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Linear Stability of Thermosolutal Convection in a Micropolar Fluid Saturating a Porous Medium

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Abstract

In the present paper, the theoretical investigation of the double-diffusive convection in a micropolar fluid layer heated and soluted from below saturating a porous medium is considered. For a flat fluid layer contained between two free boundaries, an exact solution is obtained. A linear stability analysis theory and normal mode analysis method have been used. For the case of stationary convection, the effect of various parameters like medium permeability, solute gradient and micropolar parameters (i.e., coupling parameter, spin diffusion parameter, micropolar heat conduction parameter and micropolar solute parameter arises due to coupling between spin and solute fluxes) has been analyzed and found that medium permeability, spin diffusion and micropolar solute parameter has destabilizing effect under certain conditions, whereas stable solute gradient, micropolar coupling parameter and micropolar heat conduction parameter has stabilizing effect on the system under certain conditions. The critical thermal Rayleigh number and critical wave numbers for the onset of instability are also determined numerically and results are depicted graphically. It is found that the oscillatory modes are introduced due to the presence of the micropolar viscous effects, microinertia and stable solute gradient, which were non-existence in their absence. The principle of exchange of stabilities is found to hold true for the micropolar fluid saturating a porous medium heated from below in the absence of micropolar viscous effect, microinertia and stable solute gradient. An attempt is also made to obtain sufficient conditions for the non-existence of overstability.

Keywords: Thermosolutal Convection; Porous Medium; Micropolar Fluids; Medium Permeability; Rayleigh Number

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1. Introduction

A general theory of micropolar fluids has been presented by Eringen (1964, 1966, 1980). Compared to the classical Newtonian fluids, micropolar fluids are characterised by two supplementary variables, i.e., the spin, responsible for the micro-rotations and the micro-inertia tensor describing the distributions of atoms and molecules inside the fluid elements in addition to the velocity vector. Liquid crystals, colloidal fluids, polymeric suspension, animal blood can be modeled using micropolar fluids [Labon and Perez-Garcia (1981)]. Kazakia and Ariman (1971) and Eringen (1972) extended this theory of structure continue to account for the thermal effects. A large number of references about modelling and applications aspect of micropolar fluids has been given in the book (1999). The literature concerning applications of micropolar fluids in engineering sciences is vast and still growing.

The theory of thermomicropolar convection has been studied by many authors [Datta and Sastry (1976), Ahmadi (1976), Bhattacharya and Jena (1983), Payne and Straughan (1989), Sharma and Kumar (1995,1997), Rama Rao (1980)]. They give a good understanding of thermal convection in micropolar fluids. The Rayleigh-Benard instability in a horizontal thin layer of fluid heated from below is an important particular stability problem. A detailed account of Rayleigh-Benard instability in a horizontal thin layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar (1981). Perez-Garcia et al. (1981) have extended the effects of the microstructures in the Rayleigh-Benard instability and have found that in the absence of coupling between thermal and micropolar effects, the Principle of Exchange of Stabilities (PES) holds good. Perez-Garcia and Rubi1(1982) have shown that when coupling between thermal and micropolar effect is present, the Principle of Exchange of Stabilities (PES) may not be fulfilled and hence oscillatory motions are present in micropolar fluids. The medium has been considered to be non-porous in all the above studies. In recent years, there has been a lot of interest in study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of convective flow.

The study of flow of fluids through porous media is of considerable interest due to its natural occurrence and importance in many problems of engineering and technology such as porous bearings, porous layer insulation consisting of solid and pores, porous rollers, etc. In addition, these flows are applicable to bio-mathematics particularly in the study of blood flow in lungs, arteries, cartilage and so on. The study of a layer of a fluid heated from below in porous media is motivated both theoretically as also by its practical applications in engineering. Among the applications in engineering disciplines, one find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood (1948) and Wooding (1960). Recent studies of stellar atmosphere has shown the existence and importance of porosity in astrophysical context [McDonnel (1978)]. A comprehensive review of the literature concerning convection in porous medium is available in the book of Nield and Bejan (2006).

The thermoconvective instability in a micropolar fluid saturating a porous medium has been studied by Sharma and Gupta (1995) and the effect of rotation on thermal convection in micropolar fluids in porous medium has been considered by Sharma and Kumar (1998). Siddheshwar and Krishna (2003) have studied the linear and non-linear analysis of convection in a micropolar fluid occupying a porous medium. More recently, Reena and Rana (2008a) and Mittal and Rana (2008) have studied some of the thermal convection problems in micropolar rotating fluid saturating a porous medium. Sunil et al. (2006) have studied the effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. All of them found that the rotation has a stabilizing effect.

The interesting situation arises from both a geophysical and a mathematical point of view when the layer is simultaneously heated from below and salted from below. The buoyancy force can arise not only from density difference due to variations in temperature but also those due to variations in solute concentration. Brakke (1955) explained a double diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Themosolutal convection problems arise in oceanography, limnology and engineering. Double-diffusive convection in fluids in porous media is also of interest in geophysical systems, electrochemistry, metallurgy, chemical technology, geophysics and biomechanics, soil sciences, astrophysics, ground water hydrology. Examples of particular interest are provided by ponds built to trap solar heat [Tabor and Matz (1965)] and some Antarctic lakes [Shirtcliffe (1964)]. Particularly, the case involving a temperature field and sodium chloride referred to a thermohaline convection. Veronis (1965) has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. For three or greater field case, it is referred to as multi-component convection. There are many recent studies involving three or more fields, such as temperature and two salts such as NaCl, KCl. O'Sullivan et al. (2001) have reviewed and studied numerical techniques and their applications in geothermal reservoir simulation. The Salton sea geothermal system in southern California is specifically interesting as it involves convection of hypersaline fluids. For example, Oldenburg and Pruess (1998) have developed a model for convection in a Darcy's porous medium, to model the Salton geothermal system, where the mechanism involves temperature, NaCl, $CaCl_2$ and KCl. Other applications include the oceans, the Earth's magma. Drainage in a mangrove system is yet another area enclosing double-diffusive flows. Solar ponds are a specifically promising means of harnessing energy from the sun by preventing convective overturning in a thermohaline system by salting from below. The survey of double-diffusive convection in porous medium given in third edition (2006) of Nield and Bejan. Sharma and Sharma (2000) have studied the thermosolutal convection of micropolar fluids in hydromagnetics in porous medium. They found that Rayleigh number increases with magnetic field and solute parameter. The thermosolutal convection in a ferromagnetic fluid in porous and non-porous medium has been considered by Sunil et al. (2004,2005) and the double-diffusive convection in a micropolar ferromagnetic fluid in porous and non-porous medium has been studied by Sunil et al. (2007a, b). They found the stabilizing effect of stable solute gradient. The driving force for many studies in double-diffusive or multi-component convection has largely physical applications. More recently, Reena and Rana (2008b) have studied the thermosolutal convection of micropolar fluids in porous medium in the presence of rotation. They found the stability effect of rotation in the presence of salinity. Thus, the study of thermosolutal convection in porous medium in a fluid is of great importance.

In view of the above investigations and keeping in mind the usefulness of double-diffusive convection of micropolar fluids saturating a porous medium in various fields, the present problem deals with the linear stability of thermosolutal convection in micropolar fluid saturating a porous medium. It is attempted to discuss the effect of solute gradient and how micropolar parameters affects the stability in micropolar fluid heated and soluted from below saturating a porous medium of very low permeability using generalized Darcy's model [Walker and Honsy (1977)] including the inertial forces. The present problem, to the best of our knowledge, has not been investigated yet and can serve as a theoretical support for an experimental investigation.

2. Mathematical Formulation of the Problem

Here, we consider an infinite horizontal layer of thickness d of an incompressible thin micropolar fluid heated and soluted from below saturating a porous medium. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1 and acted on by a gravity field $\mathbf{g} = (0, 0, -g)$. The temperature T and solute concentration C at the bottom and top surfaces $Z = 0$ and $Z = d$ are T_0, T_1 and C_0, C_1, respectively, and a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a solute concentration gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained (see Figure 1). The temperature gradient thus maintained is qualified as adverse since, on the account of thermal expansion, the fluid at the bottom will be higher than the fluid at the top, and this is a top heavy arrangement, which is potentially unstable. On the other hand, the heavier salt at the lower part of the layer has exactly the opposite effect and this acts to prevent motion through convection overturning. Thus, these two physical effects are competing against each other. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid. Here, both the boundaries are taken to be free and perfect conductors of heat. Here, the porosity is defined as the fraction of the total volume of the medium that is occupied by void space. Thus $1-\varepsilon$ is the fraction that is occupied by solid. For an isotropic medium, the surface porosity (i.e., the fraction of void area to total area of a typical cross section) will normally be equal to ε. Here, we adopt the Boussinesq approximation [Chandrasekhar (1981)] which implies that the density can be treated as constant everywhere except when multiplied by gravity. When the fluid flows through a porous medium, the gross effect is represented by Darcy's law.

