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Bianchi Type I Bulk Viscous Fluid String Dust Cosmological Model with Magnetic Field in Bimetric Theory of Gravitation

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Abstract

In the presence of both magnetic field and bulk viscosity, Bianchi Type I bulk viscous fluid string dust cosmological model in Rosen's bimetric theory of gravitation have been investigated by using the technique of Letelier and Stachel. The nature of the model is discussed in the absence of both magnetic field and bulk viscosity. To get a determinate solution, we have assumed the condition that σ is proportional to θ and $\zeta\theta$ = constant where σ is the shear, θ is the expansion in the model and ζ is the coefficient of bulk viscosity. Further the physical and geometrical significance of the model are discussed. Here, we compared between the case in the presence of magnetic field and bulk viscosity and the case in the absence of magnetic field and bulk viscosity.

Keywords: Bimetric theory; bulk viscous; cosmic string, magnetic field, Bianchi type-I

MSC 2000: 83 CXX, 83C50, 83 F 05

1. Introduction

Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein's theory of gravitation. The most important among them are Rosen's bimetric theory of gravitation and Scalar-tensor theory of gravitation. The Rosen's bimetric theory is the theory of gravitation based on two metrics, see Rosen (1974). One is the fundamental metric tensor g_{ij} describes the gravitational potential and the second metric γ_{ij} refers to the flat space-time and describes the inertial forces associated with the acceleration of the frame of reference. The metric tensor g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and describe the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Therefore it interacts with g_{ij} but not directly with matter. One can regard γ_{ij} as giving the geometry that would exists if there were no matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$. Moreover, the bimetric theory also satisfied the covariance and equivalence principles; the formation of general relativity. The theory agrees with the present observational facts pertaining to general relativity [for details one may refer Karade (1980), Katore and Rane (2006), and Rosen (1974, 1977)]. Thus, at every point of space-time, there are two metrics

$$ds^2 = g_{ij}\, dx^i dx^j \tag{1}$$

and

$$d\eta^2 = \gamma_{ij}\, dx^i dx^j . \tag{2}$$

The field equations of Rosen (1974)'s bimetric theory of gravitation are

$$K_i^j = N_i^j - \frac{1}{2} N \delta_i^j = -8\pi k T_i^j \tag{3}$$

where $N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj} g_{si|p}\right)_{|r}$, $N = N_i^i$, and $k = \sqrt{\frac{g}{\gamma}}$ together with $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$. Here, the vertical bar (|) stands for γ-covariant differentiation and T_i^j is the energy-momentum tensor of matter fields.

The several aspects of bimetric theory of gravitation have been studied by Rosen (1974), Karade (1980), Israelit (1981), Katore and Rane (2006), and Khadekar and Tade (2007). In particular Reddy and N. V. Rao (1998) have obtained some Bianchi type cosmological models in bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that

occur in general relativity that was appearing in the big-bang in cosmological models and therefore recently, there has been a lot of interest in cosmological model on the basis of Rosen's bimetric theory of gravitation.

In bimetric theory, the background metric tensor γ_{ij} should not be taken as describing an empty universe but it should rather be chosen on the basis of cosmological consideration. Hence Rosen proposed that the metric γ_{ij} be taken as the metric tensor of a universe in which perfect cosmological principle holds. In accordance with this principle, the large scale structure of universe presents the same aspect from everywhere in space and at all times. The fact, however, is that while taking the matter actually present in the universe, this principle is not valid on small scale structure due to irregularities in the matter distribution and also not valid on large scale structure due to the evolution of the matter. Therefore, we adopt the perfect cosmological principle as the guiding principle. It does not apply to g_{ij} and the matter in the universe but to the metric γ_{ij}. Hence, γ_{ij} describes a space-time of constant curvature.

In the context of general relativity cosmic strings do not occur in Banchi type models, see Krori et al. (1994). Some Bianchi type cosmological models – two in four and one in higher dimensions- are studied by Krori et al. (1994). They have shown that the cosmic strings do not occur in Bianchi type V cosmology. Bali and Dave (2003); Bali and Upadhaya (2003), Bali and Singh (2005), Bali and Pareek (2007) have investigated Bianchi type IX, I and V string cosmological models under different physical conditions in general relativity. The magnetic field is due to an electric current produced along x-axis. Raj Bali and Anjali (2004) have investigated Bianchi Type I bulk viscous fluid string dust magnetized cosmological model in general relativity. They have assumed that the eigenvalue (σ_1^1) of shear tensor (σ_i^j) is proportional to the expansion (θ) which is physically plausible condition. The string dust condition leads to $\in = \lambda$, where $\in$ is the rest energy density and λ the string tension density.

