Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 6, Issue 2 (December 2011), pp. 510 – 521

# Stability of Stratified Couple-Stress Dusty Fluid in the Presence of Magnetic Field through Porous Medium

**Vivek Kumar**

Department of Mathematics
College of Engineering Studies
University of Petroleum & Energy Studies (UPES)
P.O. Bidholi
Dehradun-248007, (Uttarakhand), India
vivek_shrawat@yahoo.co.in; vivek.shrawat@gmail.com

Received: April 06, 2011; Accepted: November 15, 2011

## Abstract

The combined effect of magnetic field and dust particles on the stability of a stratified couple-stress fluid through a porous medium is considered. For stable stratification, the system is found to be stable for disturbances of all wave numbers. The magnetic field succeeds in stabilizing the potentially unstable stratifications for a certain wave-number range which were unstable in the absence of the magnetic field. Discussions of oscillatory and non-oscillatory modes are also made.

**Keywords:** Stratified couple-stress fluid, dust particles, magnetic field

**MSC 2010 No.:** 76E06, 76E07, 76E15, 76U05

## 1. Introduction

Several authors have studied the instability of a two plane interface separating two Newtonian fluids where one is accelerated towards the other or when one is superposed over the other. A comprehensive account of the problem of Rayleigh-Taylor instability was studied by

Chandrasekhar (1981). Rayleigh (1883) demonstrated that the system is stable or unstable according as the density decreases everywhere or increases anywhere. The stability of a horizontal layer of an electrically conducting fluid with continuous density and viscosity stratification in the presence of a horizontal magnetic field is discussed by Gupta (1963). Kumar (2000) have studied the problem of Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluid in the presence of suspended particles through porous medium and found that in the case of two uniform elastico-viscous fluids separated by a horizontal boundary and exponentially varying density, the perturbation decay with time for potentially stable configuration/stable stratification and grow with time for potentially unstable configuration/unstable stratification.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigation of such fluids is desirable. Stokes (1966) has formulated the theory of couple-stress fluid. The theory of Stokes (1966) allows for polar effects such as the presence of couple-stress and body couples and has been applied to the study of some simple lubrication problems. According to Stokes, couple-stresses appear in fluids with very large molecules since the long chain hyaluronic acid molecules are found as additives in synovial fluids. In recent years, the problems of the fluid flow through a porous medium have grown in importance due to the recovery of crude oil from the pores of reservoir rocks. When a fluid layer flows through a porous medium, the gross effect is represented by Darcy's law. As a result, the usual viscous and couple-stress viscous terms in equation of motion are replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\nabla^2\right)q\right]$, where $\mu$ and $\mu'$ are the viscosity and couple-stress viscosity respectively. Recently, Sunil, Sharma and Chandel (2002) have studied the Rayleigh-Taylor instability of two superposed couple-stress fluids of uniform densities in a porous medium in the presence of a uniform horizontal magnetic field and found that the magnetic field stabilizes a certain wave number range $k > k^*$, which is unstable in the absence of the magnetic field. Kumar and Abhilasha (2009) have studied the stability of two superposed Rivlin-Ericksen viscoelastic dusty fluids in the presence of magnetic field and found that the magnetic field succeeds in stabilizing the potentially unstable stratifications for a certain wave number range which were unstable in the absence of the magnetic field. Kumar et.al (2009) has studied the problem of thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field.

Recent spacecraft observations have confirmed that the dust particles play an important role in the dynamics of the atmosphere as well as in the diurnal and surface variations in the temperature of the Martin weather. It is, therefore, of interest to study the presence of dust particles in astrophysical situations. Sharma and Sharma (2004) have studied the effect of suspended particles on couple-stress fluids heated from below in the presence of rotation and magnetic field and found that dust particles have a destabilizing effect on the system. The instability of two rotating viscoelastic fluids with suspended particles in a porous medium is considered by Kumar and Singh (2007). Kumar and Kumar (2010) have studied the problem on a couple-stress fluid heated from below in hydromagnetics and found that the magnetic field has a stabilizing effect under a condition while dust particles have a destabilizing effect on the system. Kumar and Singh (2010) have studied the problem on the stability of two stratified Walters B' viscoelastic superposed fluids and found that, for the stable stratifications, the system is found to be stable or unstable under certain conditions. Singh and Dixit (2010) have discussed the stability of stratified Oldroydian fluid in hydromagnetics in the presence of suspended particles in a porous medium and found that the magnetic field succeeds in stabilizing wave numbers in the given range.