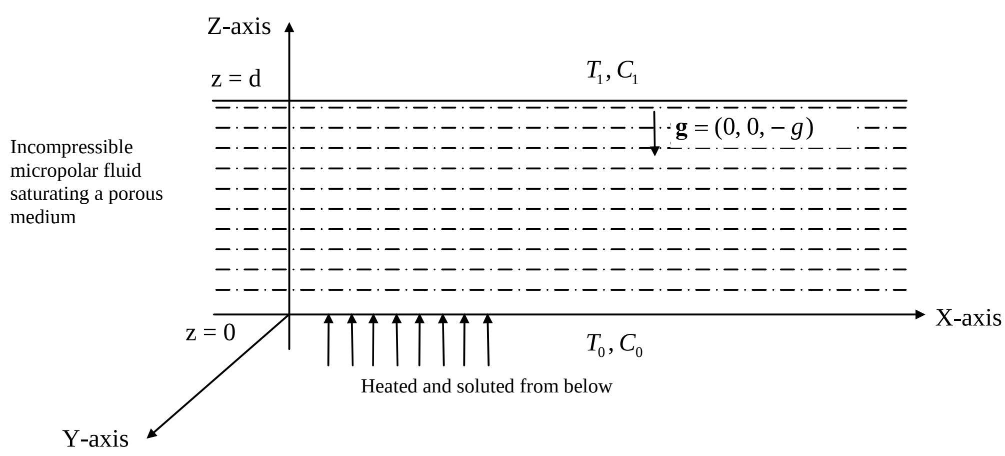

Figure 1. Geometrical configuration

The mathematical equations governing the motion of a micropolar fluid saturating a porous medium following Boussinesq's approximation for the above model [Lukaszewicz (1999), Chandrasekhar (1981), Sunil et al. (2000)] are as follows

The continuity equation for an incompressible fluid is

$$\nabla.\mathbf{q} = 0 \tag{1}$$

The momentum and internal angular momentum equations for the generalized Darcy model including the inertial forces are

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\right]\mathbf{q} = -\nabla p + \rho\mathbf{g} - \frac{1}{k_1}(\mu + k)\,\mathbf{q} + k(\nabla\times\mathbf{v}), \tag{2}$$

and

$$\rho_0 j\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\right]\mathbf{v} = (\varepsilon' + \beta'')\,\nabla(\nabla.\mathbf{v}) + \gamma'\nabla^2\mathbf{v} + \frac{k}{\varepsilon}\nabla\times\mathbf{q} - 2k\mathbf{v}, \tag{3}$$

where $\rho, \rho_0, \mathbf{q}, \mathbf{v}, \mu, k, p, \varepsilon', \beta'', \gamma', j$ and t are the fluid density, reference density, filter velocity, spin (microrotation), shear kinematic viscosity coefficient (constant), coupling viscosity coefficient or vortex viscosity, pressure, bulk spin viscosity coefficient, shear spin viscosity coefficient, micropolar coefficients of viscosity, microinertia constant and time, respectively. When the fluid flows through a porous medium, the gross effect is represented by Darcy's law. As a result, the usual viscous terms is replaced by the resistance term $-\left[\frac{\mu + k}{k_1}\right]\mathbf{q}$. When the permeability of porous material is low, then the inertial force becomes relatively insignificant as compared with the viscous drag force when flow is considered.
Internal energy balance equations and analogous solute equations are

$$\left[\rho_0 C_v\varepsilon + \rho_s C_s(1-\varepsilon)\right]\frac{\partial(T,C)}{\partial t} + \rho_0 C_v(\mathbf{q}.\nabla)(T,C) = (K_T, K_T^{'})\nabla^2(T,C) + (\delta,\delta^{'})(\nabla\times\mathbf{v}).\nabla(T,C), \tag{4}$$

and the density equation of state is given by

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(C - C_0)\right], \tag{5}$$

where $C_v, C_s, K_T, K_T^{'}, \delta, \delta', \rho_s, \alpha, \alpha', T, C, T_0$ and C_0 are the specific heat at constant volume, heat capacity of solid (porous material matrix), thermal conductivity, solute conductivity, coefficients giving account of coupling between the spin flux with heat flux and spin flux with solute flux, density of solid matrix, thermal expansion coefficient, an analogous solvent coefficient of expansion, temperature, solute concentration, reference temperature and reference solute concentration at the lower boundary, respectively.

3. Basic State and Perturbation Equations

Now we are interested in studying the stability of the rest state by giving small perturbations on the rest (initial) state and examine the reactions of the perturbations on the system. The initial state is characterised by $\mathbf{q} = (0,0,0)$, $\mathbf{v} = (0,0,0)$, $p = p(z)$, $T = T(z)$ defined as $T = -\beta z + T_0$, where $\beta = -\dfrac{dT}{dz}$ is the uniform adverse temperature gradient, $C = C(z)$ defined as $C = -\beta' z + C_0$, where $\beta' = -\dfrac{dC}{dz}$ is the solute concentration gradient and $\rho = \rho_0\left[1 + \alpha\beta z - \alpha'\beta' z\right]$.

Now, we shall analyze the stability of the basic (initial) state by introducing the perturbations, $\mathbf{u}'(u, \mathrm{v}, w)$, $\boldsymbol{\omega}, \rho', p', \theta$ and γ in velocity $\mathbf{q}$, spin $\mathbf{v}$, density ρ, pressure p, temperature T and solute concentration C, respectively. The change in density ρ' caused mainly by the perturbation θ and γ in temperature and solute concentration, is given by

$$\rho' = -\rho_0(\alpha\theta - \alpha'\gamma)\,. \tag{6}$$

Then, the linearized perturbation equations of the micropolar fluid become

$$\nabla\,.\,\mathbf{u}' = 0 \tag{7}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x} - \frac{1}{k_1}(\mu + k)\,u + k\Omega_1^{'}, \tag{8}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathrm{v}}{\partial t} = -\frac{\partial p'}{\partial y} - \frac{1}{k_1}(\mu + k)\,\mathrm{v} + k\Omega_2^{'}, \tag{9}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{\partial p'}{\partial z} - \frac{1}{k_1}(\mu + k)\,w + k\Omega_3^{'} + g\rho_0(\alpha\theta - \alpha'\gamma)\,, \tag{10}$$

$$\rho_0\, j\,\frac{\partial \boldsymbol{\omega}}{\partial t} = (\varepsilon' + \beta'')\,\nabla(\nabla.\boldsymbol{\omega}) + \gamma'\nabla^2\boldsymbol{\omega} + \frac{k}{\varepsilon}\nabla\times\mathbf{u}' - 2k\boldsymbol{\omega}\,, \tag{11}$$

$$\left[\rho_0 C_v\varepsilon + \rho_s C_s(1-\varepsilon)\right]\frac{\partial\theta}{\partial t} = k_T\nabla^2\theta - \delta(\nabla\times\boldsymbol{\omega})_z\,\beta + \rho_0 C_v\beta w\,, \tag{12}$$

and

$$\left[\rho_0 C_v\varepsilon + \rho_s C_s(1-\varepsilon)\right]\frac{\partial\gamma}{\partial t} = k_T^{'}\nabla^2\gamma - \delta'(\nabla\times\boldsymbol{\omega})_z\,\beta' + \rho_0 C_v\beta' w\,, \tag{13}$$

where, the non-linear terms $(\mathbf{u}'.\nabla)\,\mathbf{u}', (\mathbf{u}'.\nabla)\theta, (\mathbf{u}'.\nabla)\gamma, \nabla\theta.(\nabla\times\boldsymbol{\omega}), \nabla\gamma.(\nabla\times\boldsymbol{\omega})$ and $(\mathbf{u}'.\nabla)\boldsymbol{\omega}$ in equations (8)-(13) are neglected (using the first order approximations) as the perturbations applied on the system are assumed to be small, the second and higher order perturbations are negligibly small and only linear terms are retained. Also we have assumed

$$\boldsymbol{\Omega}' = (\Omega_1^{'}, \Omega_2^{'}, \Omega_3^{'}) = (\nabla\times\boldsymbol{\omega})\,.$$

Now, it is usual to write the balance equations in a dimensionless form, scaling as

$$(x, y, z) = (x^*, y^*, z^*)d,\ t = \frac{\rho_0 d^2}{\mu}t^*,\ \theta = \beta d\theta^*,\ \gamma = \beta' d\gamma^*, \mathbf{u}' = \frac{x_T}{d}(\mathbf{u}')^*,\ p' = \frac{\mu x_T}{d^2}(p')^*$$

and $\boldsymbol{\omega} = \frac{x_T}{d^2}\boldsymbol{\omega}^*$.