Recently people like Bali et al. (2003), Pradhan et al. (2007), Pradhan (2009) and Wang (2004, 2006) developed the models in the field of bulk viscous fluid solutions and Bianchi type string models which are the most useful models in general relativity. In an attempt to achieve our bulk viscous model in bimetric theory of gravitation, we used the terminology and the notations of Bali et al. (2003).

In this paper, we have investigated Bianchi Type I bulk viscous fluid string dust cosmological model with and without magnetic field in Rosen's bimetric theory of gravitation as there has been a lot of interest in cosmological model on the basis of Rosen's bimetric theory of gravitation. To get determinate solution we have assumed that σ is proportional to θ and $\zeta\theta$ = constant where σ is shear, θ is the expansion in the model and ζ is the coefficient of bulk viscosity. Also the physical and geometrical significance of the model are discussed.

In our models, bulk viscosity plays important role in the presence of magnetic field as well as in the absence of magnetic field. This agreed with the findings of Tripathy et al. (2008) that the bulk viscosity plays role in the evolution of the universe.

We consider Bianchi Type I metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \ , \tag{4}$$

where A, B and C are functions of t alone. The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 . \tag{5}$$

The energy momentum tensor T_i^j for string dust is given by

$$T_i^j = \in \upsilon_i \upsilon^j - \lambda x_i x^j - \zeta \upsilon^{\ell}_{;\ell} (g_i^j + \upsilon_i \upsilon^j) + E_i^j \tag{6}$$

with

$$\upsilon_i \upsilon^i = -x_i x^i = -1 \tag{7}$$

and

$$\upsilon^i x_i = 0 . \tag{8}$$

In this model, $\in$ is the rest energy density for a cloud of strings and is given by $\in = \in_p + \lambda$ where $\in_p$ and λ denote the particle density and the string tension density of the system of strings respectively, x^i is the direction of strings and ζ is the coefficient of bulk viscosity.

The electromagnetic field E_{ij} is given by Lichnerowicz (1967)

$$E_{ij} = \overline{\mu} \left[\ |h|^2 \left(\upsilon_i \upsilon_j + \frac{1}{2} g_{ij} \right) - h_i h_j \right] , \tag{9}$$

where four velocity vector υ_i is given by

$$g_{ij} \upsilon^i \upsilon^j = -1 \tag{10}$$

and $\overline{\mu}$ is the magnetic permeability and the magnetic flux vector h_i defined by

$$h_i = \frac{\sqrt{-g}}{2\overline{\mu}} \in_{ijkl} F^{kl} \upsilon^j , \tag{11}$$

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levi Civita tensor density.

Assume the comoving coordinates system, so that $\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \upsilon^4 = 1$. Further, we assume that the incident magnetic field is taken along x-axis so that $h_1 \neq 0$ and $h_2 = h_3 = h_4 = 0$. The first set of Maxwell's equation

$$F_{[ij,k]} = 0 \tag{12}$$

Yields $F_{23} =$ constant H (*say*). Due to the assumption of infinite electrical conductivity, we have

$$F_{14} = F_{24} = F_{34} = 0 .$$

The only non-vanishing component of F_{ij} is F_{23}. So that

$$h_1 = \frac{AH}{\bar{\mu}BC} \tag{13}$$

and

$$|h|^2 = \frac{H^2}{\bar{\mu}^2 B^2 C^2} . \tag{14}$$

From equation (9) we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\bar{\mu}B^2C^2} . \tag{15}$$

From equation (6) we obtain

$$T_1^1 = \left(-\lambda - \frac{H^2}{2\bar{\mu}B^2C^2} - \zeta\upsilon^{\ell}_{;\ell} \right), \; T_2^2 = T_3^3 = \left(\frac{H^2}{2\bar{\mu}B^2C^2} - \zeta\upsilon^{\ell}_{;\ell} \right),$$

$$T_4^4 = -\left(\in + \frac{H^2}{2\bar{\mu}B^2C^2} \right) . \tag{16}$$

Substituting these values of T_i^j [equation (16)] in the Rosen's field equations (3), we write

