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Reliability Analysis of a Series and Parallel Network using Triangular Intuitionistic Fuzzy Sets

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Abstract

This paper describes a novel approach, based on intuitionistic fuzzy set theory for reliability analysis of series and parallel network. The triangular intuitionistic fuzzy sets are used to represent the failure possibility of each basic (terminal) event to get more comprehensive results for the failure possibility of the top event. The proposed technique is demonstrated on a web server LOG data used to illustrate HTTP (Hyper Text Transfer Protocol) failure.

Keywords: Intuitionistic fuzzy set, Triangular intuitionistic fuzzy set, System reliability, parallel and series system

MSC 2010 No.: 03E72, 90C05, 90C70

1. Introduction

It is well recognized that the classical set theoretic approach in reliability analysis fails to address the uncertainty up to a significant level. This problem was overcome by using the notion of the fuzzy set introduced by L. A. Zadeh (1965). Using the fuzzy set theoretic approach Cai et al. (1993) introduced the fuzzy state assumption and possibility assumption to give Profust and Possbist. Singer (1990) presented fuzzy set theoretic approach to fault tree analysis. Chen (1994) used the arithmetic of fuzzy numbers to evaluate system reliability. Z. X. Yang et al. (1995) constructed a fuzzy fault diagnostic system which uses the fuzzy fault tree analysis to represent knowledge of the causal relationships in process operation and control system. Their proposed method is applied successfully to a nitric acid cooler process plant. Bowles and Palaez (1995) obtained the profust reliability estimates for a single unit gracefully degradable system. Fuzzy set theoretic approach for estimating failure rate parameters developed by Pandey and Tyagi (2007) provides comprehensive results that can be applied to a variety of parameters involving human judgment, unreported times, vague operating conditions, etc.

The application of fuzzy sets in reliability analysis depends on acquiring adequate membership for fuzzy sets. In fuzzy set theory, the non-membership for fuzzy sets of an element *x* of the universe is considered to one minus the membership degree. In real life situation, it is assumed that a certain object may or may not be in a set *A* to a certain degree, but it is possible to entertain some doubt about it. In other words, some hesitation about the degree of belongingness exists. This hesitation in the membership degree may be modeled by intuitionistic fuzzy sets defined by Attanassov (1986).

Intuitionistic fuzzy sets provide us the opportunity to model hesitation and uncertainty by introducing a non-membership function, in addition to the membership function. In intuitionistic fuzzy sets the non-membership of an element *x* of the universe need not be one minus the membership degree. Rather, it might be any number lying between 0 and 1. An intuitionistic fuzzy set (IFS) $\tilde{A}^i$ in *X* is characterized by a membership function $\mu_{\tilde{A}^i}(x)$ and a non membership function $\nu_{\tilde{A}^i}(x)$. Here $\mu_{\tilde{A}^i}(x)$ and $\nu_{\tilde{A}^i}(x)$ are associated with each point in *X*, a real value in [0, 1]. It may be proved that the closer the value of $\mu_{\tilde{A}^i}(x)$ to unity and the value of $\nu_{\tilde{A}^i}(x)$ to zero, the higher the grade of membership and the lower the grade of non-membership of *x*. De et al. (2000, 2001) studied the Sanchez's approach for medical diagnosis and extended this concept to the notion of intuitionistic fuzzy set theory, and defined some operations on intuitionistic fuzzy sets.

In this paper a fault tree is depicted, aiming to illustrate the interrelation among these HTTP basic events. A fault tree is a graphical model of pathways within a system that can lead to a foreseeable, undesirable loss event. The pathways interconnect contributory events and conditions using series and parallel networks. In a fault tree, the rectangle defines an intermediate or top event that is the output of these networks. The circle indicates a basic event that is a primary failure of a system element. The symbols "+" and "**.**" respectively mean that the systems are connected in parallel and series. The vague and imprecise data, like LOG data, web tools results etc are the integral part of Web Server reliability analysis. Many times the HTTP failure caused by the users (status code 4xx) and servers (status code 5xx) is illustrated by LOG files.

