\sum_{k=0}^{m}\frac{2^kE_\alpha(y^\alpha)}{s^{(2k+2)\alpha}},$$
$$Ł_\alpha\{v_m(x,y)\}=-\sum_{k=0}^{m}\frac{2^kE_\alpha(y^\alpha)}{s^{(2k+2)\alpha}}. \tag{3.10}$$

Consequently, the local fractional series solution is:

$$u(x,y)=\lim_{m\to\infty}Ł_\alpha^{-1}\left(Ł_\alpha\{u_m(x,y)\}\right)=Ł_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}+\frac{2E_\alpha(y^\alpha)}{s^{4\alpha}}+\frac{4E_\alpha(y^\alpha)}{s^{6\alpha}}+\cdots\right)$$
$$v(x,y)=\lim_{m\to\infty}Ł_\alpha^{-1}\left(Ł_\alpha\{v_m(x,y)\}\right)=Ł_\alpha^{-1}\left(-\frac{E_\alpha(y^\alpha)}{s^{2\alpha}}-\frac{2E_\alpha(y^\alpha)}{s^{4\alpha}}-\frac{4E_\alpha(y^\alpha)}{s^{6\alpha}}-\cdots\right)$$
$$=E_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{2x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)=E_\alpha(y^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}x^\alpha\right)}{\sqrt{2}},$$
$$=-E_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{2x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)=-E_\alpha(y^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}x^\alpha\right)}{\sqrt{2}}. \tag{3.11}$$

Example 2.

Consider the system of local fractional coupled Burger's equations with local fractional derivative Yang et al. (2014):

$$\frac{\partial^\alpha u(x,y)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}-2u(x,y)\frac{\partial^\alpha u(x,y)}{\partial y^\alpha}+\frac{\partial^\alpha[u(x,y)v(x,y)]}{\partial y^\alpha}=0,$$
$$\frac{\partial^\alpha v(x,y)}{\partial x^\alpha}+\frac{\partial^{2\alpha}v(x,y)}{\partial y^{2\alpha}}-2v(x,y)\frac{\partial^\alpha v(x,y)}{\partial y^\alpha}+\frac{\partial^\alpha[u(x,y)v(x,y)]}{\partial y^\alpha}=0, \tag{3.12}$$

subject to the initial conditions

$$\begin{aligned} u(0,y) &= \cos_\alpha(y^\alpha), \\ v(0,y) &= \cos_\alpha(y^\alpha). \end{aligned} \tag{3.13}$$

From Equation (2.12) and Equation (3.12), we obtain

$$\begin{aligned} Ł_\alpha\{u_{m+1}(x,y)\} &= Ł_\alpha\{u_m(x,y)\} - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^\alpha u_m}{\partial x^\alpha} + \frac{\partial^{2\alpha} u_m}{\partial y^{2\alpha}} - 2u_m \frac{\partial^\alpha u_m}{\partial y^\alpha} + \frac{\partial^\alpha [u_m v_m]}{\partial y^\alpha}\right\}, \\ Ł_\alpha\{v_{m+1}(x,y)\} &= Ł_\alpha\{v_m(x,y)\} - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^\alpha v_m}{\partial x^\alpha} + \frac{\partial^{2\alpha} v_m}{\partial y^{2\alpha}} - 2v_m \frac{\partial^\alpha v_m}{\partial y^\alpha} + \frac{\partial^\alpha [u_m v_m]}{\partial y^\alpha}\right\}, \end{aligned} \tag{3.14}$$

which leads to

$$\begin{aligned} Ł_\alpha\{u_{m+1}(x,y)\} &= \frac{1}{s^\alpha} u_m(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^{2\alpha} u_m}{\partial y^{2\alpha}} - 2u_m \frac{\partial^\alpha u_m}{\partial y^\alpha} + \frac{\partial^\alpha [u_m v_m]}{\partial y^\alpha}\right\}, \\ Ł_\alpha\{v_{m+1}(x,y)\} &= \frac{1}{s^\alpha} v_m(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^{2\alpha} v_m}{\partial y^{2\alpha}} - 2v_m \frac{\partial^\alpha v_m}{\partial y^\alpha} + \frac{\partial^\alpha [u_m v_m]}{\partial y^\alpha}\right\}, \end{aligned} \tag{3.15}$$