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^{\;2}}{A^2} - \frac{B_4^{\;2}}{B^2} - \frac{C_4^{\;2}}{C^2} = 16\pi ABC\left(\lambda + \frac{H^2}{2\bar{\mu}B^2C^2} + \zeta\upsilon^{\ell}_{;\ell} \right) \tag{17}$$

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4^{\ 2}}{A^2}+\frac{B_4^{\ 2}}{B^2}-\frac{C_4^{\ 2}}{C^2}=16\pi ABC\left(-\frac{H^2}{2\overline{\mu}B^2C^2}+\zeta\upsilon_{;\ell}^{\ell}\right) \tag{18}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}-\frac{C_{44}}{C}-\frac{A_4^{\ 2}}{A^2}-\frac{B_4^{\ 2}}{B^2}+\frac{C_4^{\ 2}}{C^2}=16\pi ABC\left(-\frac{H^2}{2\overline{\mu}B^2C^2}+\zeta\upsilon_{;\ell}^{\ell}\right) \tag{19}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4^{\ 2}}{A^2}-\frac{B_4^{\ 2}}{B^2}-\frac{C_4^{\ 2}}{C^2}=16\pi ABC\left(\in+\frac{H^2}{2\overline{\mu}B^2C^2}\right), \tag{20}$$

where

$$A_4=\frac{dA}{dt}, B_4=\frac{dB}{dt}, C_4=\frac{dC}{dt}, \text{ etc.}$$

Equations (17) to (20) are four equations in five unknowns A, B, C, λ and $\in$. Therefore, to deduce a determinate solution; we assume a supplementary condition

$$A=(BC)^n, \quad n>0, \tag{21}$$

for which the shear (σ) is proportional to the scalar of expansion θ .

The universe is filled with Zel'dovich matter string dust and perfect fluid and therefore we are using Zel'dovich (1980) condition

$$\in=\lambda \tag{22}$$

in our model. From equations (19) and (20), we obtain

$$2\frac{C_4^2}{C^2}-2\frac{C_{44}}{C}=16\pi ABC\left(\zeta\upsilon_{;\ell}^{\ell}-\in-\frac{H^2}{\overline{\mu}B^2C^2}\right). \tag{23}$$

Adding equations (17) and (23) together and using the condition ε = λ, we get

$$\frac{B_{44}}{B}-\frac{A_{44}}{A}-\frac{C_{44}}{C}+\frac{A_4^{\ 2}}{A^2}-\frac{B_4^{\ 2}}{B^2}+\frac{C_4^{\ 2}}{C^2}=16\pi ABC\left(2\zeta\,\upsilon_{;\ell}^{\ell}-\frac{H^2}{2\overline{\mu}B^2C^2}\right). \tag{24}$$

From equations (21) and (24), we write

$$(n-1)\frac{B_4^{\ 2}}{B^2}+(n+1)\frac{C_4^{\ 2}}{C^2}+(1-n)\frac{B_{44}}{B}-(n+1)\frac{C_{44}}{C}=-16\pi K\left(BC\right)^{n-1}+32\pi\left(BC\right)^{n+1}\zeta\upsilon_{;\ell}^{\ell}, \tag{25}$$

where $K = \frac{H^2}{2\bar{\mu}}$.

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_4^{\ 2}}{C^2} - \frac{B_4^{\ 2}}{B^2}. \tag{26}$$

On simplifying above equation, we get

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = \frac{(BC)_4}{BC}, \tag{27}$$

which on integrating, yield

$$C^2\left(\frac{B}{C}\right)_4 = LBC \ , \tag{28}$$

where L is the constant of integration.