A LOG file consists of information regarding the service request from a system, the upcoming response and the genesis of information. According to the LOG file the status code (4xx) are intended for cases in which the user has committed a mistake, while accessing a particular website. On the other hand, the status code (5xx) indicates cases in which the server is aware that it has caused an error or incapable of performing the user request. Although a LOG file provides the frequency of the accesses failure within a certain period. It usually appears with imprecise and vague information. To address the vague and unreliable resource of the data, the basic events are assigned intuitionistic fuzzy sets. For the sake of simplicity of computation and efficiency of evaluation, the triangular intuitionistic fuzzy sets are used to represent the reliability of basic events. The triangular intuitionistic fuzzy set is also capable of capturing the uncertainty inherited with the data collected regarding the occurrence of a basic event.

2. Intuitionistic Fuzzy Set

An intuitionistic fuzzy set $\tilde{A}^i$ [Attanassov, 1986] on X is given by $\tilde{A}^i = \left\{ < x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x) >: x \in X \right\}$ with $\mu_{\tilde{A}^i}(x): X \to [0, 1]$ and $\nu_{\tilde{A}^i}(x): X \to [0, 1]$ such that $0 \le \mu_{\tilde{A}^i}(x) + \nu_{\tilde{A}^i}(x) \le 1$ for all $x \in X$.

The value $\mu_{\tilde{A}^i}(x)$ is a lower bound on the degree of membership of *x* derived from the evidence for *x* and $\nu_{\tilde{A}^i}(x)$ is a lower bound on the negation of *x* derived from the evidence against *x*. We will call $\pi_{\tilde{A}^i}(x) = 1 - \mu_{\tilde{A}^i}(x) - \nu_{\tilde{A}^i}(x), x \in X$, the intuitionistic index of x. It is the hesitancy of x in $\tilde{A}^i$, and expressed lack of knowledge of whether $x \in X$ or not.

When the universe of discourse *X* is discrete, an intuitionistic fuzzy set $\tilde{A}^i$ can be written as

$$\tilde{A}^i = \sum_{j=1}^{n} \left\{ \mu_{\tilde{A}^i}(x_j), 1 - \nu_{\tilde{A}^i}(x_j) \right\} / x_j \quad \text{for all } x_j \in X.$$

An intuitionistic fuzzy set $\tilde{A}^i$ with continuous universe of discourse *X* can be written as

$$\tilde{A}^i = \int_X \left\{ \mu_{\tilde{A}^i}(x_j), 1 - \nu_{\tilde{A}^i}(x_j) \right\} / x_j \text{ for all } x_j \in X,$$

