



## Applying GMDH-type Neural Network and Particle swarm Optimization for Prediction of Liquefaction Induced Lateral Displacements

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### Abstract

Lateral spreading and flow failure are amongst the most destructive effects of liquefaction. Estimation of the peril of lateral spreading requires characterization of subsurface conditions, principally soil density, fine content, groundwater conditions, site topography and seismic characteristics. In this paper a GMDH-type neural network and particle swarm optimization is developed for prediction of liquefaction induced lateral displacements. Using this method, a new model was proposed that is suitable for predicting the liquefaction induced lateral displacements. The proposed model was tested before the requested calculation. The data set which is contains 250 data points of liquefaction-induced lateral ground spreading case histories from eighteen different earthquakes was divided into two parts: 70% were used as training and 30% were used as a test set, which were randomly extracted from the database. After initially testing on the input\_output process, the predicted values were compared with experimental values to evaluate the performance of the group method of data handling neural network method.

**Keywords:** Artificial neural network; GMDH model; Soil liquefaction; Lateral displacement.

**MSC (2010) No.:** 93A30, 68A05

## 1. Introduction

Liquefaction occurs in saturated sand deposits, due to excess pore water pressure increase, during earthquake induced cyclic shear stresses. Thus, can cause serious to destructive damage to structures. The liquefaction mechanism includes ground subsidence, flow failure and lateral spreading, among other effects. Perhaps one of the earliest observed cases of lateral spreading is the San Francisco 1906 earthquake, Youd et al. (2002). Lateral spreading involves the movement of relatively intact soil blocks on a layer of liquefied soil toward a free face or incised channel. It can also induce different forms of ground deformation, which can be very destructive, in the vicinity of natural and artificial slopes. A number of approaches have been proposed for prediction of the magnitude of lateral ground displacements under various conditions. Al Bawwab (2005) has categorized the methods into the following four groups:

- (1) Numerical analyses in the form of finite element and/or finite difference techniques.
- (2) Simplified analytical methods.
- (3) Soft computing techniques.
- (4) Empirical methods, developed, based on the assessment of either laboratory test data or statistical analyses of lateral spreading case histories.

Numerical and analytical methods have been widely used in geo-mechanics to simulate the patterns of kinematic behavior under various loadings. The success of such methods is highly dependent on the constitutive model and the input parameters. The finite element or finite difference methods are perhaps the most widely used numerical methods. However, these procedures are highly dependent on material parameters that are usually difficult to estimate, and, as a result, limited success has been achieved in producing results that are comparable to field observations, Javadi et al. (2006). Analytical models have also contributed to the development of knowledge in this field. A number of simplified analytical models have been utilized to simulate liquefaction induced lateral spreading. The Sliding Block Model, Newmark (1965), Yegian et al. (1991), Baziar et al. (1992), Jibson (1994), Minimum Potential Energy Model, Towhata et al. (1992), Tokida et al. (1993), Shear Strength Loss and Strain Re-hardening Model, Bardet et al. (1999), and the Viscous Model, Hamada et al. (1994) are examples of this approach. In Hamada et al. (1986), Youd and Perkins (1987), Bardet et al. (1999), Youd et al. (2002) and Kanibir (2003) the authors have introduced empirical correlations and Multi-Linear Regression (MLR) models for the assessment of liquefaction induced lateral spreading. In Zhang et al. (2004), the authors have introduced a “Lateral Displacement Index (LDI)” calculated by integration of the maximum shear strain over potentially liquefiable layers, and then have used it in a couple of simple correlations for “free-face” and “ground slope” cases. Liu and Tesfamariam (2012) have investigated different types of models to predict the lateral spread displacement over a free-face and ground-slope conditions. Jafarian et al. (2012) have used computational fluid dynamic to predict liquefaction-induced lateral deformation of an infinite earth slope. In Kalantary et al. (2013), the authors have proposed the robust counterpart of the least squares model to quantify the effect of uncertainties on the evaluation of model parameters.