where the initial value are:

$$\begin{aligned} Ł_\alpha\{u_0(x,y)\} &= Ł_\alpha\{\cos_\alpha(y^\alpha)\} = \frac{\cos_\alpha(y^\alpha)}{s^\alpha}, \\ Ł_\alpha\{v_0(x,y)\} &= Ł_\alpha\{\cos_\alpha(y^\alpha)\} = \frac{\cos_\alpha(y^\alpha)}{s^\alpha}. \end{aligned} \tag{3.16}$$

Applying Equation (3.15) and Equation (3.16), we get the first approximation, namely:

$$\begin{aligned} Ł_\alpha\{u_1(x,y)\} &= \frac{1}{s^\alpha} u_0(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^{2\alpha} u_0}{\partial y^{2\alpha}} - 2u_0 \frac{\partial^\alpha u_0}{\partial y^\alpha} + \frac{\partial^\alpha [u_0 v_0]}{\partial y^\alpha}\right\}, \\ Ł_\alpha\{v_1(x,y)\} &= \frac{1}{s^\alpha} v_0(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^{2\alpha} v_0}{\partial y^{2\alpha}} - 2v_0 \frac{\partial^\alpha v_0}{\partial y^\alpha} + \frac{\partial^\alpha [u_0 v_0]}{\partial y^\alpha}\right\}. \end{aligned} \tag{3.17}$$

Hence, we have

$$\begin{aligned} Ł_\alpha\{u_1(x,y)\} &= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha} Ł_\alpha\{-\cos_\alpha(y^\alpha)\} = \frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}}, \\ Ł_\alpha\{v_1(x,y)\} &= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha} Ł_\alpha\{-\cos_\alpha(y^\alpha)\} = \frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}}. \end{aligned} \tag{3.18}$$

In view of Equation (3.15) and Equation (3.18), we arrive the second approximation:

$$
\begin{aligned}
Ł_\alpha\{u_2(x,y)\} &= \frac{1}{s^\alpha}u_1(0,y) - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}} - 2u_1\frac{\partial^\alpha u_1}{\partial y^\alpha} + \frac{\partial^\alpha[u_1v_1]}{\partial y^\alpha}\right\},\\
Ł_\alpha\{v_2(x,y)\} &= \frac{1}{s^\alpha}v_1(0,y) - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^{2\alpha}v_1}{\partial y^{2\alpha}} - 2v_1\frac{\partial^\alpha v_1}{\partial y^\alpha} + \frac{\partial^\alpha[u_1v_1]}{\partial y^\alpha}\right\}.
\end{aligned} \tag{3.19}
$$

Therefore, we have

$$
\begin{aligned}
Ł_\alpha\{u_2(x,y)\} &= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha}Ł_\alpha\left\{-\cos_\alpha(y^\alpha) - \frac{x^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(y^\alpha)\right\}\\
Ł_\alpha\{v_2(x,y)\} &= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha}Ł_\alpha\left\{-\cos_\alpha(y^\alpha) - \frac{x^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(y^\alpha)\right\}\\
&= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\cos_\alpha(y^\alpha)}{s^{3\alpha}},\\
&= \frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\cos_\alpha(y^\alpha)}{s^{3\alpha}},\\
&\vdots
\end{aligned} \tag{3.20}
$$

$$
\begin{aligned}
Ł_\alpha\{u_m(x,y)\} &= \sum_{k=0}^{m}\frac{\cos_\alpha(y^\alpha)}{s^{(k+1)\alpha}},\\
Ł_\alpha\{v_m(x,y)\} &= \sum_{k=0}^{m}\frac{\cos_\alpha(y^{\alpha)}}{s^{(k+1)\alpha}}.
\end{aligned} \tag{3.21}
$$