$$\mu_{\tilde{A}^i}(x), 1 - \nu_{\tilde{A}^i}(x).$$

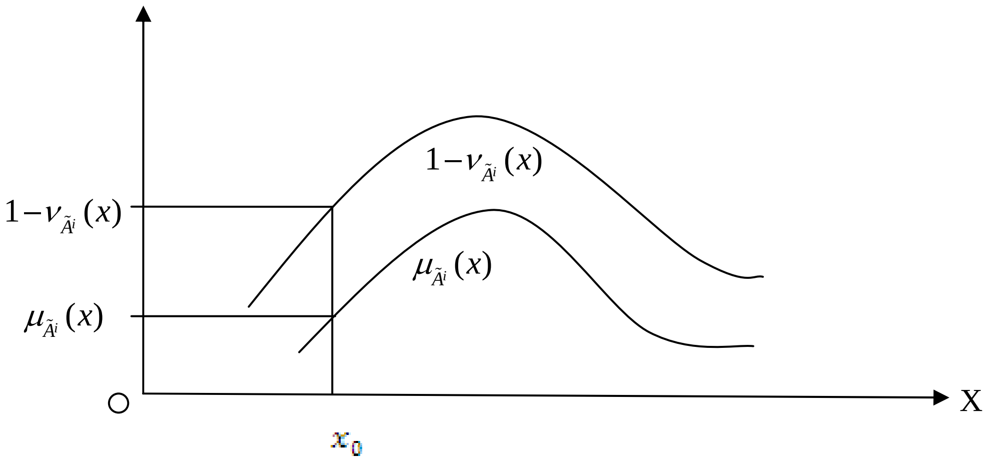

Figure 1. Intuitionistic fuzzy set explanation of real number *R*

2.1. Triangular Intuitionistic Fuzzy Set

A triangular intuitionistic fuzzy set $\tilde{A}^i$ over the universe of discourse X shown in Figure 2 may be denoted as $\tilde{A}^i = <\left[(a_1, a_2, a_3); \mu_{\tilde{A}^i}(x)\right], \left[(a_1', a_2, a_3'); \nu_{\tilde{A}^i}(x)\right]>$. The membership function $\mu_{\tilde{A}^i}(x)$ and non-membership function $\nu_{\tilde{A}^i}(x)$ are defined as below

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}\mu_{\tilde{A}^i}(a_2) & if\ a_1 \le x < a_2 \\ \dfrac{a_3-x}{a_3-a_2}\mu_{\tilde{A}^i}(a_2) & if\ a_2 \le x < a_3 \end{cases}$$

and

$$\nu_{\tilde{A}^i}(x) = \begin{cases} \dfrac{x-a_1'}{a_2-a_1'}\nu_{\tilde{A}^i}(a_2) & if\ a_1' \le x < a_2 \\ \dfrac{a_3'-x}{a_3'-a_2}\nu_{\tilde{A}^i}(a_2) & if\ a_2 \le x < a_3' \end{cases}$$

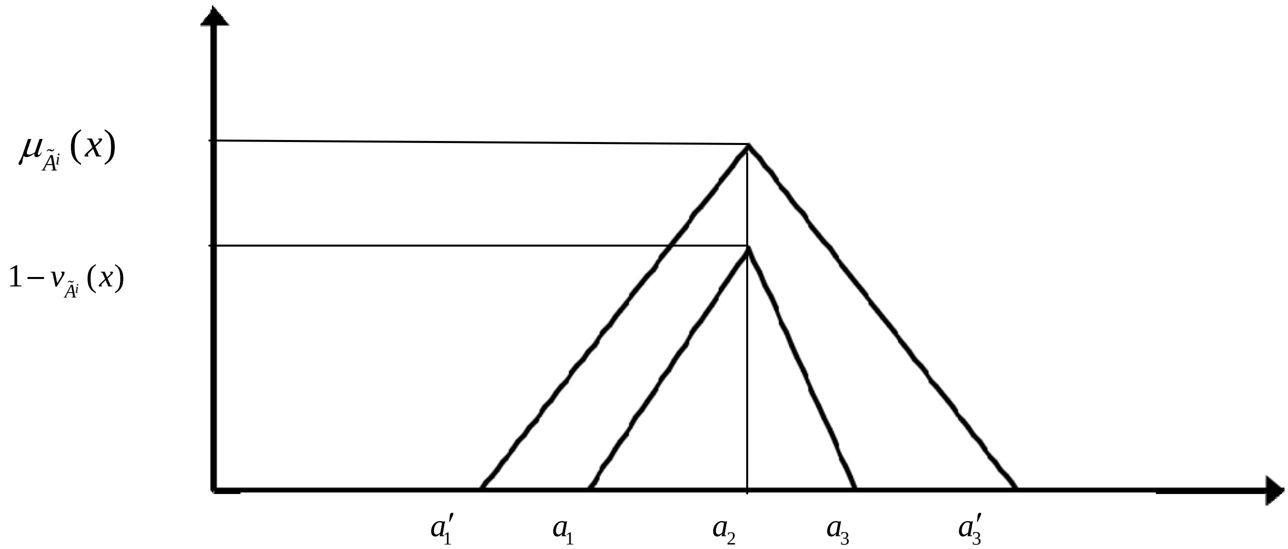

Figure 2. Triangular Intuitionistic Fuzzy Set

3. Operations on Triangular Intuitionistic Fuzzy Sets

Here we define some basic operations on triangular intuitionistic fuzzy sets [De, S.K et al., 2000]. Let $\tilde{A}^i$ and $\tilde{B}^i$ be two intuitionistic fuzzy sets on universe of discourse X as shown in Figure 3 and defined as