Artificial Neural Networks (ANNs) have been used for modeling induced displacement by Bartlet and Youd (1992). ANNs are nonlinear and highly flexible models that have been successfully used in many complicated systems. The ANNs can be considered as universal function approximators. Giving enough data, they can approximate the underlying function with accuracy. However, the main disadvantage of traditional NNs is that the detected dependencies are hidden within the NN structure, Nariman-Zadeh and Jamali (2007).

Conversely, the group method of data handling (GMDH), Ivakhnenko (1971) is aimed at identifying the functional structure of a model hidden in the empirical data. The main idea of the GMDH is the use of feed-forward networks based on short-term polynomial transfer functions whose coefficients are obtained using regression combined with emulation of the self-organizing activity behind neural network (NN) structural learning, Farlow (1984). The GMDH was developed in complex systems for the modeling, prediction, identification, and approximation. It has been shown that, the GMDH is the best optimal simplified model for inaccurate, noisy, or small data sets, with a higher accuracy and a simpler structure than typical full physical models, Ghanadzadeh et al. (2011, 2012), Shooshpasha and MolaAbasi (2012).

In this work, a model for prediction of liquefaction induced lateral displacements was developed using the GMDH algorithm. Using existing experimental data the proposed network was trained. The trained network was used to predict the liquefaction induced lateral displacements. Then, the predicted data was compared with the experimental data which have previously reported. In order to investigate the reliability of the proposed method, the accuracy of the model was determined using coefficient of determination ( $R^2$ ), mean square error (MSE), root mean square error (RMSE) and mean absolute deviation (MAD).

## 2. Group Method of Data Handling (GMDH)

Using the GMDH algorithm, a model can be represented as a set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and, therefore, produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function,  $\hat{f}$ , that can be approximately used instead of the actual one,  $f$ , in order to predict output  $\hat{y}$  for a given input vector  $X = (x_1, x_2, x_3, \dots, x_n)$  as close as possible to its actual output  $y$ . Therefore, given number of observations ( $M$ ) of multi-input, single output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad (i = 1, 2, 3, \dots, M). \quad (1)$$

It is now possible to train a GMDH-type-NN to predict the output values  $\hat{y}_i$  for any given input vector  $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ , that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad (i = 1, 2, 3, \dots, M). \quad (2)$$

In order to determine a GMDH type-NN, the square of the differences between the actual output and the predicted one is minimized, that is

$$\sum_{i=1}^M \left[ \hat{f}(x_{i1}, x_{i2}, \dots, x_i) - y_i \right]^2 \rightarrow \min. \tag{3}$$

The general, connection between the inputs and the output variables can be expressed by a complicated discrete form of the Volterra functional series, Ivakhnenko (1971) in the form of

$$y = a_o + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots, \tag{4}$$

which is known as the Kolmogorov-Gabor polynomial, Ivakhnenko (1971). The general form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of

$$\hat{y} = G(x_i, x_j) = a_o + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \dots \tag{5}$$

In this way, such partial quadratic description is recursively used in a network of connected neurons to build the general mathematical relation of the inputs and output variables given in equation (4). The coefficients  $a_i$  in equation (5) are calculated using regression techniques. It can be seen that a tree of polynomials is constructed using the quadratic form given in equation (5). In this way, the coefficients of each quadratic function  $G_i$  are obtained to fit optimally the output in the whole set of input–output data pairs, that is

$$E = \frac{\sum_{i=1}^M (y_i - G_i())^2}{M} \rightarrow \min. \tag{6}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of the total  $n$  input variables are taken in order to construct the regression polynomial in the form of equation (5) that best fits the dependent observations  $(y_i, i=1,2,\dots,M)$  in a least squares sense. Using the quadratic sub-expression in the form of equation (5) for each row of  $M$  data triples, the following matrix equation can be readily obtained as

$$Aa = Y, \tag{7}$$

where  $a$  is the vector of unknown coefficients of the quadratic polynomial in equation (5),

$$a = \{a_o, a_1, a_2, a_3, a_4, a_5\}, \tag{8}$$

and

$$Y = \{y_1, y_2, y_3, \dots, y_M\}^T. \tag{9}$$

Here,  $Y$  is the vector of the output’s value from observation. It can be readily seen that