Keeping in mind the importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and the petroleum industry, we propose to study the stability of stratified Stokes (1966) incompressible couple-stress dusty fluid in the presence of a magnetic field through a porous medium.

## 2. Formulation of the Problem

Consider a static state in which an incompressible Stokes couple-stress fluid layer containing dust particles of variable density is arranged in horizontal strata and the pressure $p$, density $\rho$, viscosity $\mu$ and viscoelasticity $\mu'$ are functions of vertical coordinate $z$ only. The fluid layer is under the action of gravity $\mathbf{g}(0,0,-g)$ and the horizontal magnetic field $\mathbf{H}(H,0,0)$. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\varepsilon$ and medium permeability $k_1$. The particles are assumed to be non-conducting.

Let $\mathbf{q}(u,v,w)$, $\rho$ and $p$ denote respectively the velocity, density and pressure of the hydromagnetic fluid. $\mathbf{q}_d(\bar{x},t)$ and $\mathbf{N}(\bar{x},t)$ denote the velocity and number density of particles, respectively. $K=6\pi\mu\eta$, where $\eta$ particle radius, is a constant and $\bar{x}=(x,y,z)$. Then the equation of motion and continuity for the Stokes (1966) couple-stress fluid are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\nabla p+\mathbf{g}\rho-\frac{\rho}{k_1}\left(\nu-\nu'\nabla^2\right)\mathbf{q}+\frac{KN}{\varepsilon}(\mathbf{q}_d-\mathbf{q})+\frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right], \tag{1}$$

$$\nabla.\mathbf{q}=0, \tag{2}$$

$$\varepsilon\frac{\partial \rho}{\partial t}+(\mathbf{q}.\nabla)\rho=0, \tag{3}$$

$$\varepsilon\frac{d\mathbf{H}}{dt}=(\mathbf{H}.\nabla)\mathbf{q}-(\mathbf{q}.\nabla)\mathbf{H} \tag{4}$$

and

$$\nabla.\mathbf{H}=0, \tag{5}$$

where $\mu_e$, the magnetic permeability is assumed to be constant and the fluid is assumed to be infinitely conducting.

The presence of particles adds an extra force term, proportional to the velocity difference between particles and appears in equations of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to the exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large as compared to their diameters. The equations of motion and continuity for the particles [Kumar (2000)], under the above approximation, are

$$mN\left[\frac{\partial \mathbf{q}_d}{\partial t}+(\mathbf{q}_d.\nabla)\mathbf{q}_d\right]=KN(\mathbf{q}-\mathbf{q}_d) \tag{6}$$

and

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.(N\mathbf{q}_d)=0 \tag{7}$$

where $mN$ is the mass of the particles per unit volume.

## 3. Perturbation Equations and Normal Mode Analysis

The time independent solution of (1) to (7) known as the basic state, whose stability we wish to examine is that of an incompressible, couple-stress fluid layer of variable density arranged in horizontal strata. The basic motionless solution is

$$\mathbf{q}=(0,0,0)\,,\ \mathbf{q}_d=(0,0,0),\ N=N_0=\text{Constant}.$$

The character of equilibrium is examined by supposing that the system is slightly disturbed and then by following its further evolution.