$$\tilde{A}^i = < \left[(a_1, a_2, a_3); \mu_{\tilde{A}^i}(x_i)\right], \left[(a_1', a_2, a_3'); \nu_{\tilde{A}^i}(x_i)\right] >$$

and

$$\tilde{B}^i = < \left[(b_1, b_2, b_3); \mu_{\tilde{B}^i}(x_i)\right], \left[(b_1', b_2, b_3'); \nu_{\tilde{B}^i}(x_i)\right] >,$$

where $s = 1 - \nu_{\tilde{A}^i}(x_i), t = 1 - \nu_{B^i}(x_i), u = \mu_{A^i}(x_i)$ and $v = \mu_{\tilde{B}^i}(x)$.

Then,

(i) $\tilde{A}^i \otimes \tilde{B}^i = < \left[(a_1 b_1, a_2 b_2, a_3 b_3) : \min(\mu_{\tilde{A}^i}, \mu_{\tilde{B}^i})\right], \left[(a_1' b_1', a_2 b_2, a_3' b_3') : \max(\nu_{\tilde{A}^i}, \nu_{\tilde{B}^i})\right] >$

(ii) $\tilde{A}^i \oplus \tilde{B}^i = < \left[(a_1 + b_1, a_2 + b_2, a_3 + b_3) : \min(\mu_{\tilde{A}^i}, \mu_{\tilde{B}^i})\right], \left[(a_1' + b_1', a_2 + b_2, a_3' + b_3') : \max(\nu_{\tilde{A}^i}, \nu_{\tilde{B}^i})\right] >$

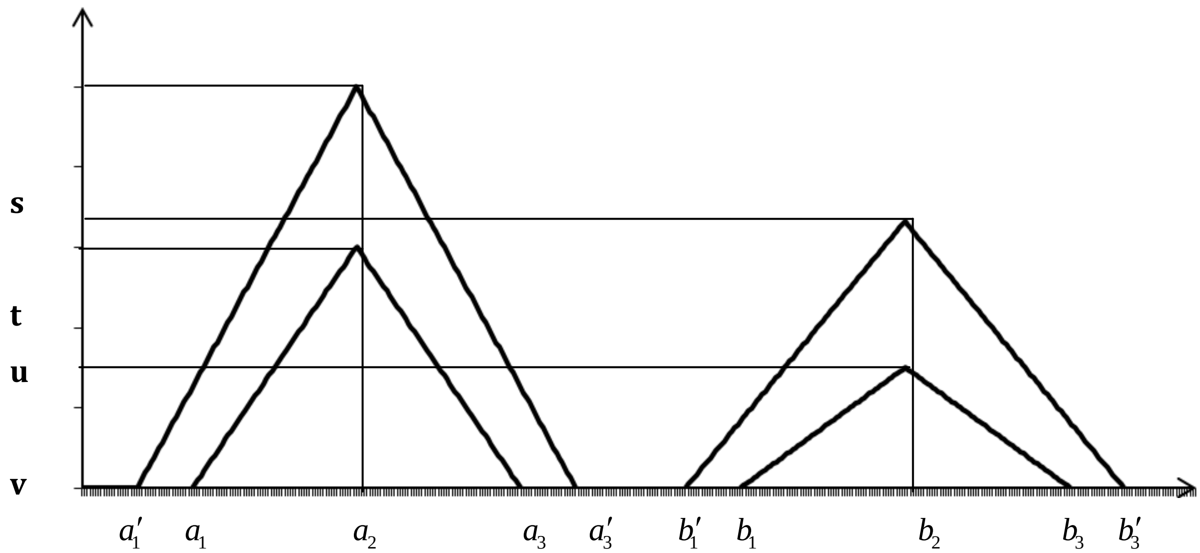

Figure 3. Triangular Intuitionistic Fuzzy Sets

4. Reliability Analysis of Series and Parallel Network

In this section, taking the reliability of each component to be a triangular intuitionistic fuzzy set we have evolved a fuzzy reliability evaluation technique for series and parallel systems.