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix}. \quad (10)$$

The least squares technique from multiple regression analysis leads to the solution of the normal equations in the form of

$$a = (A^T A)^{-1} A^T Y. \quad (11)$$

### 3. Descriptive Variables for the Proposed Models

In our study we have used three kind of descriptive variables:

#### 1. Seismological Variables

This group includes variables that are directly related to durational and intensity-related issues of the strong ground motion shaking. They are listed as:

$M_W$ : Moment magnitude of the earthquake, representing duration of shaking.

$a_{\max}$ : Maximum horizontal ground acceleration (g), representing intensity of shaking.

#### 2. Topographical Variables

The variables of this category describe the site boundary conditions, and define the location of the point where lateral spreading deformations were mapped relative to the boundaries. As show in Figure (1), these variables are:

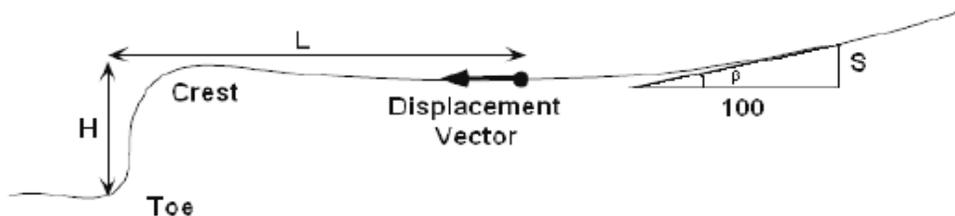
W: Free-face ratio =  $H/L$  (%),

L: Distance to the free face from the point of displacement (m),

H: Height of free face (m),

S: Ground surface slope (%),

$\beta$ : Ground surface slope angle (degrees) =  $\tan^{-1}(S/100)$ .



**Figure 1.** Topography-related descriptive variables

### 3. Geotechnical Variables

These descriptive variables are subdivided into three main groups.

i) static soil stability:

These variables are adopted as safety measures for the stability of gently sloping sites against sliding under earthquake shaking. They are defined as:

$\tan \phi' / \tan \beta$ : Factor of safety measure against gravitational forces for the most critical sub-layer, where  $\phi'$  is the equivalent mobilized angle of internal friction of liquefied or potentially liquefiable soils.

ii) Descriptive Variables for Inertial Forces:

These descriptive variables are safety measures for the resistance of sites against inertial impact effects due to the ground acceleration produced during earthquake shaking. They are defined as:

$a_y/a_{max}$ : Factor of safety against sliding for the most critical sub-layer, where  $a_y$  is the yield acceleration with finite slope assumption.

iii) Liquefaction Severity:

This probabilistic variable, LSI, represents a measure for the seismic-induced liquefaction potential of a given site rather than the potential failure of a particular soil sub-layer. The other descriptive variable within this group is the depth from ground surface to the most critical potentially liquefiable soil sub-layer,  $Z_{cr}$ .

### 4. The displacements prediction using the GMDH-type neural network

The feed-forward GMDH-type neural network for liquefaction induced lateral displacements was constructed using an experimental data set which is containing 250 data points of liquefaction-induced lateral ground spreading case histories from eighteen different earthquakes including 1906 San Francisco – USA, 1964 Prince William Sound – Alaska, 1964 Niigata–Japan, 1971 San Fernando – USA, 1979 Imperial Valley – USA, 1983 Borah Peak–USA, 1983 Nihonkai – hubu – Japan, 1987 Superstition Hills – USA, 1989 Loma Prieta – USA, 1995 Hyogoken-Nanbu–Japan, 1976 Guatemala, 1977 San Juan - Argentina, 1990 Luzon – Philippines, 1994 Northridge – USA, 1999 Kocaeli (Izmit) – Turkey, 1999 Chi Chi–Taiwan, 2003 San Simeon- USA earthquake and 2003 Tokachi-Oki–Japan earthquakes, Youd et al. (2002) and Al Bawwab (2005). A random sample selection of database is given in Table 1. The data was divided into two parts: 70% was used as training data, and 30% was used as test data.