Let $\delta\rho, \delta p, \mathbf{q}(u,v,w), \mathbf{q}_d(l,r,s)$ and $\mathbf{h}(h_x,h_y,h_z)$ denote respectively the perturbations in the hydromagnetic fluid density $\rho$, pressure $p$, velocity $\mathbf{q}(0,0,0)$, particles velocity $\mathbf{q}_d(0,0,0)$ and the magnetic field $\mathbf{H}(H,0,0)$. Then the linearized perturbation equations are

$$\frac{\rho}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}=-\nabla\delta p+\mathbf{g}\delta\rho-\frac{\rho}{k_1}\left(\nu-\nu'\nabla^2\right)\mathbf{q}+\frac{KN_0}{\varepsilon}(\mathbf{q}_d-\mathbf{q})+\frac{\mu_e}{4\pi}[(\nabla\times\mathbf{h})\times\mathbf{H}], \tag{8}$$

$$\nabla.\mathbf{q}=0\,, \tag{9}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho=-w(D\rho)\,, \tag{10}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\mathbf{q}_d=\mathbf{q}\,, \tag{11}$$

$$\frac{\partial M}{\partial t}+\nabla.\mathbf{q}_d=0\,, \tag{12}$$

$$\varepsilon\frac{\partial \mathbf{h}}{\partial t}=(\mathbf{H}.\nabla)\,\mathbf{q} \tag{13}$$

and

$$\nabla.\mathbf{h} \;=\; 0\,, \tag{14}$$

where $M=\varepsilon N/N_0$ and $N, N_0$ respectively stands for the initial uniform number density and the perturbation in the number density.

Analyzing the perturbations into normal modes, we seek the solution whose dependence on $x, y$ and $t$ is given by

$$\exp(ik_x x+ik_y y+nt)\,, \tag{15}$$

where, $k_x$ and $k_y$ are the horizontal components of the wave number, $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number and $n$ is the growth rate, which is, in general, a complex constant.

With the dependence of physical variables on $x, y$ and $t$ and following the usual procedure, we get

$$\frac{n}{\varepsilon}[D(\rho Dw)-\rho k^2 w]+\frac{1}{k_1}\Big[D\left\{\mu-\mu'(D^2-k^2)\right\}Dw-k^2\left\{\mu-\mu'(D^2-k^2)\right\}w\Big]$$

$$+\frac{gk^2}{\varepsilon n}(D\rho)w+\frac{n}{\varepsilon(1+\tau n)}[D(mN_0Dw)-mN_0k^2w]\;+\frac{\mu_e H^2k_x^2}{4\pi\varepsilon n}(D^2-k^2)w=0\,, \tag{16}$$

where

$$\tau=m\,/\,K \;\text{ and}\, D=d\,/\,dz\,.$$

**Case of Exponentially Varying Density, Viscosity, Viscoelasticity, Magnetic Field and Particle Number Density**

Let us assume that

$$\rho = \rho_0 e^{\beta z},\ N_0 = N_1 e^{\beta z},\ \mu = \mu_0 e^{\beta z},\ H^2 = H_1^2 e^{\beta z} \text{ and } \mu' = \mu_0' e^{\beta z}, \tag{17}$$

where $\rho_0, N_1, \mu_0, H_1, \mu_0'$ and $\beta$ are constants. Substituting the values of $\rho, N_1, \mu, \mu'$ and $H$ in equation (16), we obtain

$$\left[n + \frac{mnN_1}{\rho_0(1+\tau n)} + \frac{\varepsilon}{k_1}(v_0 - v_0'(D^2 - k^2)) + \frac{k_x^2 V_A^2}{n}\right](D^2 - k^2)w + \frac{g\beta k^2}{n}w = 0, \tag{18}$$

where

$$v_0 = \frac{\mu_0}{\rho_0},\ v_0' = \frac{\mu_0'}{\rho_0} \text{ and } V_A^2 = \frac{\mu_e H^2}{4\pi}.$$