Then, removing the stars (*) for convenience, the non-dimensional form of equations (7)-(13) become

$$\nabla . \mathbf{u}' = 0 \tag{14}$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{u}'}{\partial t} = -\nabla p' - \frac{1}{P_l}(1+K)\mathbf{u}' + K(\boldsymbol{\Omega}') + \left(R\theta - S\gamma\frac{p_1}{q_1}\right)\hat{e}_z\,, \tag{15}$$

$$\bar{j}\frac{\partial \boldsymbol{\omega}}{\partial t} = C_1'\nabla(\nabla . \boldsymbol{\omega}) - C_0'\nabla\times(\nabla\times\boldsymbol{\omega}) + K\left(\frac{1}{\varepsilon}\nabla\times\mathbf{u}' - 2\boldsymbol{\omega}\right), \tag{16}$$

$$Ep_1\frac{\partial\theta}{\partial t} = \nabla^2\theta + w - \bar{\delta}(\nabla\times\boldsymbol{\omega})_z \tag{17}$$

and

$$Eq_1\frac{\partial\gamma}{\partial t} = \nabla^2\gamma + w - \bar{\delta}'(\nabla\times\boldsymbol{\omega})_z \tag{18}$$

where, the new dimensionless coefficients are

$$\bar{j} = \frac{j}{d^2},\ \bar{\delta} = \frac{\delta}{\rho_0 C_v d^2},\ \bar{\delta}' = \frac{\delta'}{\rho_0 C_v d^2}, P_l = \frac{k_1}{d^2}, K = \frac{k}{\mu}, C_0' = \frac{\gamma'}{\mu d^2}, C_1' = \frac{\varepsilon' + \beta'' + \gamma'}{\mu d^2}, E = \varepsilon + \frac{\rho_s C_s}{\rho_0 C_v}(1-\varepsilon) \tag{19}$$

and $\hat{e}_z$ is a unit vector along z-axis and the dimensionless Rayleigh number R, analogous solute number S, Prandtl number p_1 and the analogous Schmidt number q_1 are

$$R = \frac{g\alpha\beta\rho_0 d^4}{\mu x_T},\ S = \frac{g\alpha'\beta'\rho_0 d^4}{\mu x_T'},\ p_1 = \frac{\mu}{\rho_0 x_T},\ q_1 = \frac{\mu}{\rho_0 x_T'}\,, \tag{20}$$

where, $x_T = \frac{K_T}{\rho_0 C_v}$ and $x_T' = \frac{K_T'}{\rho_0\ C_v}$ are thermal diffusivity and solute diffusivity, respectively. Here, we consider both the boundaries to be free and perfectly heat conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The dimensionless boundary conditions are

$$w = 0,\ \frac{\partial^2 w}{\partial z^2} = 0,\ \boldsymbol{\omega} = 0,\ \theta = 0 = \gamma \ \text{ at } z = 0 \text{ and } 1. \tag{21}$$

4. Mathematical Analysis and Dispersion Relation

Applying the curl operator twice to equation (15) and taking the z-component, we get

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{P_l}(1+K)\right]\nabla^2 w = R(\nabla_1^2\theta) - S(\nabla_1^2\gamma)\frac{p_1}{q_1} + K\nabla^2\Omega_3^{'}, \tag{22}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},\ \nabla_1^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ \Omega_3^{'} = (\nabla\times\boldsymbol{\omega})_z = \left(\frac{\partial\omega_y}{\partial x}-\frac{\partial\omega_x}{\partial y}\right). \tag{23}$$

Again applying curl operator once to equation (16) and taking z -component, we get

$$\bar{j}\,\frac{\partial\Omega_3^{'}}{\partial t} = C_0^{'}\nabla^2\Omega_3^{'} - K\left[\frac{1}{\varepsilon}\nabla^2 w + 2\Omega_3^{'}\right]. \tag{24}$$

The linearized form of equation (17) and (18) are

$$Ep_1\frac{\partial\theta}{\partial t} = \nabla^2\theta + w - \bar{\delta}\Omega_3^{'} \tag{25}$$

and

$$Eq_1\frac{\partial\gamma}{\partial t} = \nabla^2\gamma + w - \bar{\delta}'\Omega_3^{'}\ . \tag{26}$$

Now, the boundary conditions are

$$w = 0 = \frac{\partial^2 w}{\partial z^2},\ \Omega_3^{'} = 0 = \theta = \gamma \ \text{ at } z = 0 \text{ and } 1 \tag{27}$$

$\Omega_3^{'} = 0$ at $z = 0$ and 1 are the boundary conditions for the spin. In the equation (24) for spin, the coefficient $C_0^{'}$ and K account for spin diffusion and coupling between vorticity and spin effects, respectively.

Analyzing the disturbances into the normal modes, we assume that the solutions of equations (22)-(26) are given by

$$\left[w, \Omega_3^{'}, \theta, \gamma\right] = \left[W(z), G(z), \Theta(z), \Gamma(z)\right]\exp(ik_x x + ik_y y + \sigma t)\,, \tag{28}$$

where k_x, k_y are the wave numbers along with x and y directions respectively, $a = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and σ is the stability parameter which is, in general a complex constant.

For solutions having the dependence of the form (28), equation (22) and (24)-(26) yield

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+K)\right](D^2 - a^2)W = -Ra^2\Theta + \frac{Sp_1}{q_1}a^2\Gamma + K(D^2 - a^2)G \tag{29}$$

$$\left[\bar{j}\sigma + 2K - C_0^{'}(D^2 - a^2)\right]G = -K\varepsilon^{-1}(D^2 - a^2)W\,, \tag{30}$$

$$\left[Ep_1\sigma - (D^2 - a^2)\right]\Theta = W - \bar{\delta}G\,, \tag{31}$$

and

$$\left[Eq_1\sigma - (D^2 - a^2)\right]\Gamma = W - \bar{\delta}'G\,, \tag{32}$$

where, $\bar{j}, K, \bar{\delta}, \bar{\delta}'$ and $C_0^{'}$ are the non-dimensional micropolar parameters and $D \equiv \frac{d}{dz}$.

The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus, the exact solution of the system (29)-(32) subject to the boundary conditions

$$W = D^2W = 0,\ \Theta = \Gamma = G = 0 \text{ at } z = 0 \text{ and } z = 1 \tag{33}$$

is written in the form

$$W = A_1\, e^{\sigma t}\, \sin\pi z,\ \Theta = B_1 e^{\sigma t}\, \sin\pi z,\ G = C_1 e^{\sigma t} \sin\pi z,\ \Gamma = D_1 e^{\sigma t} \sin\pi z\,, \tag{34}$$

where, A_1, B_1, C_1 and D_1 are constants and σ is the growth rate which is, in general, a complex constant. Substituting equation (34) in equation (29)-(32), we get following equations

$$\left[\left(\frac{\sigma}{\varepsilon} + \frac{1+K}{P_l}\right)(\pi^2 + a^2)\right]A_1 - Ra^2B_1 + \frac{Sp_1}{q_1}a^2D_1 - K(\pi^2 + a^2)C_1 = 0\,, \tag{35}$$

$$-\frac{K}{\varepsilon}(\pi^2 + a^2)A_1 + \left[\bar{j}\sigma + 2K + C_0^{'}(\pi^2 + a^2)\right]C_1 = 0\,, \tag{36}$$

$$A_1 - (Ep_1\sigma + \pi^2 + a^2)B_1 - \bar{\delta}C_1 = 0\,, \tag{37}$$

and

$$A_1 - \left[Eq_1\sigma + (\pi^2 + a^2)\right]D_1 - \bar{\delta}'C_1 = 0\,. \tag{38}$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1 and D_1 in equations (35)-(38) must vanish. This determinant on simplification yields

$$b'\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right]\left[Ep_1\sigma + b'\right]\left[Eq_1\sigma + b'\right]\left[\bar{j}\sigma + 2K + C_0^{'}b'\right] = Ra^2\left[Eq_1\sigma + b'\right]$$

$$\left[\bar{j}\sigma + 2K + C_0^{'}b' - \bar{\delta}K\varepsilon^{-1}b'\right] + K^2\varepsilon^{-1}b'^2\left[Ep_1\sigma + b'\right]\left[Eq_1\sigma + b'\right]$$

$$-\frac{Sp_1}{q_1}a^2\left[Ep_1\sigma + b'\right]\left[\bar{j}\sigma + 2K + C_0^{'}b' - \bar{\delta}'K\varepsilon^{-1}b'\right] \tag{39}$$

where, $b' = \pi^2 + a^2$.