Earthquake moment magnitude ( $M_w$ ), the ratio of maximum horizontal ground acceleration over acceleration of gravity ( $a_{max}/g$ ), Slope of ground surface (S), Free face ratio (W), liquefaction

severity index (LSI), Critical potentially liquefiable soil sub-layer depth ( $Z_{cr}$ ), the ratio of yield acceleration over maximum horizontal ground acceleration ( $a_y/a_{max}$ ) and the ratio of Ground surface slope angle over the equivalent mobilized angle of internal friction of liquefied ( $\tan \beta / \tan \phi'$ ) were used as inputs of the GMDH-type network. The horizontal displacement (DH) was used as desired output of the neural network.

In the present study, liquefaction induced lateral displacements was predicted using GMDH-type-NNs. Such a NN identification process needs a suitable optimization method to find the best network architecture. In this way, particle swarm optimization (PSO) is arranged in a new approach to design the whole architecture of the GMDH-type-NNs. PSO is a global search strategy that can handle efficiently arbitrary optimization problems. It is one of the evolutionary computing methods that has elements inspired by the social behavior of natural swarms and is introduced by Kennedy and Eberhart (Kennedy and Eberhart, 1995) for the first time in 1995.

In a PSO algorithm, population (set of solutions of the problem) is initiated randomly with particles and they are evaluated to compute fitnesses together with finding the particle best (best value of each individual so far) and global best (best particle in the whole swarm). PSO algorithm provides the optimal number of neurons in each hidden layer and their connectivity configuration to find the optimal set of appropriate coefficients of quadratic expressions to model liquefaction induced lateral displacements. The swarm size is set to 150 and the maximum number of iterations is set to 100 as stopping criteria. Computations are performed in MATLAB 7.13.0 on a 2.3GHz laptop with 4 GB of RAM.

The developed GMDH neural network was successfully used to obtain a model for calculate liquefaction induced lateral displacements (Table 2). In the GMDH architecture, the selection of nodes with the best predictive capability is decided by the PSO and subsequently the network construction with the corresponding layers are realized based on the search results. For each layer the best node is found based on the objective function (which is simply the external criterion used for solving the problem at hand).

The nodes in the preceding layer connected to the best node in the current layer are marked for realizing the network as search progresses from layer to layer as shown in Figure 2. In these networks, the most important input variables, number of layers, neurons in hidden layers and optimal model structure are determined automatically. The polynomial terms are created by using linear and non-linear regressions. The initial layer is simply the input layer. The first layer created is made by computing regressions of the input variables and then choosing the best ones. The second layer is created by computing regressions of the values in the first layer along with the input variables. This means that the algorithm essentially builds polynomials of polynomials. The optimal structures of the developed neural network with 2-hidden layers are shown in Figure 2. For instance “ccagdgbg” and “acgfcfad” are corresponding genome representations of displacements for sloping sites without a free face and level sites with a free face, in which a, b, c, d, e, f and g stand for  $M_w$ ,  $a_{max}/g$ ,  $S(W)$ , LSI,  $Z_{cr}$ ,  $a_y/a_{max}$  and  $\tan \beta / \tan \phi'$  respectively. The GMDH-type-NN provides an automated selection of essential input variables, and builds polynomial equations for the modeling. This polynomial equation shows the quantitative relationship between input and output variables (Table (2)). Our proposed models behavior in prediction is demonstrated in Figures. (3) and (4).