We consider the case of two free boundaries. Let us assume that $\beta << 1$, i.e., the variation of density at two neighboring points in the velocity field which is much less than the average density has a negligible effect on the inertia of the fluid. The boundary conditions for the case of two free surfaces are

$$w = D^2 w = 0 \quad \text{at} \quad z = 0 \text{ and } z = d. \tag{19}$$

The proper solution of equation (18) satisfying equation (19) is given by

$$w = A \sin\frac{s\pi z}{d}, \tag{20}$$

where $A$ is a constant and $s$ is any integer. Using equation (20), equation (18) gives

$$\tau n^3 + n^2\left[1 + \frac{mN_1}{\rho_0} + \frac{\tau\varepsilon}{k_1}(v_0 + v_0' L)\right] + n\left[\frac{\varepsilon}{k_1}(v_0 + v_0' L) + \tau\left(k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right)\right] + \left[k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right] = 0, \tag{21}$$

where

$$L = \left(\frac{s\pi}{d}\right)^2 + k^2.$$

## 4. Results and Discussion

**Theorem 1:** For stable density stratification $(\beta < 0)$, the system is always stable.

***Proof:***

For stable stratification $(\beta < 0)$, the roots of the equation (21) are either real or negative or one root is real and negative and two roots are complex conjugate with negative real parts. In each case, the system is stable for disturbances of all wave numbers.

**Theorem 2:** For $\beta > 0$, the system is stable or unstable provided $k_x^2 V_A^2 >$ or $< \dfrac{g\beta k^2}{L}$.

***Proof:***

If $\beta > 0$ and $k_x^2 V_A^2 > \dfrac{g\beta k^2}{L}$, then equation (21) does not involve any change of sign and so does not admit any positive value of real part of $n$. Therefore, the system is stable for disturbances of all wave numbers. On the other hand, if $\beta > 0$ and $k_x^2 V_A^2 < \dfrac{g\beta k^2}{L}$, then the constant term in equation (21) is negative. Therefore, allow at least one change of sign and so has at least one positive root. The occurrence of a positive root implies that the system is unstable.

## 5. Discussion of Oscillatory Modes

Equation (21) can be written as

$$A_1 n^3 + A_2 n^2 + A_3 n + A_4 = 0, \tag{22}$$

where

$$A_1 = \tau,\ \ A_2 = 1 + \frac{mN_1}{\rho_0} + \frac{\tau\varepsilon}{k_1}(v_0 + v_0' L),\ \ A_3 = \frac{\varepsilon}{k_1}(v_0 + v_0' L) + \tau\left(k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right)$$

and

$$A_4 = k_x^2 V_A^2 - \frac{g\beta k^2}{L}.$$

After dividing by $n$, the real and imaginary parts of equation (22) are

$$A_1(n_r^2 - n_i^2) + A_2 n_r + A_3 + \frac{A_4 n_r}{|n|^2} = 0 \tag{23}$$

and

$$n_i\left[2A_1 n_r + A_2 - \frac{A_4}{|n|^2}\right] = 0. \tag{24}$$

**Theorem 3:** For $\beta < 0$, the estimate of $n$ for the growth rate of oscillatory stable modes is given by $|n|^2 > \frac{A_4}{A_2}$.

***Proof:***

If $\beta < 0$, then the value of $A_2$ and $A_4$ are definite positive. Since modes are oscillatory $(n_i \neq 0)$ and if $n_r$ is negative (for stable mode), then for the consistency of equation (24), we must have $|n|^2 > \frac{A_4}{A_2}$. Hence, for $\beta < 0$, the estimate of $n$ for the growth rate of oscillatory stable modes is given by $|n|^2 > \frac{A_4}{A_2}$.

**Theorem 4:** For $\beta < 0$, the estimate of $n$ for the growth rate of oscillatory unstable modes is given by $|n|^2 < \frac{A_4}{A_2}$.

***Proof:***

If $\beta < 0$, then the value of $A_2$ and $A_4$ are definite positive. Since the modes are oscillatory $(n_i \neq 0)$ and if $n_r$ is positive (for unstable mode), then for the consistency of equation (24), we must have $|n|^2 < \frac{A_4}{A_2}$. Hence, for $\beta < 0$, the estimate of $n$ for the growth rate of oscillatory unstable modes is given by $|n|^2 < \frac{A_4}{A_2}$.

**Theorem 5:** For $\beta > 0$ and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$, the estimate of $n$ for the growth rate of oscillatory stable or unstable modes are respectively given by $|n|^2 > \frac{A_4}{A_2}$ or $|n|^2 < \frac{A_4}{A_2}$.