In the absence of solute parameter $(S = 0,$ i.e., $\bar{\delta}' = 0$), equation (39) reduces to

$$b'\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right]\left[Ep_1\sigma + b'\right]\left[l\sigma + 2A + b'\right] = Ra^2\left[l\sigma + 2A + b' - \bar{\delta}A\varepsilon^{-1}b'\right] + KA\varepsilon^{-1}b'^2\left[Ep_1\sigma + b'\right], \quad (40)$$

where

$$l = \bar{j}A/K, \ A = \frac{K}{C_0^{'}},$$

a result derived by Sharma and Gupta (1995). Equation (39) is the required dispersion relation studying the effect of medium permeability, solute parameter and micropolar parameters on the system.

For simplification of further calculations, equation (39) may also be written in the form:

$$P_4\sigma_i^4 - iP_3\sigma_i^3 - P_2\sigma_i^2 + iP_1\sigma_i + P_0 = 0 \quad (41)$$

where,

$$\left.\begin{aligned}
P_0 &= b^3\left(L_0L_1 - \frac{K^2b}{\varepsilon}\right) + \frac{bS_1p_1x}{q_1}L_3 - bxL_2R_1 \\
P_1 &= b^2E(p_1+q_1)\left(L_0L_1 - \frac{K^2b}{\varepsilon}\right) + b^3\left(I_1L_0 + \frac{L_1}{\varepsilon}\right) - xR_1\left[bI_1 + Eq_1L_2\right] + \frac{S_1p_1x}{q_1}\left[bI_1 + Ep_1L_3\right] \\
P_2 &= E^2p_1q_1b\left(L_0L_1 - \frac{K^2b}{\varepsilon}\right) + b^2E(p_1+q_1)\left(I_1L_0 + \frac{L_1}{\varepsilon}\right) + \frac{b^3I_1}{\varepsilon} + x\left(\frac{S_1p_1}{q_1}\right)Ep_1I_1 - xR_1\left(Eq_1I_1\right) \\
P_3 &= \left(\frac{bE}{\varepsilon}\right)\left[bI_1(p_1+q_1) + Ep_1q_1\left(\varepsilon I_1L_0 + L_1\right)\right] \\
P_4 &= b\left[\frac{E^2p_1q_1I_1}{\varepsilon}\right]
\end{aligned}\right\}, \quad (42)$$

where,

$$R_1 = \frac{R}{\pi^4},\ S_1 = \frac{S}{\pi^4},\ x = \frac{a^2}{\pi^2},\ i\sigma_i = \frac{\sigma}{\pi^2},\ b = 1 + x,\ I_1 = \pi^2\bar{j},\ N_3 = C_0^{'}\pi^2,\ N_5 = \bar{\delta}\pi^2,\ N_6 = \bar{\delta}'\pi^2,$$

$$P_l^{'} = \pi^2P_l,\ L_0 = \left(\frac{1+K}{P_l^{'}}\right),\ L_1 = (2K + N_3b),\ L_2 = 2K + b\left(N_3 - \frac{N_5K}{\varepsilon}\right)$$

and

$$L_3 = 2K + b\left(N_3 - \frac{N_6K}{\varepsilon}\right).$$

6. The Case of Stationary Convection

Let the marginal state be stationary, so that it is characterized by putting $\sigma_i = 0$ (Chandrasekhar [1981]). Hence, for the stationary convection, putting $\sigma_i = 0$ in equation (41), Rayleigh number is given by

$$R_1 = \left[\frac{S_1 p_1}{q_1} \left\{ 2K + b\left(N_3 - \frac{N_6 K}{\varepsilon} \right) \right\} + \frac{b^2}{x} \left\{ \left(\frac{1+K}{P_l^{'}} \right)(2K + N_3 b) - \frac{K^2 b}{\varepsilon} \right\} \right] \left[2K + b\,(N_3 - N_5 K \varepsilon^{-1}) \right]^{-1}, \quad (43)$$

which leads to the marginal stability curve in stationary conditions. Equation (43) expresses the Rayleigh number R_1 as a function of dimensionless wave number, medium permeability parameter $P_l^{'}$ (Darcy number), solute gradient parameter S_1, coupling parameter K (coupling between vorticity and spin effects), spin diffusion (couple stress) parameter N_3, micropolar heat conduction parameter N_5 (arises due to coupling between spin and heat fluxes) and micropolar solute parameter N_6 (arises due to coupling between spin and solute fluxes). The parameters K and N_3 measures the micropolar viscous effects and micropolar diffusion effects, respectively. The classical results in respect of Newtonian fluids can be obtained as the limiting case of present study.

Setting $K = 0$ and $S_1 = 0$ and keeping N_3 arbitrary in equation (43), we obtained

$$R_1 = \frac{(1+x)^2}{x P_l^{'}}, \quad (44)$$

which is the classical Rayleigh-Benard result in porous medium for the Newtonian fluid case.

To investigate the effect of medium permeability, stable solute gradient, coupling parameter, spin diffusion parameter micropolar heat conduction parameter and micropolar solute parameter, we examine the behaviour of $\frac{dR_1}{dP_l^{'}}, \frac{dR_1}{dS_1}, \frac{dR_1}{dN_3}, \frac{dR_1}{dN_5}$ and $\frac{dR_1}{dN_6}$ analytically.

From equation (43),

$$\frac{dR_1}{dP_l^{'}} = -\frac{b^2 (bN_3 + 2K)(1+K)}{x (P_l^{'})^2 \left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon} \right) \right]}, \quad (45)$$

which is always negative if

$$N_3 > \frac{N_5 K}{\varepsilon}. \quad (46)$$

This shows that, for the stationary convection, the medium permeability has a destabilizing effect when condition (46) holds. In the absence of micropolar viscous effect (coupling parameter, K), equation (45) yields that the medium permeability always has destabilizing effect on the system.

It can easily be found from equation (43) that

$$\frac{dR_1}{dS_1} = \frac{p_1 \left\{ 2K + b\left(N_3 - \frac{N_6 K}{\varepsilon} \right) \right\}}{q_1 \left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon} \right) \right]}, \tag{47}$$

which is always positive if

$$N_3 > \max\left[\frac{N_6 K}{\varepsilon},\ \frac{N_5 K}{\varepsilon} \right]. \tag{48}$$

This shows the stabilizing effect of stable solute gradient when condition (48) holds. In the absence of micropolar viscous effect, equation (47) yields that the stable solute gradient always has a stabilizing effect. Equation (43) also gives

$$\frac{dR_1}{dK} = \frac{b^2 \left[\frac{N_3 b^2}{P_l'} \left\{ \frac{N_5}{\varepsilon} + N_3 \right\} + K\left(\frac{2}{P_l'} - \frac{b}{\varepsilon} \right) \left\{ 2K + b\left(2N_3 - \frac{N_5 K}{\varepsilon} \right) \right\} + \frac{S_1 p_1}{q_1} \frac{N_3 x}{\varepsilon} (N_5 - N_6) \right]}{x \left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon} \right) \right]^2}, \tag{49}$$

which is always positive if

$$\frac{1}{P_l'} > \frac{b}{\varepsilon},\ N_3 > \frac{N_5 K}{\varepsilon},\ \text{ and }\ N_5 > N_6. \tag{50}$$

This indicates that coupling parameter has a stabilizing effect when condition (50) hold. Equation (49) also yields that $(dR_1)/(dK)$ is always positive in the absence of micropolar solute parameter (coupling between spin and solute fluxes) and in a non-porous medium, implying thereby the stabilizing effect of coupling parameter. Thus, the salinity, the medium permeability and porosity have a significant role in developing the condition for the stabilizing behavior of coupling parameter.

It follows from equation (43) that

$$\frac{dR_1}{dN_3} = -\frac{b^2 K \left[b^2 \left\{ N_5 + K(N_5 - P_l') \right\} + \frac{S_1 p_1}{q_1} x P_l' (N_5 - N_6) \right]}{\varepsilon x P_l' \left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon} \right) \right]^2}, \tag{51}$$

which is always negative if

$N_5 > P_l'$ and $N_5 > N_6$,

which implies that $N_5 > \max\,(P_l', N_6)$. (52)

This shows that spin diffusion has a destabilizing effect when condition (52) holds. Equation

(43) also gives

$$\frac{dR_1}{dN_5} = \frac{Kb\left[\frac{S_1 p_1}{q_1}\left\{2K + b\left(N_3 - \frac{N_6 K}{\varepsilon}\right)\right\} + \frac{1}{x}b\left\{\left(\frac{1+K}{P_l^{'}}\right)N_3 b + \left(\frac{2K}{P_l^{'}}\right) + K^2\left(\frac{2}{P_l^{'}} - \frac{b}{\varepsilon}\right)\right\}\right]}{\varepsilon\left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon}\right)\right]^2}, \tag{53}$$

which is always positive if

$$\frac{1}{P_l^{'}} > \frac{b}{\varepsilon}, \quad N_3 > \frac{N_6 K}{\varepsilon}. \tag{54}$$

This gives that the micropolar heat conduction has a stabilizing effect when condition (54) holds. Equation (53) also yields that $\frac{dR_1}{dN_5}$ is always positive in a non-porous medium, implying thereby the stabilizing effect of micropolar heat conduction parameter.