**Table 1.** Experimental and GMDH estimated tie-line data for the liquefaction induced lateral displacements.

$M_w$	$a_{max}/g$	S(%)	W(%)	LSI	$Z_{cr}$ (m)	$a_y/a_{max}$	$\tan \beta / \tan \phi'$	$D_H$ (m)
7.9	0.6	0.6	0	3.55	3.5	0.07	0.124	1
7.5	0.19	0.31	0	5.8	1.09	0.264	0.058	1.09
9.2	0.21	0.7	7.03	6.18	4.57	0.594	0.053	1.37
6.4	0.55	1.23	0	0.01	11.89	1.488	0.015	1.68
7.5	0.12	3.5	1.03	1.25	5.56	0.864	0.252	0
7.4	0.2	1	0	0.96	12.19	0.537	0.085	1
6.5	0.51	2	4.69	1.65	3.36	0.406	0.088	0.86
6.9	0.6	11	0	2.06	1.74	0	1.392	0
7.7	0.25	0.56	0	3.01	3	0.567	0.038	1.14
6.6	0.21	0.47	41.38	1.94	3.05	0.345	0.061	0.2
7	0.13	0	10	1.11	6.34	0.717	0	0.5
7.6	0.2	0.5	50	7.73	9.23	0.249	0.091	5
6.7	0.52	1	0	0	14.1	2.718	0.007	1
6.9	0.6	0.1	0	5.08	13.5	0.358	0.005	0.4
7.6	0.43	1	5	3.47	2.8	0.118	0.165	5.84
7.4	0.4	1.6	0	4.54	9.8	0.138	0.225	2.2
6.5	0.12	1	0	0.56	2.75	0.371	0.183	0.3
7.9	0.31	0.1	0	2.29	11	0.368	0.009	0.2
7.9	0.6	0.6	0	3.55	3.5	0.07	0.124	1
7.5	0.19	0.31	0	5.8	1.09	0.264	0.058	1.09

The results of the developed models give a close agreement between observed and predicted values. Some statistical measures are given in Table (3), in order to determine the accuracy of the models. These statistical values are based on  $R^2$  as absolute fraction of variance, RMSE as root-mean squared error, MSE as mean squared error, and MAD as mean absolute deviation which are defined as follows:

$$R^2 = 1 - \left[ \frac{\sum_{i=1}^M (Y_{i(model)} - Y_{i(actual)})^2}{\sum_{i=1}^M (Y_{i(actual)})^2} \right] \tag{12}$$

$$RMSE = \left[ \frac{\sum_{i=1}^M (Y_{i(model)} - Y_{i(actual)})^2}{M} \right]^{1/2} \tag{13}$$

$$MSE = \frac{\sum_{i=1}^M (Y_{i(model)} - Y_{i(actual)})^2}{M} \tag{14}$$

$$MAD = \frac{\sum_{i=1}^M |Y_{i(model)} - Y_{i(actual)}|}{M} \tag{15}$$

**Table 2.** Polynomial equations of the GMDH model

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$D_H (W = 0)$

$Y_1 = -115346 + 3.5050 (M_w) - 68.9404 (\tan \beta / \tan \phi') - 0.2432 (M_w)^2 - 0.9335 (\tan \beta / \tan \phi')^2 + 10.0645 (M_w) (\tan \beta / \tan \phi')$

$Y_2 = 1.3397 - 0.5783 (LSI) - 5.8479 (\tan \beta / \tan \phi') + 0.0726 (LSI)^2 - 3.6651 (\tan \beta / \tan \phi')^2 + 5.1493 (LSI) (\tan \beta / \tan \phi')$

$Y_3 = -1.7571 + 19.2485 (a_{max}/g) + 1.2785 (\tan \beta / \tan \phi') - 27.2681 (a_{max}/g)^2 - 11.3716 (\tan \beta / \tan \phi')^2 + 24.2619 (a_{max}/g) (\tan \beta / \tan \phi')$