Let us consider a system consisting of n components, the intuitionistic fuzzy sets $\tilde{R}_j^{\,i}$, $j=1,2,3\ldots\ldots n$, are taken to represent the reliability of each component. If the components are connected as a series system as shown in Figure 4, the reliability $\tilde{R}_S^{\,i}$ of the series system is defined as follows:

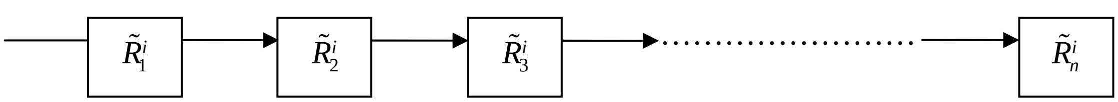

Figure 4. Systems in Series

$$\tilde{R}_s^{\,i} = \otimes_{j=1}^{n} \tilde{R}_j^{\,i}$$
$$= < \left[\left(\prod_{j=1}^{n} a_{1j}, \prod_{j=1}^{n} a_{2j}, \prod_{j=1}^{n} a_{3j} \right) : \min_{j=1}^{n} (\mu_{\tilde{A}_j^i}(x)) \right], \left[\left(\prod_{j=1}^{n} a_{1j}, \prod_{j=1}^{n} a_{2j}, \prod_{j=1}^{n} a_{3j} \right) : \max_{j=1}^{n} (\nu_{\tilde{A}_j^i}(x)) \right] > .$$

If the components are supposed to be in parallel as shown in Figure 5, the reliability $\tilde{R}_P^{\,i}$ of the parallel system can be defined by using the expression:

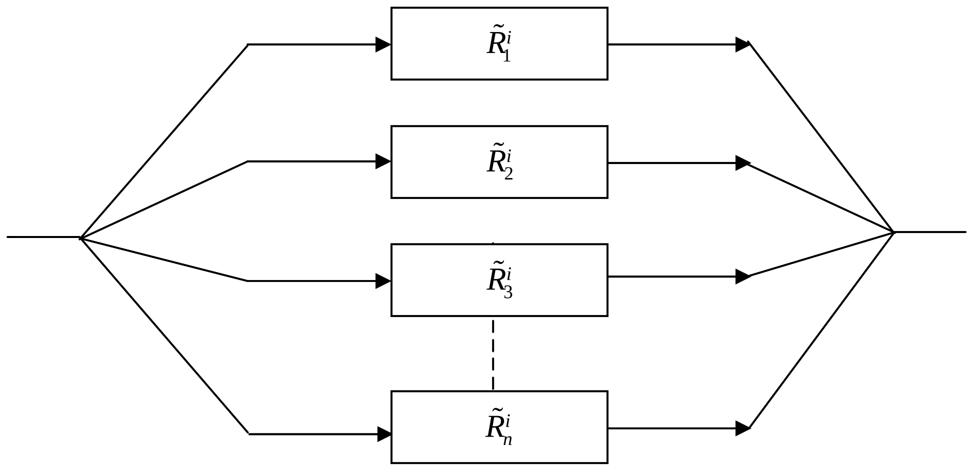

Figure 5. Parallel System

$$\tilde{R}_p{}^i = 1 - \prod_{i=1}^{n}(1 - \tilde{R}_i^i)$$

$$= < \left[\left(1 - \prod_{i=1}^{n}(1 - a_{1i}), 1 - \prod_{i=1}^{n}(1 - a_{2i}), 1 - \prod_{i=1}^{n}(1 - a_{3i})\right) : \min_{i=1}^{n}(\mu_{\tilde{A}_i^i}(x)\right],$$
$$\left[\left(1 - \prod_{i=1}^{n}(1 - a_{1i}), 1 - \prod_{i=1}^{n}(1 - a_{2i}), 1 - \prod_{i=1}^{n}(1 - a_{3i})\right) : \max_{i=1}^{n}(\nu_{\tilde{A}_i^i}(x)\right] > .$$

5. Intuitionistic Fuzzy Fault Tree Analysis of Real Web Server LOG Data

The HTTP failure (Top Event) occurs due to some intermediate and/or basic events as shown in Figure 6.