We can also find from equation (43) that

$$\frac{dR_1}{dN_6} = -\frac{S_1 p_1}{q_1}\frac{bK}{\varepsilon}.\left[2K + b\left(N_3 - \frac{N_5 K}{\varepsilon}\right)\right]^{-1}, \tag{55}$$

which is always negative if

$$N_3 > \frac{N_5 K}{\varepsilon}, \tag{56}$$

implying thereby the destabilizing effect of micropolar solute parameter under condition (56).

Now, the critical thermal Rayleigh number for the onset of instability is determined numerically using Newton-Raphson method by the condition $\frac{dR_1}{dx} = 0$.

As a function of x, R_1 given by equation (43) attains its minimum when

$$T_4 x^4 + T_3 x^3 + T_2 x^2 + T_1 x + T_0 = 0, \tag{57}$$

where

$$T_4 = DI,\ T_3 = 2DH,\ T_2 = \frac{S_1 p_1}{q_1}(HB - IA) + C(H - 2I) + 3D(H - I),\ T_1 = -2I(C + D)$$

and

$$T_0 = -H(C + D),$$

where

$$A = 2K + \left(N_3 - \frac{N_6 K}{\varepsilon} \right),$$

$$B = \left(N_3 - \frac{N_6 K}{\varepsilon} \right),$$

$$C = \left(\frac{1+K}{P_l^{'}} \right) 2K \ ,$$

$$D = \left(\frac{1+K}{P_l^{'}} \right) N_3 - \frac{K^2}{\varepsilon} \ ,$$

$$H = \left[2K + \left(N_3 - \frac{N_5 K}{\varepsilon} \right) \right],$$

$$I = \left(N_3 - \frac{N_5 K}{\varepsilon} \right),$$

with x_c determined as a solution of equation (57), equation (43) will give the required critical thermal Rayleigh number (R_c) for various values of critical wave number (x_c). The critical thermal Rayleigh number (R_c), depends on medium permeability $P_l^{'}$, stable solute gradient S_1 and micropolar parameters K, N_3, N_5 and N_6. The numerical values of critical thermal Rayleigh number (R_c) and critical wave number (x_c) determined for various values of $P_l^{'}, S_1$ and micropolar parameters K, N_3, N_5 and N_6 are given in Tables I-V and values of R_c are illustrated in figures 2-7. Here, in Figures 2 and 3, we have plotted the graphs for the critical thermal Rayleigh number R_c versus $P_l^{'}$ (for various values of solute parameter S_1) and stable solute parameter S_1 (for various values of $P_l^{'}$), respectively, in the presence and absence of coupling parameter K. Figures 4-7 exhibit the plots of critical thermal Rayleigh number R_c versus micropolar parameters K, N_3, N_5 and N_6 for several values of $P_l^{'}$, respectively,

From Figure 2 and Table I, one may find that as $P_l^{'}$ increases, R_c decreases and hence showing the destabilizing effect of the medium permeability. This behavior can also be observed in Figures 3-7 and Tables II-V. Figure 3 and Table I indicate the stabilizing behaviour of stable solute parameter S_1, as the value of critical thermal Rayleigh number increases with the increase in the value of stable solute gradient parameter S_1. This shows that the stable solute parameter postpones the onset of convection. This leads to laterally onset of convection instability. Also, it is obvious from Figures 2 and 3 that only for small values of K, the onset of convection is delayed. This shows that higher values of R_c are needed for the onset of convection in the presence of K, hence justifying the stabilizing effect of the coupling parameter, which can also be observed from figure 4 and Table II.

Figures 4-7 represent the graphs of critical thermal Rayleigh number R_c versus micropolar parameters K, N_3, N_5 and N_6, respectively for various values of $P_l^{'}$. Figures 4 and 6 clearly show that critical thermal Rayleigh number R_c increases with increasing K and N_5, respectively, which can also be observed from Table II and Table IV, respectively, implying thereby that the coupling parameter K and the micropolar heat conduction parameter N_5 has a stabilizing effect in the presence of salinity. This leads to laterally onset of convection instability. Figures 5 and 7 indicates that the value of R_c decreases with increasing N_3 and N_6 respectively, implying thereby the destabiizing behavior of spin diffusion (couple stress) parameter N_3 and micropolar solute parameter N_6 on the system. This leads to an early onset of convection. It can also observed from Tables III and V, respectively.

7. Principle of Exchange of Stabilities

Here, we investigate the possibility of oscillatory modes, if any, on stability problem due to the presence of medium permeability, micropolar parameters and solute gradient. Equating the imaginary parts of equation (41), we obtain

$$\sigma_i\left[-\left(\frac{bE}{\varepsilon}\right)\left[bI_1(p_1+\varepsilon_1)+Ep_1q_1\left\{\varepsilon I_1\left(\frac{1+K}{P_l^{'}}\right)+(2K+bN_3)\right\}\right]\sigma_i^2\right.$$

$$+b^2E(p_1+q_1)\left\{\left(\frac{1+K}{P_l^{'}}\right)(2K+bN_3)-\frac{K^2b}{\varepsilon}\right\}+b^3\left\{I_1\left(\frac{1+K}{P_l^{'}}\right)+\frac{(2K+bN_3)}{\varepsilon}\right\}$$

$$+\left(\frac{S_1p_1}{q_1}\right)x\left[bI_1+Ep_1\left\{2K+b\left(N_3-\frac{N_6K}{\varepsilon}\right)\right\}\right]$$

$$\left.-xR_1\left\{bI_1+Eq_1\left\{2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right\}\right\}\right]=0. \tag{58}$$

It is evident from equation (58) that σ_i may be either zero or non-zero, implies that the modes may be either non-oscillatory or oscillatory.

Limiting Case: In the absence of micropolar viscous effect ($K=0$), microinertia $(I_1=0)$ and solute gradient ($S_1=0, q_1=0$ and $N_6=0$), we obtain the result as

$$\sigma_i\left[\frac{b}{\varepsilon}+\frac{Ep_1}{P_l^{'}}\right]=0. \tag{59}$$

Here, the quantity inside the bracket is positive definite. Hence,

$$\sigma_i=0, \tag{60}$$

which shows that the oscillatory modes are not possible and the principle of exchange of stabilities (PES) is satisfied for micropolar fluid heated from below, in the absence of micropolar viscous effect, microinertia and solute gradient. Thus, we conclude that the oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia and solute gradient, which were non-existence in their absence.

8. The Case of Overstability

In the present section, we have to find the possibility that the observed instability may really be overstability. Since σ is, in general, a complex constant, so we put $\sigma = \sigma_r + i\sigma_i$, where σ_r and σ_i are real. The marginal state is reached when $\sigma_r = 0$: If $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stabiities (PES) is valid otherwise we have overstability and then $\sigma = i\sigma_i$, at marginal stability.

Equating real and imaginary parts of equation (41) and eliminating R_1 between them, yields

$$A_2 C_1^2 + A_1 C_1 + A_0 = 0\,, \tag{61}$$

where, $C_1 = \sigma_i^2$.

$$A_2 = (Eq_1)^2\, I_1 \left[\frac{1}{\varepsilon}\left\{\frac{KN_5(Ep_1)}{\varepsilon} + I_1\right\}b^2 + \left\{\frac{I_1(Ep_1)(1+K)}{P_l^{'}}\right\}b\right] \tag{62}$$

$$\begin{aligned}
A_1 = b^4 &\left[\left(\frac{I_1}{\varepsilon}\right)\left\{I_1 + \left(\frac{N_5 K}{\varepsilon}\right)(Ep_1)\right\} + N_3 (Eq_1)^2 \left(N_3 - \frac{N_5 K}{\varepsilon}\right)\right] \\
&+ b^3\left[(Ep_1) I_1^2 \left(\frac{1+K}{P_l^{'}}\right) + \frac{I_1 K^2}{\varepsilon}(Eq_1)^2 + \frac{2K}{\varepsilon} N_3 (Eq_1)^2\right] \\
&+ (Eq_1)^2 \left\{\left(N_3 - \frac{N_5 K}{\varepsilon}\right)\frac{K}{\varepsilon}\{1-(Ep_1)K\} + \frac{1}{2}N_3\left(\frac{1+K}{P_l^{'}}\right)\{(Ep_1)N_3 - I_1\}\right. \\
&\left.\left. + \frac{1}{2} I_1 \left(\frac{1+K}{P_l^{'}}\right)\left(N_3 - \frac{2N_5 K}{\varepsilon}\right) + \frac{1}{2}(Ep_1)\left(\frac{1+K}{P_l^{'}}\right) N_3 \left(N_3 - \frac{2N_5 K}{\varepsilon}\right)\right\}\right] \\
&+ b^2 (Eq_1)^2\, 2K\left[(Ep_1)\left(\frac{1+K}{P_l^{'}}\right)\left(N_3 - \frac{N_5 K}{\varepsilon}\right) + \left(\frac{1+K}{P_l^{'}}\right)\{(Ep_1 N_3 - I_1\} + \left(\frac{1+K}{P_l^{'}}\right) I_1 + \frac{K}{\varepsilon}\{2-(Ep_1)K\}\right] \\
&+ b\left[(Ep_1)(Eq_1)^2\left(\frac{1+K}{P_l^{'}}\right)4K^2 + \left(\frac{S_1 p_1}{q_1}\right) x I_1 \left\{I_1\left\{Ep_1 - Eq_1\left(1 + \frac{N_3(Ep_1)}{I_1}\right)\right\}\right.\right. \\
&\left.\left. + (Ep_1)\,(Eq_1)\left\{\left(N_3 - \frac{N_5 K}{\varepsilon}\right) + \frac{N_6 K}{\varepsilon}\right\}\right\}\right]
\end{aligned} \tag{63}$$