Consequently, the local fractional series solution is:

$$
\begin{aligned}
u(x,y) &= \lim_{m\to\infty} Ł_\alpha^{-1}(Ł_\alpha\{u_m(x,y\}) = Ł_\alpha^{-1}\left(\frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\cos_\alpha(y^\alpha)}{s^{3\alpha}} + \cdots\right)\\
v(x,y) &= \lim_{m\to\infty} Ł_\alpha^{-1}(Ł_\alpha\{v_m(x,y)\}) = Ł_\alpha^{-1}\left(\frac{\cos_\alpha(y^\alpha)}{s^\alpha} + \frac{\cos_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\cos_\alpha(y^\alpha)}{s^{3\alpha}} - \cdots\right)\\
&= \cos_\alpha(y^\alpha)\left(1 + \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right)\\
&= \cos_\alpha(y^\alpha)\left(1 + \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right)\\
&= E_\alpha(x^\alpha)\cos_\alpha(y^\alpha),\\
&= E_\alpha(x^\alpha)\cos_\alpha(y^\alpha).
\end{aligned} \tag{3.22}
$$

Example 3.

Let us consider the system of coupled partial differential equations involving local fractional operator:

$$\begin{aligned}&\frac{\partial^\alpha u(x,y)}{\partial x^\alpha}+\frac{\partial^\alpha v(x,y)}{\partial y^\alpha}-u(x,y)-v(x,y)=0,\\&\frac{\partial^\alpha v(x,y)}{\partial x^\alpha}+\frac{\partial^\alpha u(x,y)}{\partial y^\alpha}-v(x,y)-u(x,y)=0,\end{aligned} \tag{3.23}$$

with initial conditions

$$\begin{aligned}&u(0,y)=\sinh_\alpha(y^\alpha),\\&v(0,y)=\cosh_\alpha(y^\alpha).\end{aligned} \tag{3.24}$$

In view Equation (2.12) and Equation (3.23) we have

$$\begin{aligned}&Ł_\alpha\{u_{m+1}(x,y)\}=Ł_\alpha\{u_m(x,y)\}-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_m}{\partial x^\alpha}+\frac{\partial^\alpha v_m}{\partial y^\alpha}-u_m-v_m\right\},\\&Ł_\alpha\{v_{m+1}(x,y)\}=Ł_\alpha\{v_m(x,y)\}-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha v_m}{\partial x^\alpha}+\frac{\partial^\alpha u_m}{\partial y^\alpha}-v_m-u_m\right\},\end{aligned} \tag{3.25}$$

which leads to

$$\begin{aligned}&Ł_\alpha\{u_{m+1}(x,y)\}=\frac{1}{s^\alpha}u_m(0,y)-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha v_m}{\partial y^\alpha}-u_m-v_m\right\},\\&Ł_\alpha\{v_{m+1}(x,y)\}=\frac{1}{s^\alpha}v_m(0,y)-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_m}{\partial y^\alpha}-v_m-u_m\right\},\end{aligned} \tag{3.26}$$

where the initial value reads:

$$\begin{aligned}&Ł_\alpha\{u_0(x,y)\}=Ł_\alpha\{\sinh_\alpha(y^\alpha)\}=\frac{\sinh_\alpha(y^\alpha)}{s^\alpha},\\&Ł_\alpha\{v_0(x,y)\}=Ł_\alpha\{\cosh_\alpha(y^\alpha)\}=\frac{\cosh_\alpha(y^\alpha)}{s^\alpha}.\end{aligned} \tag{3.27}$$

Making use of Equation (3.26) and Equation (3.27), we get the first approximation, namely:

$$\begin{aligned}&Ł_\alpha\{u_1(x,y)\}=\frac{1}{s^\alpha}u_0(0,y)-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha v_0}{\partial y^\alpha}-u_0-v_0\right\},\\&Ł_\alpha\{v_1(x,y)\}=\frac{1}{s^\alpha}v_0(0,y)-\frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_0}{\partial y^\alpha}-v_0-u_0\right\}.\end{aligned} \tag{3.28}$$