Using assumptions $BC = \mu$ and $\frac{B}{C} = v$, equation (28) leads to

$$\frac{v_4}{v} = L. \tag{29}$$

Now using equation (21) and the condition $BC = \mu$ and $\frac{B}{C} = v$, the equation (25) gives

$$-\frac{n}{\mu^{(n+1)}}\left(\frac{\mu_{44}}{\mu}\right) + \frac{n}{\mu^{n+1}}\frac{\mu_4^2}{\mu^2} = -\frac{16\pi K}{\mu^2} + 32\pi\zeta\upsilon^{\ell}_{;\ell}. \tag{30}$$

Applying the condition $\zeta\theta$ = constant to the above equation, we get

$$\mu_{44} - \frac{\mu_4^2}{\mu} + \beta\mu^{n+2} = \frac{16\pi K}{n}\mu^n \ , \tag{31}$$

where

$$\beta = \frac{32\pi}{n}\zeta\upsilon^{\ell}_{;\ell},$$

which reduces to

$$\frac{d}{d\mu}\left[f^2\right]+(-\frac{2}{\mu})f^2=2\left[\frac{16\pi K}{n}-\beta\mu^2\right]\mu^n \ , \tag{32}$$

where $\mu_4 = f(\mu)$.

The differential equation (32) has solution

$$f^2=\frac{32\pi K}{n(n-1)}\mu^{n+1}-\frac{2\beta}{n+1}\mu^{n+3}+P\mu^2 , \tag{33}$$

where P is the constant of integration. From equation (29) we write

$$\log v=\int\frac{L\ d\mu}{\sqrt{\frac{32\pi K}{n(n-1)}\mu^{n+1}-\frac{2\beta}{(n+1)}\mu^{n+3}+P\mu^2}}+\log b . \tag{34}$$

Using $\mu_4 = f(\mu)$ and expression (33), the metric (4) will be

$$ds^2=-\frac{d\mu^2}{\left[\frac{32\pi K}{n(n-1)}\mu^{(n+1)}-\frac{2\beta}{(n+1)}\mu^{(n+3)}+P\mu^2\right]}+\mu^{2n}dx^2+\mu v dy^2+\frac{\mu}{v}dz^2 \ , \tag{35}$$

where v is determined by equation (34). After suitable transformation of coordinates i.e., putting

$$\mu = T,\ x = X,\ y = Y,\ z = Z$$

the above metric (35) takes the form

$$ds^2=-\frac{dT^2}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)}-\frac{2\beta}{(n+1)}T^{(n+3)}+PT^2\right]}+T^{2n}dX^2+T v\ dY^2+\frac{T}{v}dZ^2 \ . \tag{36}$$

This is the Bianchi Type-I bulk viscous fluid string dust cosmological model with magnetic field in bimetric theory of gravitation.

In the absence of magnetic field i.e., $K = 0$, the metric (36) have the form

$$ds^2 = \frac{-dT^2}{\left[PT^2 - \frac{2\beta}{(n+1)}T^{(n+3)}\right]} + T^{2n}dX^2 + T\,v\,dY^2 + \frac{T}{v}dZ^2. \tag{37}$$

In the absence of viscosity, i.e., $\beta = 0$, the metric (36) takes the form

$$ds^2 = \frac{-dT^2}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)} + PT^2\right]} + T^{2n}dX^2 + T\,v\,dY^2 + \frac{T}{v}dZ^2. \tag{38}$$

2. Some Physical and Geometrical Features

The density (ε), the string tension density (λ), for the model (36) is given by

$$\in = \lambda = \left[\frac{(5n-4n^2-1)}{n(n-1)}\frac{K}{T^2} + \frac{(2n^2+n-1)\beta}{16\pi(n+1)}\right]. \tag{39}$$

Now, the expansion θ is given by $\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$, which has the value $\theta = \frac{(n+1)f}{T}$, or

$$\theta = (n+1)\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}}. \tag{40}$$

The components of shear tensor σ_i^j are given by

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right]$$

or

$$\sigma_1^1 = \frac{(2n-1)}{3}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}}. \tag{41}$$

Likewise, we obtain the other components of σ_i^j as

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2}, \tag{42}$$

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} - \frac{L}{2}, \tag{43}$$

and

$$\sigma_4^4 = 0. \tag{44}$$

Thus,

$$\sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}) = \frac{(2n-1)^2}{12}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right] + \frac{L^2}{4}, \tag{45}$$

and the spatial volume is

$$R^3 = \alpha T^{(n+1)}, \tag{46}$$

where $\alpha = \frac{1}{f} = \frac{1}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)} - \frac{2\beta}{(n+1)}T^{(n+3)} + PT^2\right]^{\frac{1}{2}}}$.