***Proof:***

If $\beta > 0$ and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$, the value of $A_1$, $A_2$ and $A_4$ are positive definite. Since modes are oscillatory $(n_i \neq 0)$ and stable $(n_r < 0)$, then equation (24) gives $|n|^2 > \frac{A_4}{A_2}$. Also, for oscillatory unstable modes $(n_i \neq 0,\ n_r > 0)$, we must have for the consistency of equation (24) as $|n|^2 < \frac{A_4}{A_2}$ under the given conditions.

## 6. Discussion of Non-Oscillatory Modes

For non-oscillatory modes, we must have $n_i = 0$, then equation (23) becomes

$$A_1 n_r^3 + A_2 n_r^2 + A_3 n_r + A_4 = 0, \tag{25}$$

where

$$A_1 = \tau,\ \ A_2 = 1 + \frac{mN_1}{\rho_0} + \frac{\tau\varepsilon}{k_1}(v_0 + v_0' L),\ \ A_3 = \frac{\varepsilon}{k_1}(v_0 + v_0' L) + \tau\left(k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right)$$

and

$$A_4 = k_x^2 V_A^2 - \frac{g\beta k^2}{L}.$$

**Theorem 6:** For $\beta < 0$, the non-oscillatory modes are always stable.

***Proof:***

If $\beta < 0$, then equation (25) does not involve any change of sign and therefore does not allow any positive value of real part $n_r$. Therefore, the non-oscillatory modes are stable for all wave numbers.

**Theorem 7:** For $\beta > 0$, the non-oscillatory modes are stable provided $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$.

***Proof:***

For $\beta > 0$ and $k_x^2 V_A^2 > \dfrac{g\beta k^2}{L}$, equation (25) does not involve any change of sign and therefore does not allow any positive of real part $n_r$. Therefore, the non-oscillatory modes are stable.

**Theorem 8:** For $\beta > 0$, the non-oscillatory modes are unstable provided $k_x^2 V_A^2 < \dfrac{g\beta k^2}{L}$.

***Proof:***

For $\beta > 0$ and $k_x^2 V_A^2 < \dfrac{g\beta k^2}{L}$, the value of $A_4$ is negative. Therefore, equation (25) involves at least one change of sign so has at least one positive root. Therefore, the non-oscillatory modes are unstable.

**Theorem 9:** For $\beta > 0$ and $k_x^2 V_A^2 < \dfrac{g\beta k^2}{L}$, there are wave propagating for a given wave number.

***Proof:***

Let the roots of equation (25) are $n_{r_1}$, $n_{r_2}$, $n_{r_3}$, then using the theory of equations, we get

$$n_{r_1}.n_{r_2}.n_{r_3} = -\frac{A_4}{A_1} > 0$$

and

$$n_{r_1} + n_{r_2} + n_{r_3} = -\frac{A_2}{A_1} < 0.$$

Clearly, when $\beta > 0$ and $k_x^2 V_A^2 < \dfrac{g\beta k^2}{L}$, then $A_4$ is definite negative. Also, $A_1$ and $A_2$ are positive definite. So the product of the roots is positive and the sum of the roots is negative. Therefore, the possibility that all the three non-oscillatory modes can be unstable is ruled out. It follows that two waves of propagation are damped and one is amplified for a given wave number.

## 7. Conclusion

The stability of superposed fluids under varying assumptions of hydromagnetics has been discussed in detail by Chandrasekhar (1981). With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on couple-stress fluid are desirable. In the present paper, the stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium is considered. For stable stratification, the system is found to be stable for disturbances of all wave numbers. The magnetic field succeeds in stabilizing the potentially unstable stratifications for a certain wave-number range which were unstable in the absence of the magnetic field. It is also found that for $\beta < 0$ the non-oscillatory modes are always stable and for $\beta > 0$ the non-oscillatory modes are stable or unstable under certain conditions.