The interrelation of HTTP failure basic events as shown in Figure 6 may be explained below:

The HTTP failure (top event) occurs due to client error or server error. The client error is caused by a basic event E_1 (malformed syntax) or two intermediate events, code 401 (bad request) and code 403 (forbidden). The basic events E_2 (expired time access) or E_3 (invalid password or username) lead to the occurrence of code 401 (unauthorized). The occurrence of code 403 (forbidden) is due to the basic events E_4 (Server Policy) and E_5 (No other response available). Similarly the server error frequently occurs due to a basic event E_6 (Internal server error) or an intermediate event code 501 (service unavailable). The intermediate event code 501 is caused by the basic events E_7 (Unable to understand the request) or E_8 (Does not support the functionality.

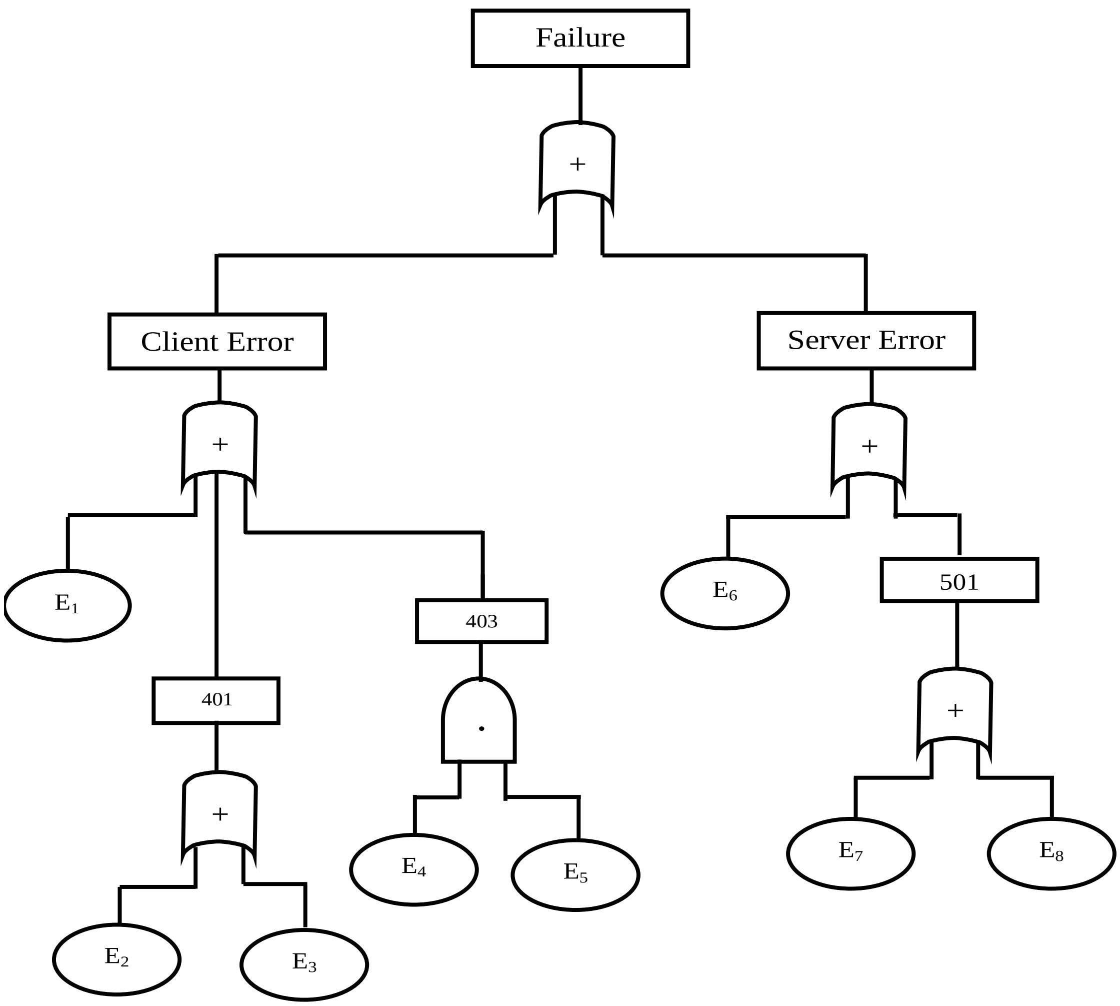

Figure 6. Fault Tree Structure

The basic events E_1, E_2, E_3, E_3 E_4, E_5, E_6, E_7 and E_8 are represented by the following triangular intuitionistic fuzzy sets.

$$E_1 =< \left[(0.0809,\ 0.4064,\ 0.8128,);\ 0.88\right], \left[(0.0603,\ 0.4064,\ 0.923,);\ 0.04\right] >$$

$$E_2 =< \left[(0.0055,\ 0.0273,\ 0.0546,);\ 0.87\right], \left[(0.0043,\ 0.0273,\ 0.0647,);\ 0.06\right] >$$

$$E_3 =< \left[(0.0055,\ 0.0273,\ 0.0546,);\ 0.86\right], \left[(0.0043,\ 0.0273,\ 0.0647,);\ 0.05\right] >$$

$$E_4 =< \left[(0.0224,\ 0.1118,\ 0.2236,);\ 0.87\right], \left[(0.0123,\ 0.1118,\ 0.3235,);\ 0.04\right] >$$