$Y_4 = -2.3125 + 2.0762 (S) + 2.3967 Y_1 - 0.1728 (S)^2 + 0.1482 Y_1^2 - 1.0988 (S) Y_1$

$Y_5 = -0.3709 + 3.2831 Y_2 - 2.6836 Y_3 + 0.1899 Y_2^2 + 1.9872 Y_3^2 - 2.0152 Y_2 Y_3$

$D_H = 0.0508 + 1.0800 Y_4 - 0.3596 Y_5 - 0.2741 Y_4^2 + 0.1539 Y_5^2 + 0.1826 Y_4 Y_5$

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$D_H (S = 0)$

$Y_1 = -42.1453 + 11.698 (M_w) - 0.7023 (W) - 0.7899 (M_w)^2 - 0.0032 (W)^2 + 0.1239 (M_w) (W)$

$Y_2 = 0.3046 + 0.9488 (\tan \beta / \tan \phi') + 20.0143 (a_y/a_{max}) - 0.1876 (\tan \beta / \tan \phi')^2 - 14.5415 (a_y/a_{max})^2 - 24.9465 (\tan \beta / \tan \phi') (a_y/a_{max})$

$Y_3 = 0.7001 + 1.0824 (W) + 0.4840 (a_y/a_{max}) - 0.0122 (W)^2 - 0.1186 (a_y/a_{max})^2 - 1.6209 (W) (a_y/a_{max})$

$Y_4 = -136.3152 + 40.9184 (M_w) - 9.8062 (LSI) - 3.0427 (M_w)^2 - 0.0485 (LSI)^2 + 1.4306 (M_w) (LSI)$

$Y_5 = 2.1905 - 1.5655 Y_1 - 1.4131 Y_2 - 0.0897 Y_1^2 - 0.0418 Y_2^2 + 1.8398 Y_1 Y_2$

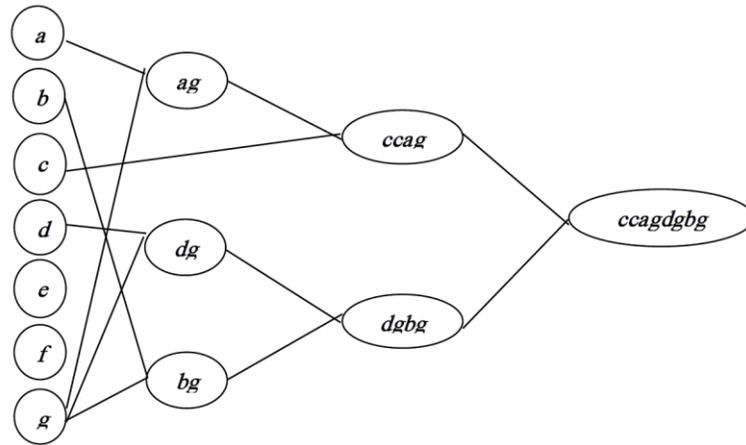
$Y_6 = -0.4177 + 0.1373 Y_3 + 1.1825 Y_4 + 0.1203 Y_3^2 - 0.2482 Y_4^2 + 0.1287 Y_3 Y_4$

$D_H = -0.2393 + 0.5031 Y_5 + 0.7207 Y_6 + 0.3337 Y_5^2 + 0.2761 Y_6^2 - 0.6405 Y_5 Y_6$

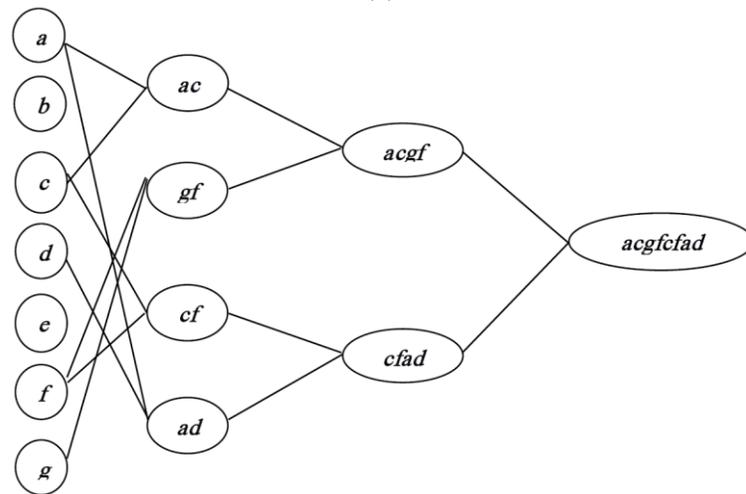
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**Table 3.** Model statistics and information of the group method of data handling-type neural network model for predicting of liquefaction induced lateral displacements