## NOTATIONS

| | |
|---|---|
| $\rho$ | Density of fluid, |
| $\mu$ | Coefficient of viscosity, |
| $\mu'$ | Coefficient of viscoelasticity, |
| $\mu_e$ | Magnetic permeability, |
| $\partial$ | Curly operator, |
| $\delta$ | Perturbation in respective physical quantity, |
| $\nabla$ | Del operator, |
| $\beta$ | Constant, |
| $\nu$ | Kinematic viscosity $(\mu / \rho)$ , |
| $\nu'$ | Kinematic viscoelasticity, $(\mu'/ \rho)$ |
| $p$ | Fluid pressure, |
| $g(0,0,\mathbf{g})$ | Acceleration due to gravity, |
| $\mathbf{H}(H,0,0)$ | Magnetic field vector having components $(H,0,0)$, |
| $\delta\rho$ | Perturbation in density $\rho(z)$, |
| $\delta p$ | Perturbation in pressure $p(z)$, |
| $\mathbf{q}\ (u,v,w)$ | Perturbation in fluid velocity $\mathbf{q}(0, 0, 0)$, |
| $\mathbf{q}_d\ (l,r,s)$ | Perturbations in particle velocity $\mathbf{q}_d\,(0, 0, 0)$ |
| $\mathbf{h}(h_x, h_y, h_z)$ | Perturbation in magnetic field $\mathbf{H}(H,0,0)$, |
| $k_x, k_y$ | Wave numbers in $x$ and $y$ directions respectively, |
| $k = \sqrt{k_x^2 + k_y^2}$ | Wave number of the disturbance, |
| $n$ | Growth rate of disturbance, |
| $V_A^2$ | Square of the Alfven velocity $\left( V_A^2 = \dfrac{\mu_e H^2}{4\pi} \right)$, |
| $\rho_0, \nu_0, \nu_0', \mu_0, \mu_0', N_0$ | Constants, |
| $\pi$ | Constant value, |
| $D$ | Derivative with respect to $z \left( = \dfrac{d}{dz} \right)$. |

## *Acknowledgement*

*The author is grateful to the referees for their technical comments and valuable suggestions, resulting in a significant improvement of this paper.*

## REFERENCES

Chandrasekhar, S. (1981). Hydrodynamic and Hydromagnetic stability, Dover Publications, New York.

Gupta, A.S. (1963). Rayleigh-Taylor instability of viscous electrically conducting fluid in the presence of a horizontal magnetic field, J. Phys. Soc. Japan, **18**, 1073.

Kumar, P. (2000). Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluids in presence of suspended particles through porous medium, Indian J. Pure Appl. Math., **31**(5), 533-539.

Kumar, P. and Singh, G.J. (2010). On the stability of two stratified Walters B' viscoelastic superposed fluids, Studia Geotechnica et Mechanica, XXXII(4), 29-37.

Kumar, P. and Singh, M. (2007). Instability of two rotating viscoelastic fluids with suspended particles in porous medium, BIBLID, 11(1), 93-102.

Kumar, V. and Abhilasha (2009). Stability of two superposed Rivlin-Ericksen viscoelastic dusty fluids in the presence of magnetic field, Journal of Informatics and Mathematical Sciences, 1(2), 99-112

Kumar, V. and Kumar, S. (2010). On a couple-stress fluid heated from below in hydromagnetics, Applications and Applied Mathematics, 5(10), 1529-1542.

Kumar, V., Abhilasha and Pundir, S. (2009). Thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field, International Transactions in Applied Sciences, 1(1), 113-124.

Rayleigh, L. (1883). Investigation of the character of an incompressible heavy fluid of variable density, Proc. London Math. Soc., **14**, 170.

Sharma, R. C. and Sharma, M. (2004). Effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field, Indian J. Pure and Appl. Math., 35(8), 973-989.

Singh, V. and Dixit, S. (2010). Stability of stratified Oldroydian fluid in hydromagnetics in presence of suspended particles in porous medium, Studia Geotechnica et Mechanica, XXXII(4), 55-65.

Stokes, V.K. (1966). Phys. Fluids, **9**, 1709.

Sunil, Sharma R. C. and Chandel R. S. (2002). On superposed couple-stress fluid in a porous medium in hydromagnetics, Z. Naturforch., 57a, 955-60.