and the coefficient A_0 being quite lengthy and not needed in the discussion of overstability, have not been written here. Since σ_i is real for overstability, the three values of $C_1 (= \sigma_i^2)$ are positive. The sum of roots of equation (61) is $-A_1 / A_2$, must be positive and if this is to be negative, then $A_1 > 0$ and $A_2 > 0$. Since $A_2 > 0$ (from equation (62)), $A_1 > 0$ gives the sufficient conditions for non-existence of overstability.

It is clear from equation (63) that A_1 is positive if

$$N_3 > \frac{2N_5K}{\varepsilon},\ Ep_1N_3 > I_1,\ \frac{1}{Ep_1} > K \text{ and } Ep_1 > Eq_1\left\{1+\frac{N_3(Ep_1)}{I_1}\right\},$$

which implies that

$$N_3 > \max\left\{\frac{2KN_5}{\varepsilon},\ \frac{I_1}{(Ep_1)}\right\},\ \frac{1}{K} > (Ep_1) \text{ and } K_T < K_T^{'}\left[\frac{1}{\left(1+\frac{N_3(Ep_1)}{I_1}\right)}\right].$$

Thus, for $N_3 > \max\left\{\frac{2KN_5}{\varepsilon},\frac{I_1}{(Ep_1)}\right\},\ \frac{1}{K} > (Ep_1)$ and $K_T < K_T^{'}\left[\frac{1}{\left(1+\frac{N_3(Ep_1)}{I_1}\right)}\right]$, overstability can not occur and the principle of the exchange of stabilities is valid. Hence, the conditions mentioned above are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability, whereas in the absence of micropolar parameters in non-porous medium, the above conditions, as expected, reduces to $K_T < K_T^{'}$, i.e., the thermal conductivity is less than the solute conductivity.

9. Discussion of Results and Conclusions

The principal conclusions from the analysis of this paper are as follows:

(i) The results show that for the case of stationary convection, the medium permeability has destabilizing effect under condition(46), whereas in the absence of micropolar viscous effect (coupling parameter), medium permeability always has destabilizing effect, as it is evident from equation(45). The solute gradient, coupling parameter and micropolar heat conduction parameter has a stabilizing effect under condition(s), (48), (50) and (54), respectively, whereas the spin diffusion parameter and micropolar solute parameter has a destabilizing effect under conditions, (52) and (56), respectively. In the absence of micropolar viscous effect, stable solute gradient always has the stabilizing effect on the system, whereas in the absence of micropolar solute parameter (coupling between spin and solute fluxes) and in a non-porous medium, coupling parameter has a stabilizing effect on the system. Here, we also observe that in a non-porous medium, the micropolar heat conduction always has a stabilizing effect.

(ii) The critical thermal Rayleigh number and wave numbers for the onset of instability are also determined numerically (using Newton-Raphson method) and the sensitiveness of the critical Rayleigh number R_C to the changes in the medium permeability parameter, $P_l^{'}$, stable solute gradient parameter S_1, micropolar fluid parameters K, N_3, N_5 and N_6 is depicted graphically in figures 2-7. The effects of governing parameters on the stability of the system are discussed below:

- Figure 2 and figure 3 demonstrate the influence of medium permeability parameter

(Darcy number), $P_l^{'}$, for various values of S_1, and solute gradient parameter, S_1 for various values of $P_l^{'}$, in the presence and absence of the coupling parameter K. This can also be observed from Table I. Figure 2 illustrates, as $P_l^{'}$ increases, R_C decreases implying thereby the destabilizing of medium permeability. This behavior can also be observed from figures 3-7. The physical explanations behind it is: "when the fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, then the medium permeability has the destabilizing effect. This is because, as medium permeability increases, the void space increases and as a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increase in heat transfer is responsible for early onset of convection. Hence, increasing $P_l^{'}$ leads to decrease in R_c. Figure 3 indicates that the stable solute gradient has a stabilizing effect, as the critical Rayleigh number R_C increases with the increase in S_1. This shows that the stable solute gradient postpones the onset of convection. This can also be observed from the Table I. Also, it is observed from figure 2 and figure 3 that only for small values of K, onset of convection is delayed. This shows that higher values of R_c are needed for onset of convection in the presence of K, hence justifying the stabilizing effect of coupling parameter, which can also be observed from Figure 4 and Table II.

- Figure 4 illustrates the influence of coupling parameter K on critical thermal Rayleigh number R_C for various values of $P_l^{'}$. Figure 4 and Table II illustrates that the coupling parameter has a stabilizing effect. Clearly R_C increases with increasing K. As K increases, concentration of micro elements also increases, and as a result of this, a greater part of the energy of the system is consumed by these elements in developing gyrational (twist) velocities in the fluid, and as a result, onset of convection is delayed.

- Figure 5 represents the plot of critical thermal Rayleigh number R_C versus N_3 for various values of $P_l^{'}$. This graph exhibits a destabilizing effect of spin diffusion (couple stress) parameter on the system, as R_c decreases with increasing N_3. As N_3 increases, the couple stress of the fluid increases, which causes the microrotation to decrease and makes the system more unstable. Nevertheless, the above phenomenon is true in porous or non-porous medium. This can also be observed in Table III.

- Figure 6 represents the plot of critical thermal Rayleigh number R_C versus N_5 for various values of $P_l^{'}$. Figure 6 and Table IV illustrates that as N_5 increases, R_C increases, implying thereby that micropolar heat conduction parameter has a stabilizing effect in the presence of salinity. When N_5 increases, the heat induced into the fluid due to microelements is also increased, thus reducing the heat transfer from the bottom to the top. The decrease in heat transfer is responsible for delaying the onset of instability. Thus, increasing N_5 leads to increase in R_C. In other words, N_5 stabilizes the flow.

- In Figure 7 and Table V, we have also looked into the effect of micropolar solute parameter (N_6) (arises due to the coupling between spin and solute fluxes). Figure 7 and Table V illustrate that as N_6 increase, R_C decreases. In other words, N_6 destabilizes the flow. This leads to the conclusion that micropolar solute parameter leads to an early onset of convection in a micropolar fluid. Thus, the system is destabilized by micropolar solute parameter N_6 .

(iii)The principle of exchange of stabilities is found to hold true for the micropolar fluid heated and soluted from below in the absence of micropolar viscous effect (coupling between vorticity and spin effect), micropolar inertia and solute gradient. Thus, oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia and solute gradient, which were non-existence in their absence.

(iv) If $N_3 > \max\left\{\frac{2KN_5}{\varepsilon},\ \frac{I_1}{(Ep_1)}\right\}$, $\frac{1}{Ep_1} > K$ and $K_T < K_T^{'}\left[\frac{1}{\left\{1+\frac{N_3(Ep_1)}{I_1}\right\}}\right]$.

(v) Overstability can not occur and the principle of the exchange of stabilities is valid. Hence, the above conditions are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. In the absence of micropolar parameters in non-porous medium, the above conditions, as expected, reduces to $K_T < K_T^{'}$, i.e., the thermal conductivity is less than the solute conductivity, which is in good agreement with the results obtained earlier by Sunil et al. (2007).

(vi) Finally, from the above analysis, we conclude that the micropolar parameters and solute gradient have a deep effect on the double- diffusive convection in a micropolar fluid layer heated and soluted from below saturating a porous medium. The micropolar fluid stabilities do deserve a fresh look as related to microgravity environmental applications. It is hoped that the present work will be helpful for understanding more complex problems involving the various physical effects investigated in the present problem.