$$E_5 =< \left[(0.0224,\ 0.1118,\ 0.2236,);\ 0.88\right], \left[(0.0123,\ 0.1118,\ 0.3235,);\ 0.05\right] >$$

$$E_6 =< \left[(0.0603,\ 0.3016,\ 0.6032,);\ 0.85\right], \left[(0.0502,\ 0.3016,\ 0.7252,);\ 0.04\right] >$$

$$E_7 =< \left[(0.0019,\ 0.0056,\ 0.0168,);\ 0.88\right], \left[(0.0009,\ 0.0056,\ 0.0268,);\ 0.05\right] >$$

$$E_8 =< \left[(0.0019,\ 0.0056,\ 0.0168,);\ 0.89\right], \left[(0.0009,\ 0.0056,\ 0.0268,);\ 0.05\right] >$$

Using the operations on intuitionistic fuzzy sets, we get the failure possibility of intermediate events **401, 403** and **501** are as follows.

$$401 =< \left[(0.01097,\ 0.053855,\ 0.106219,);\ 0.86\right], \left[(0.008582,\ 0.053855,\ 0.125214,);\ 0.06\right] >$$

$$403 =< \left[(0.000502,\ 0.012499,\ 0.049997,);\ 0.87\right], \left[(0.000151,\ 0.012499,\ 0.104652,);\ 0.05\right] >$$

$$501 =< \left[(0.003796,\ 0.011169,\ 0.033318,);\ 0.88\right], \left[(0.001799,\ 0.011169,\ 0.052882,);\ 0.05\right] > .$$

Using the above calculated values for the failure possibilities of the intermediate events 401, 403 and 501, and the given value of failure possibility of E_1 and E_6, we obtain the failure possibility of client error and server error.

$$Server\,Error =< \left[(0.063867,\ 0.3094,\ 0.61642,);\ 0.85\right], \left[(0.051909,\ 0.3010,\ 0.739732,);\ 0.05\right] >$$

$$ClientError =< \left[(0.091438,\ 0.445388,\ 0.841049,);\ 0.86\right], \left[(0.068505,\ 0.445388,\ 0.939691,);\ 0.06\right] > .$$