Ground condition	Statistic	R <sup>2</sup>	RMSE	MSE	MAD
W = 0	Training	0.99	0.26	0.06	0.15
	Testing	0.98	0.26	0.06	0.19
S = 0	Training	0.98	0.34	0.11	0.23
	Testing	0.99	0.10	0.01	0.08

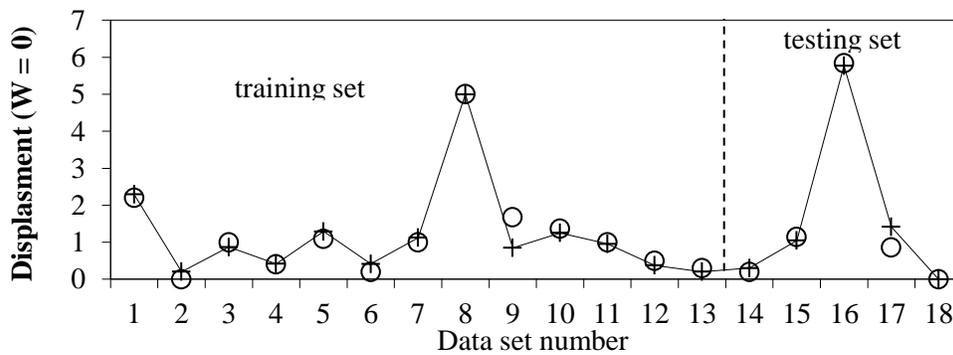


(a)

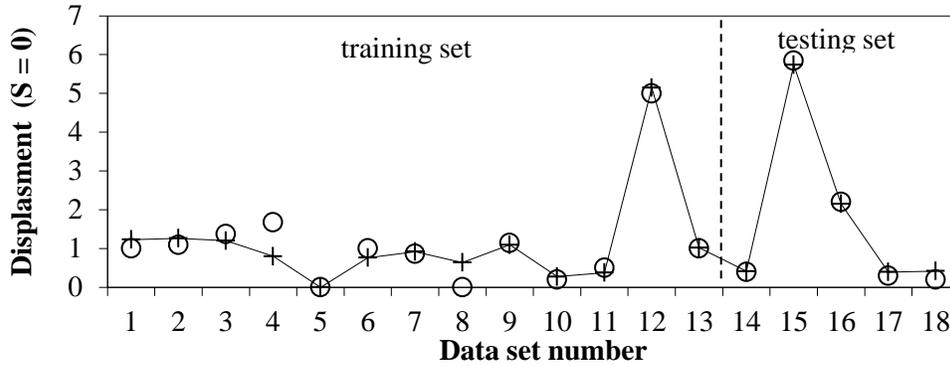


(b)

**Figure 2.** Developed structure of GMDH-Type-NN model, a) sloping sites without a free face, b) level sites with a free Face



**Figure 3.** GMDH-type NN model-predicted displacements for the sites without a free face in comparison with experimental data; (o) Experimental points; (+) Calculated points(GMDH).



**Figure 4.** GMDH-type NN model-predicted displacements for the sites with a free face in comparison with experimental data; (o) Experimental points; (+) Calculated points(GMDH).

## 5. Conclusions

In this study, a feed-forward GMDH-type neural network model was developed using experimental data. The liquefaction induced lateral displacements were predicted by the GMDH model and the results compared with the experimental data. Despite the complexity of the system studied, the GMDH model permits a good prediction. Thus, the GMDH model is suitable for predicting displacements. Although the agreements between the experimental and calculated data were found to be excellent, empirical correlations derived from a local dataset should not be implemented for different sites with significantly varying features. Therefore, these proposed relationships should be used with caution in geotechnical engineering and should be checked against measured lateral displacements.

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