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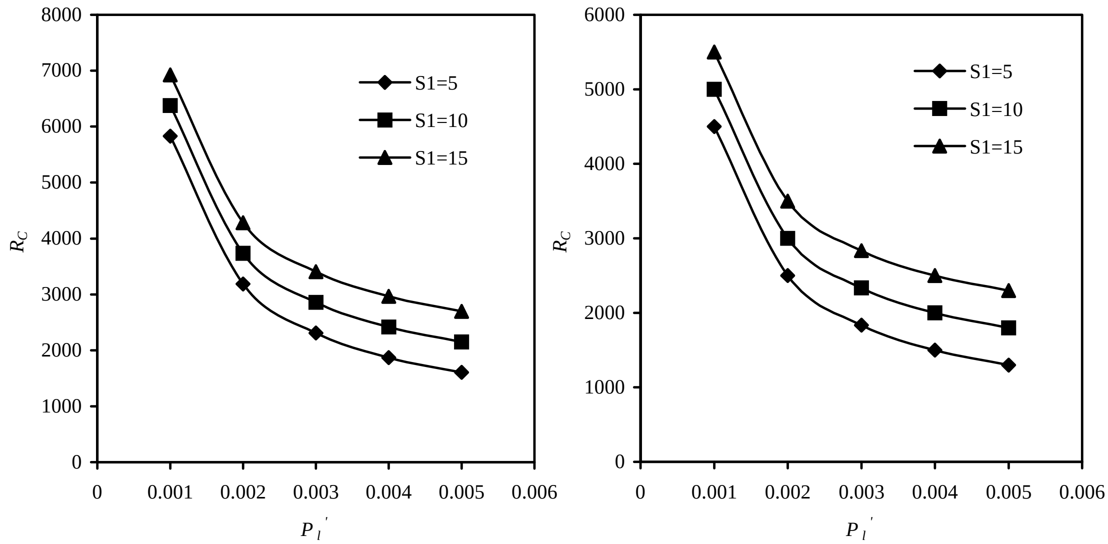
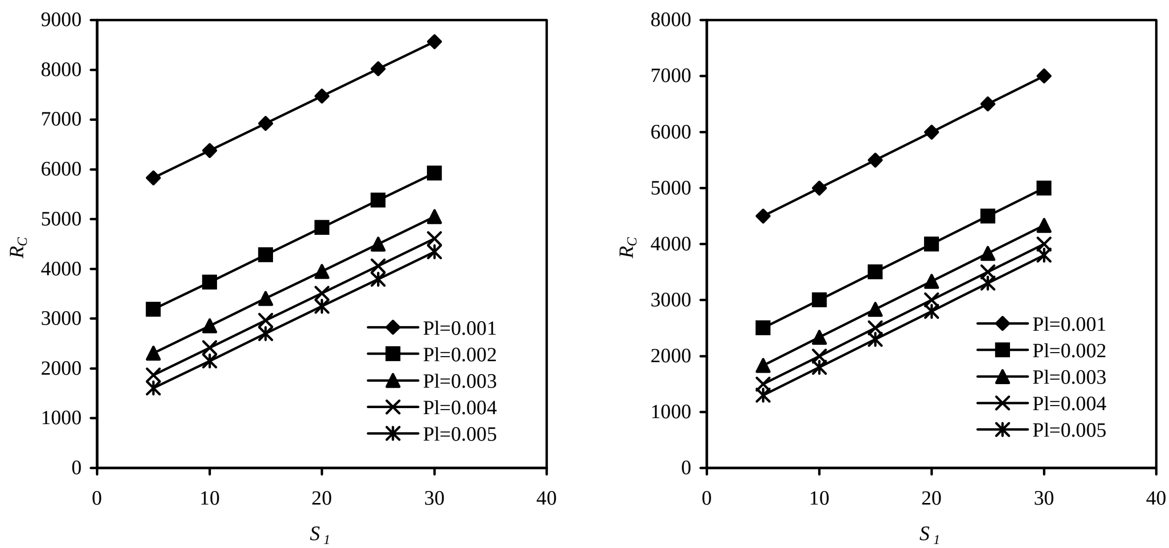
Table I. Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal instability for the onset of stationary convection for various values of medium permeability parameter ($P_l^{'}$) and stable solute gradient parameter (S_1).

S_1	K	$P_l^{'}=0.001$		$P_l^{'}=0.002$		$P_l^{'}=0.003$		$P_l^{'}=0.004$		$P_l^{'}=0.005$	
		x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
5	0.2	0.9900621	5827.708	0.9891670	3187.761	0.9882728	2307.779	0.9873794	1867.767	0.9864871	1603.791
	0	1	4500	1	2500	1	1833.333	1	1500	1	1300
10	0.2	0.9891644	6375.683	0.9873742	3735.734	0.9855881	2855.747	0.9838057	2415.752	0.9820272	2151.753
	0	1	5000	1	3000	1	2333.234	1	2000	1	1800
15	0.2	0.9882675	6923.656	0.9855854	4283.701	0.9829122	3403.709	0.9802478	2963.708	0.9775920	2699.704
	0	1	5500	1	3500	1	2833.334	1	2500	1	2300
20	0.2	0.9873717	7471.627	0.9838006	4831.664	0.9802454	3951.664	0.9767057	3511.656	0.9731818	3247.644
	0	1	6000	1	4000	1	3333.334	1	3000	1	2800
25	0.2	0.9864768	8019.595	0.9820198	5379.622	0.9775874	4499.615	0.9731796	4059.595	0.9887962	3795.574
	0	1	6500	1	4500	1	3833.334	1	3500	1	3300
30	0.2	0.9855829	8567.561	0.980243	5927.576	0.9749384	5047.556	0.9696693	4607.526	0.9644358	4343.493
	0	1	7000	1	5000	1	4333.334	1	4000	1	3800



(i) (ii)

Figure 2. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus medium permeability parameter (P_l') for $\varepsilon = 0.5,\ p_1 = 1,\ q_1 = 0.01,\ N_3 = 2,\ \ N_5 = 0.5,\ \ N_6 = 0.02$ and (i) $K = 0.2$, (ii) $K = 0$.



(i) (ii)

Figure 3. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus stable solute gradient parameter (S_1) for $\varepsilon = 0.5,\ p_1 = 1,\ q_1 = 0.01,\ N_3 = 2,\ \ N_5 = 0.5,\ \ N_6 = 0.02$ and (i) $K = 0.2$ and (ii) $K = 0$.

Table II. Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal instability for the onset of stationary convection for various values of coupling parameter (K).

K	$P_l^{'} = 0.001$		$P_l^{'} = 0.003$		$P_l^{'} = 0.005$		$P_l^{'} = 0.007$		$P_l^{'} = 0.009$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.2	0.9900621	5827.708	0.9882728	2307.778	0.9864871	1603.791	0.9847051	1302.081	0.982927	1134.463
0.3	0.9789272	6550.959	0.9754362	2564.664	0.971961	1767.397	0.9685016	1425.706	0.9650578	1235.873
0.4	0.9647086	7313.177	0.9593234	2834.387	0.9539795	1938.608	0.9486771	1554.687	0.9434161	1341.387
0.5	0.9480924	8113.607	0.940791	3116.639	0.9335746	2117.2	0.9264419	1688.837	0.9193926	1450.833
0.6	0.9296985	8951.288	0.9205851	3411.039	0.9116142	2302.907	0.9027861	1827.935	0.8940991	1564.017
0.7	0.9100889	9825.071	0.8993441	3717.142	0.8888169	2495.423	0.8785041	1971.736	0.868402	1680.727
0.8	0.8897531	10733.64	0.8776071	4034.453	0.8657635	2694.421	0.8542142	2119.983	0.8429528	1800.745

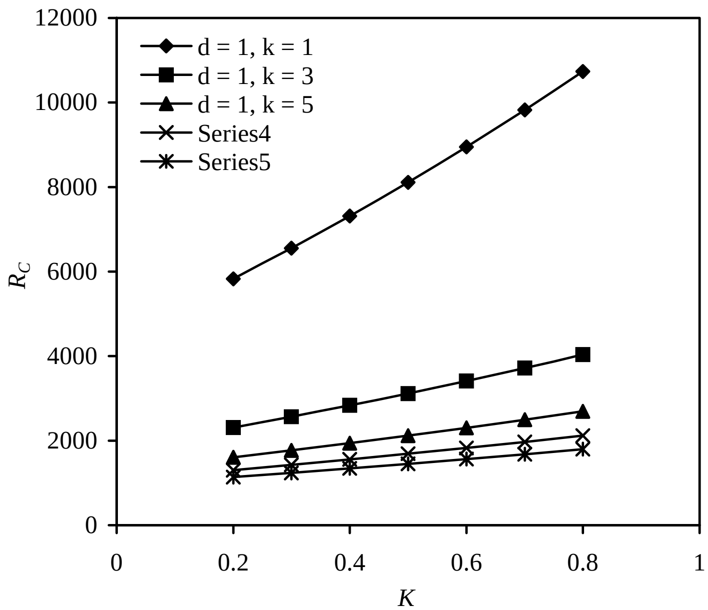


Figure 4. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus coupling parameter (K) for $\varepsilon = 0.5,\ p_1 = 1,\ q_1 = 0.01,\ S_1 = 5$, $N_3 = 2,\ N_5 = 0.5,\ N_6 = 0.02$.