From these results, it is learned that in the presence of magnetic field and bulk viscosity, the rest energy density $\in$ and string tension density λ both are infinite initially, whereas both are depends on viscosity coefficient β at infinite time. The expansion θ in the model increases as the coefficient of viscosity β decreases and it becomes maximum for $\beta = 0$. For very very large value of T, the expansion θ, the shear σ and the spatial volume R^3, are infinite and our model (36) does not approach isotropy.

3. Discussion

From the results of earlier section-2, it is seen that in presence of magnetic field and bulk viscosity, our model (36) is expanding, when the coefficient of viscosity β decreasing, and the maximum expansion is

$$(n+1)\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}}.$$

The rest energy density $\in$ and string tension density λ both are infinite initially, whereas both are depends on viscosity coefficient β at infinite time. The expansion θ in the model increases as the coefficient of viscosity β decreases and it becomes maximum for $\beta = 0$. For very very

large value of T, the expansion θ, the shear σ and the spatial volume R^3 are infinite and our model (36) does not approach isotropy.

For $n = \frac{1}{2}$, there is shear $\sigma = \frac{L}{2}$. Hence, the model (36) does not approaches isotropy for large values of T.

In the absence of magnetic field, K, the equation (39) leads to

$$\in = \lambda = \frac{\left(2n^2 + n - 1\right)\beta}{16\pi\left(n+1\right)}, \tag{47}$$

from which we conclude that the rest energy density $(\in)$, string tension density (λ) depends only on viscosity coefficient β.

The expressions for θ, σ_i^j and R^3, in the absence of magnetic field K are given by

$$\theta = \left(n+1\right) \left[P - \frac{2\beta}{\left(n+1\right)} T^{n+1}\right]^{\frac{1}{2}} \text{ sp}, \tag{48}$$

$$\sigma_1^1 = \frac{\left(2n-1\right)}{3}\left[P - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}}, \tag{49}$$

$$\sigma_2^2 = \frac{\left(1-2n\right)}{6}\left[P - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2}, \tag{50}$$

$$\sigma_3^3 = \frac{\left(1-2n\right)}{6}\left[P - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} - \frac{L}{2}, \tag{51}$$

$$\sigma_4^4 = 0, \tag{52}$$

$$\sigma^2 = \frac{\left(2n-1\right)^2}{12}\left[P - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L^2}{4}, \tag{53}$$

and

$$R^3 = \alpha_1 T^{(n+1)}, \tag{54}$$

where $\alpha_1 = \dfrac{1}{\left[PT^2 - \dfrac{2\beta}{(n+1)} T^{(n+3)}\right]^{\frac{1}{2}}}$.

In the absence of viscosity, the rest energy density $\in$, string tension density λ for the model (36) is given by

$$\in = \lambda = \frac{(5n - 4n^2 - 1)}{n(n-1)} \frac{K}{T^2}, \tag{55}$$

and it becomes zero initially and infinite for very very large value of T.

The expressions for θ, σ_i^j and R^3, in the absence of viscosity, are given by

$$\theta = (n+1)\left[P + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}},$$

$$\sigma_1^1 = \frac{(2n-1)}{3}\left[P + \frac{32\,\pi\,K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}}, \tag{56}$$

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[P + \frac{32\,\pi\,K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L}{2}, \tag{57}$$

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[P + \frac{32\,\pi\,K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} - \frac{L}{2}, \tag{58}$$

$$\sigma_4^4 = 0, \tag{59}$$

$$\sigma^2 = \frac{(2n-1)^2}{12}\left[P + \frac{32\,\pi\,K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L^2}{4}, \tag{60}$$

and

$$R^3 = \alpha_2 T^{(n+1)}, \tag{61}$$

where $\alpha_2 = \dfrac{1}{\left[PT^2 + \dfrac{32\pi K}{n(n-1)} T^{(n+1)}\right]^{\frac{1}{2}}}$.

In the absence of magnetic field, our model (36) contracting as T and β increases, and it is expanding in the absence of bulk viscosity. The model does not approaches isotropy for large values of T. The spatial volume of our model is zero initially and it is infinite for very very large value of T.

We compared between the case in the presence of magnetic field and bulk viscosity and the case in the absence of magnetic field and bulk viscosity and it is realized that our model is expanding as well as shearing in presence of magnetic field and bulk viscosity (for decreasing β), whereas it is neither expanding nor shearing in the absence of both magnetic field and bulk viscosity.

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