Hence, we have

$$\begin{aligned}
Ł_\alpha\{u_1(x,y)\} &= \frac{\sinh_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha} Ł_\alpha\{-\cosh_\alpha(y^\alpha)\} = \frac{\sinh_\alpha(y^\alpha)}{s^\alpha} + \frac{\cosh_\alpha(y^\alpha)}{s^{2\alpha}},\\
Ł_\alpha\{v_1(x,y)\} &= \frac{\cosh_\alpha(y^\alpha)}{s^\alpha} - \frac{1}{s^\alpha} Ł_\alpha\{-\sinh_\alpha(y^\alpha)\} = \frac{\cosh_\alpha(y^\alpha)}{s^\alpha} + \frac{\sinh_\alpha(y^\alpha)}{s^{2\alpha}}.
\end{aligned} \tag{3.29}$$

In view of Equation (3.26) and Equation (3.29), we arrive the second approximation reads:

$$\begin{aligned}
Ł_\alpha\{u_2(x,y)\} &= \frac{1}{s^\alpha} u_1(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^\alpha v_1}{\partial y^\alpha} - u_1 - v_1\right\},\\
Ł_\alpha\{v_2(x,y)\} &= \frac{1}{s^\alpha} v_1(0,y) - \frac{1}{s^\alpha} Ł_\alpha\left\{\frac{\partial^\alpha u_0}{\partial y^\alpha} - v_1 - u_1\right\}.
\end{aligned} \tag{3.30}$$

Therefore, we have

$$\begin{aligned}
Ł\{u_2(x,y)\} &= \frac{\sinh_\alpha(y^\alpha)}{s^\alpha} + \frac{\cosh_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\sinh_\alpha(y^\alpha)}{s^{3\alpha}},\\
Ł\{v_2(x,y)\} &= \frac{\cosh_\alpha(y^\alpha)}{s^\alpha} + \frac{\sinh_\alpha(y^\alpha)}{s^{2\alpha}} + \frac{\cosh_\alpha(y^\alpha)}{s^{3\alpha}},\\
&\vdots
\end{aligned} \tag{3.31}$$

$$\begin{aligned}
Ł_\alpha\{u_m(x,y)\} &= \sum_{k=0}^{m} \frac{\sinh_\alpha(y^\alpha)}{s^{(2k+1)\alpha}} + \sum_{k=0}^{m} \frac{\cosh_\alpha(y^\alpha)}{s^{(2k+2)\alpha}},\\
Ł_\alpha\{v_m(x,y)\} &= \sum_{k=0}^{m} \frac{\cosh_\alpha(y^\alpha)}{s^{(2k+1)\alpha}} + \sum_{k=0}^{m} \frac{\sinh_\alpha(y^\alpha)}{s^{(2k+2)\alpha}}.
\end{aligned} \tag{3.32}$$

Consequently, the local fractional series solution is:

$$\begin{aligned}
u(x,y) &= \lim_{m\to\infty} Ł_\alpha^{-1}\left(Ł_\alpha\{u_m(x,y\}\right)\\
v(x,y) &= \lim_{m\to\infty} Ł_\alpha^{-1}\left(Ł_\alpha\{v_m(x,y)\}\right)\\
&= \sinh_\alpha(y^\alpha)\left(1 + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \cdots\right) + \cosh_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right)\\
&= \cosh_\alpha(y^\alpha)\left(1 + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \cdots\right) + \sinh_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right)\\
&= \sinh_\alpha(x^\alpha + y^\alpha),\\
&= \cosh_\alpha(x^\alpha + y^\alpha).
\end{aligned} \tag{3.33}$$

4. Conclusions

In this work we considered the coupling method of the local fractional variational iteration method and Laplace transform to solve the system of linear and nonlinear local fractional partial differential equations and their nondifferentiable solutions were obtained. The local fractional Laplace variational iteration method is proved to be an effective approach for solving system of partial differential equations with local fractional derivative operators due to the excellent agreement between the obtained numerical solution and the exact solution. A comparison is made to show that the method has small size of computation in comparison with the computational size required in other numerical methods and its rapid convergence shows that the method is reliable and introduces a significant improvement in solving linear and nonlinear partial differential equations with local fractional derivative operators.

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