Table III. Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal instability for the onset of stationary convection for various values of spin diffusion parameter (N_3).

N_3	$P_l^{'} = 0.001$		$P_l^{'} = 0.003$		$P_l^{'} = 0.005$		$P_l^{'} = 0.007$		$P_l^{'} = 0.009$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
2	0.9900621	5827.708	0.9882728	2307.779	0.9864871	1603.791	0.9847051	1302.081	0.982927	1134.463
4	0.9973855	5563.912	0.9969123	2203.916	0.9964392	1531.917	0.9959665	1243.917	0.9954939	1083.917
6	0.9988182	5475.945	0.9986042	2169.279	0.9983902	1507.946	0.9981762	1224.518	0.9979622	1067.057
8	0.9993298	5431.961	0.9992082	2151.96	0.9990869	1495.96	0.9989654	1214.817	0.9988439	1058.626
10	0.9995689	5405.569	0.9994907	2141.568	0.9994126	1488.768	0.9993344	1208.997	0.9992564	1053.568
12	0.9996996	5387.973	0.9996451	2134.64	0.9995907	1483.973	0.9995362	1205.116	0.9994818	1050.196
14	0.9997788	5375.406	0.9994907	2141.568	0.9996986	1480.549	0.9996584	1202.344	0.9996184	1047.787

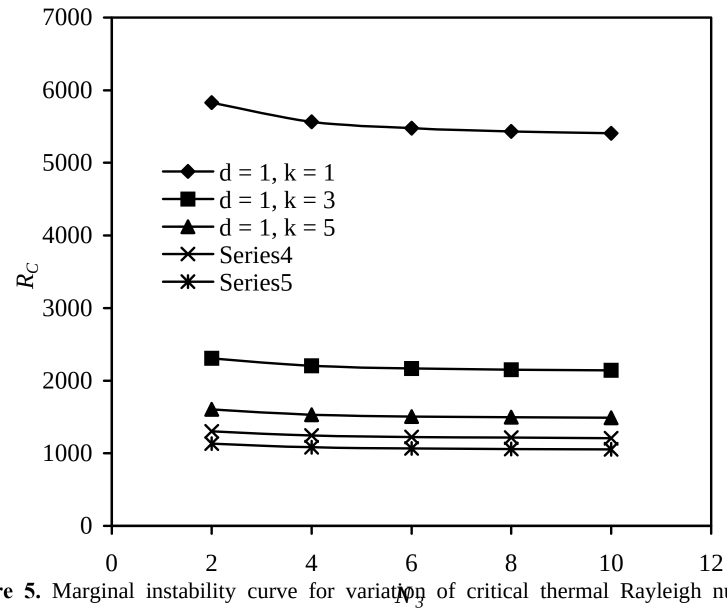


Figure 5. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar spin diffusion parameter (N_3) for $\varepsilon = 0.5$, $p_1 = 1$, $q_1 = 0.01$, $K = 0.2$, $S_1 = 5$, $N_5 = 0.5$, $N_6 = 0.02$ **.**

Table IV. Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal instability for the onset of stationary convection for various values of micropolar heat conduction parameter (N_5).

N_5	$P_l' = 0.001$		$P_l' = 0.002$		$P_l' = 0.003$		$P_l' = 0.004$		$P_l' = 0.005$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.08	0.9985572	5376.235	0.9984554	2940.81	0.9983535	2129.003	0.9982516	1723.099	0.9981497	1479.556
0.1	0.9981809	5396.144	0.9980437	2951.701	0.9979064	2136.887	0.9977692	1729.48	0.997632	1485.036
0.5	0.9900621	5827.708	0.989167	3187.761	0.9882728	2307.779	0.9873794	1867.787	0.9864871	1603.791
1.0	0.9780351	6474.653	0.976033	3541.612	0.9740359	2563.928	0.9720438	2075.082	0.9700567	1781.773
1.5	0.9632612	7282.479	0.9599268	3983.413	0.9566071	2883.712	0.9533022	2333.852	0.9500122	2003.927
2.0	0.9446994	8319.34	0.9397376	4550.386	0.9348109	3294.033	0.9299194	2665.83	0.9250629	2288.888

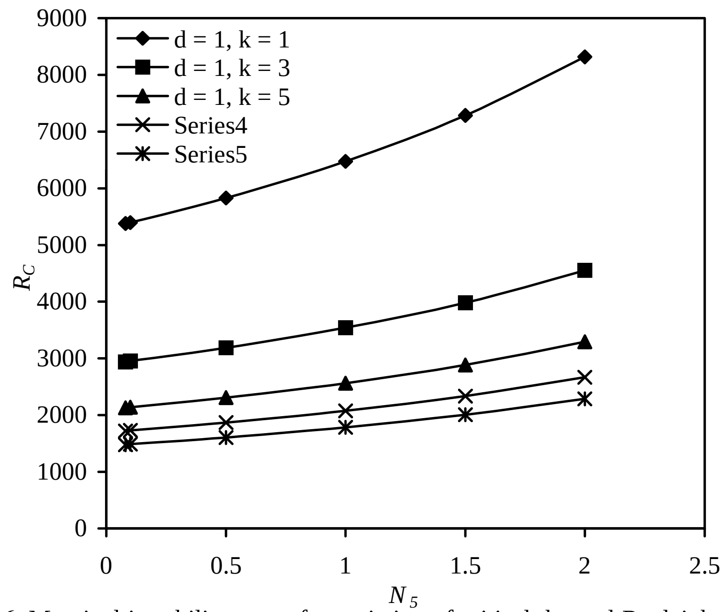


Figure 6. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar heat conduction parameter (N_5) for $\varepsilon = 0.5, K = 0.2, p_1 = 1, q_1 = 0.01, S_1 = 5, N_3 = 2, N_6 = 0.02$.

Table V. Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal instability for the onset of stationary convection for various values of micropolar solute parameter (N_6).

N_6	$P_l{}' = 0.001$		$P_l{}' = 0.002$		$P_l{}' = 0.003$		$P_l{}' = 0.004$		$P_l{}' = 0.005$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.02	0.9900621	5827.708	0.989167	3187.761	0.9882728	2307.779	0.9873794	1867.787	0.9864871	1603.791
0.05	0.9901182	5824.71	0.9892791	3184.763	0.9884409	2304.781	0.9876034	1864.789	0.9867667	1600.793
0.10	0.9902119	5819.712	0.9894662	3179.766	0.9887211	2299.783	0.9879767	1859.792	0.9872329	1595.797
0.15	0.9903054	5814.715	0.9896532	3174.768	0.9890014	2294.786	0.9883501	1854.795	0.9876992	1590.8
0.20	0.9903991	5809.717	0.9898402	3169.771	0.9892818	2289.789	0.9887238	1849.798	0.988166	1585.803
0.25	0.9904927	5804.72	0.9900274	3164.773	0.9895623	2284.791	0.9890976	1844.8	0.9886331	1580.806
0.30	0.9905862	5799.722	0.9902145	3159.776	0.9898429	2279.794	0.9894715	1839.803	0.9891002	1575.808

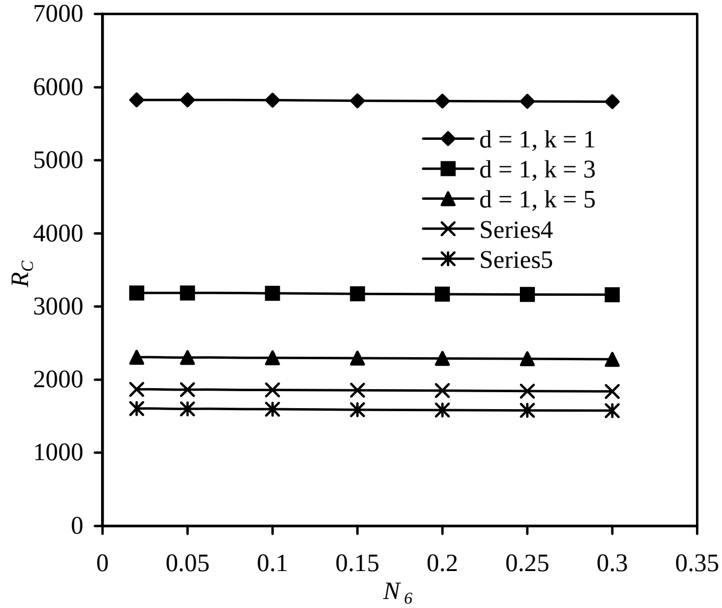


Figure 7. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar solute parameter (N_6) for $\varepsilon = 0.5, K = 0.2, p_1 = 1, q_1 = 0.01, S_1 = 5$, $N_3 = 2, N_5 = 0.5$.