Finally, using the failure possibility of client error and server error, the failure possibility of top event(Failure of Server) is computed.

$$Server\ Failure =< \left[(0.149466,\ 0.616985,\ 0.93903);\ 0.85\right], \left[(0.116858,\ 0.616985,\ 0.984303,);\ 0.06\right] > .$$

7. Result and Discussions

The triangular intuitionistic fuzzy set representing failure possibility of a server failure is,

$$Server\ Failure =< \left[(0.149466,\ 0.616985,\ 0.93903,);\ 0.85\right], \left[(0.116858,\ 0.616985,\ 0.984303,);\ 0.06\right] >$$

The resulting triangular intuitionistic fuzzy set as shown in Figure 7 for server failure can be explained as follow:

(i) The failure possibility of a server failure lies between 0.149466 and 0.939030.

(ii) 85% experts are in favor that the failure possibility of server failure is 0.616985

(iii) 6% experts are opposing that the failure possibility of server failure is 0.616985

(iv) 9% experts are in confusion that the failure possibility of server failure is 0.616985

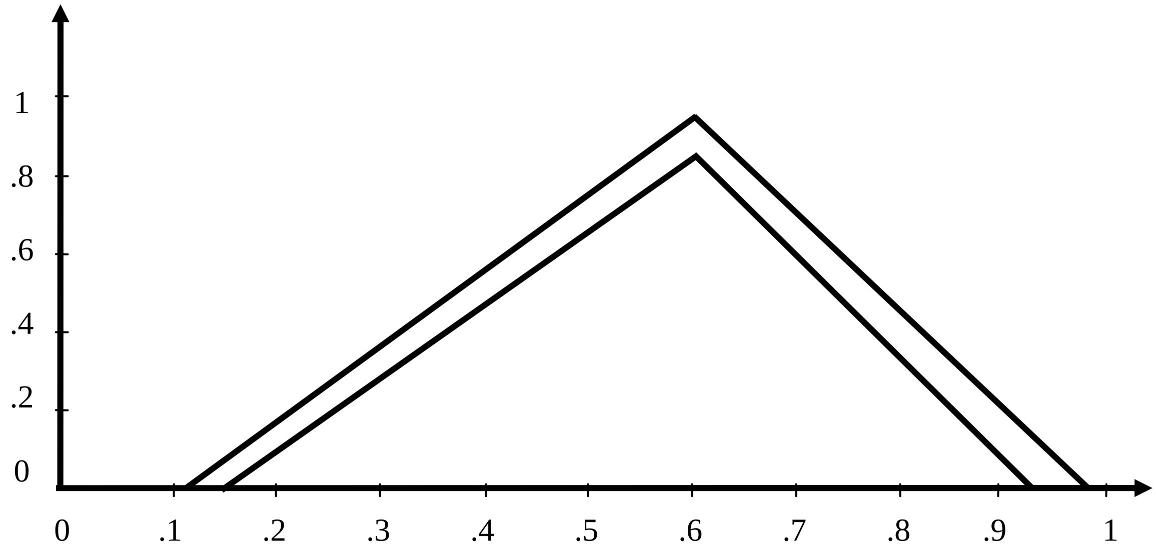

Figure 7. Failure Possibility of Top Event

8. Conclusion

Many authors used the concept of fuzzy set to deal with uncertainty inherited with the source of data used for failure possibility of basic events in fault tree analysis. But for more realistic results it is inevitable to apply the notion of intuitionistics fuzzy sets in a situation, wherein hesitation between belongingness and non-belongingness cannot be ruled out. Here, the failure of a server is modeled as a combination of different basic and intermediate events. And the triangular intuitionistic fuzzy sets are employed to represent the basic events. Using the operations on intuitionistic fuzzy sets to evaluate the possibility of series and parallel system, the failure possibility of the top event is obtained. Since intuitionistic fuzzy set represent the positive and negative evidence of membership of an element in the set. The possibility of the top event occurring as an intuitionistic triangular fuzzy set, gives a significant boost to our standpoint.

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