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Peristaltic Induced Flow of a Two-Layered Suspension in Non-Uniform Channel

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Abstract

Peristaltic transport of a two-layered particulate suspension in a non-uniform channel has been investigated. The coupled differential equations for both the fluid and the particle phases in the central as well as in the peripheral layers have been solved and the expression for the flow rate, the pressure rise and the friction force has been derived. The results obtained are discussed both qualitatively and quantitatively in brief. The significance of the particle concentration as well as the peripheral layer has been well explained.

Keywords: Particle concentration, peripheral layer, flow rate, pressure rise, friction force.

MSC (2000) No.: 76Z05

1. Introduction

For about four and half decades, the flow induced by peristaltic waves had been the subject of scientific and engineering research. Latham was probably the first to introduce the mechanism of peristaltic transport in his M. S. thesis in the year 1966. Peristalsis, as termed by the physiologists, is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible duct containing liquid or mixture. Including the vascomotion of small blood vessels, it has been found to be involved in many biological organs (Srivastava and Srivastava, 1984).Certain biomechanical systems such heart-lung machine, finger and roller pumps have been fabricated using the principles of peristalsis. Shapiro et al. (1969) and Jaffrin and Shapiro (1971) explained the basic principles of peristaltic pumping and brought out clearly the significance of the various parameters governing the flow. The literature on the subject is quite extensive by now. A review of most of the early theoretical and experimental investigations reported up to the year 1994 may be found in Srivastava and Coworkers (1984, 1995). The literature beyond this and of recent years include the investigations of Srivastava and Srivastava (1997), Mekheimer et al. (1998), Hakeem et al. (2002), Srivastava (2002), Misra and Pandey (2002), Hayat and coworkers (2002, 2003, 2004, 2005, 2006a,b; 2008a,b), Mekheimer (2003), Misra and Rao (2004), Srivastava (2007a), Ali and Hayat (2008), Medhavi and coworker (2008, 2010), and a few others.

The study of the theory of particulate suspension is very useful in many areas of technical importance (powder technology, fluidization, sedimentation, combustion, aerosol filtration, atmospheric fallout, lunar ash flows, environmental pollution, etc.). Recently, interest is increasingly developing in applying the particulate suspension theory to physiological flows as it provides an improved understanding of topics such as diffusion of protein, the rheology of blood, the swimming of microorganism, the particle deposition on respiratory tract, etc. Peristaltic pumping of multi-phase fluid has been addressed by Srivastava and Srivastava (1989, 1997), Mekheimer et al. (1998), Srivastava (2002), Medhavi and coworker (2008, 2009) and a few others.

It is known that the peripheral layer plays a significant role whenever it is required to prevent the transported fluid from coming in contact with the mechanical parts of the pumps. Flows in many of the biological organs such as chyme in the small intestines and blood through small vessels are mainly two-layered. Peristaltic transport of two-layered models have been addressed by a few investigators including Shukla et al. (1980), Srivastava and Srivastava (1982, 1984), Brasseur et al. (1987), Srivastava and Saxena (1995), Rao and Usha (1995), Misra and Pandey (2002), etc. It is to note here that the interface shape depends on the viscosity ratio of the fluids in the two (central and peripheral) layers and not on the ratio of the radii of the outer (peripheral) and the central layers, in general (Brasseur et al.,1987; Rao and Usha,1995). However, the shape of the interface is not significantly affected when the viscosity of the fluid in one of the layers is varied with respect to the viscosity of the fluid in the other layer (Misra and Pandey, 2002).

The studies mentioned above have considered the core fluid to be either a single-phase Newtonian or non-Newtonian fluid. With increasing interest in particulate suspension flows, it is highly desired to address the two-fluid peristaltic transport problem in detail when the core fluid is represented by a particle-fluid suspension. In view of the above discussion, an attempt has been made in the present work to analyze the flow of a particle-fluid suspension induced by peristaltic waves in the presence of a peripheral layer in a non-uniform channel. The theoretical model considers a two-layered flow consisting of a central layer as a particle-fluid mixture and a peripheral layer as particle-free Newtonian fluid (same fluid as suspending medium in the central layer).

2. Formulation of the Problem

Consider the flow of a two-layered particulate suspension through a two-dimensional infinite channel of non-uniform width with a sinusoidal wave traveling down its walls. The central layer (core region) consists of a particle-fluid mixture and the peripheral layer of particle free Newtonian viscous fluid (same as the suspending medium in the core region). The geometry of the wall surface is described (Figure 1) by

$$H(x,t) = d(x) + b\sin\frac{2\pi}{\lambda}(x - ct), \tag{1}$$

with

$$d(x) = d_o + kx,$$

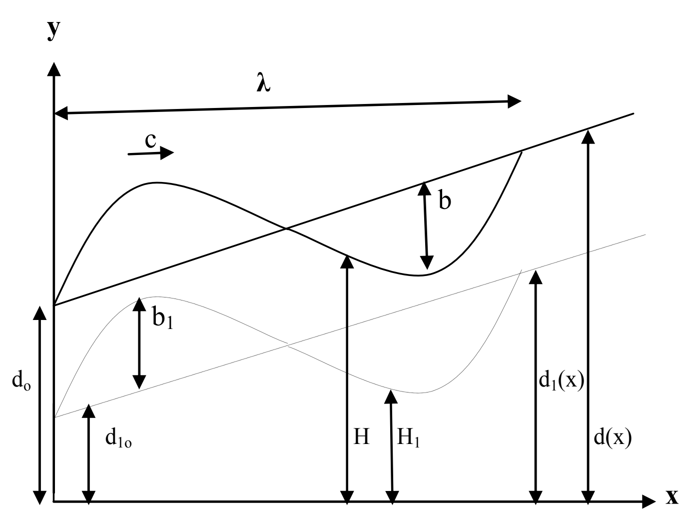

Fig. 1. Two layered flow geometry of peristaltic waves in a channel

where $d(x)$ is the half width of the channel at any axial distance x from inlet, d_o is the half width of the channel at the inlet, k (<<1) is a constant whose magnitude depends on the length of the channel and exist and inlet dimensions, b is the amplitude of the wave, λ is the wavelength, c is the wave propagation velocity and t is the time.

The equations governing the linear momentum and the conservation of mass for both the fluid and particle phases in the peripheral and the central regions are expressed (Srivastava and Coworker, 1989, 2007b)

$$\rho_f\left\{\frac{\partial u_o}{\partial t}+u_o\frac{\partial u_o}{\partial x}+v_o\frac{\partial u_o}{\partial y}\right\}=-\frac{\partial p}{\partial x}+\mu_o\nabla^2 u_o \quad ,H_1\le y\le H\,, \tag{2}$$

$$\rho_f\left\{\frac{\partial v_o}{\partial t}+u_o\frac{\partial v_o}{\partial x}+v_o\frac{\partial v_o}{\partial y}\right\}=-\frac{\partial p}{\partial y}+\mu_o\nabla^2 v_o \quad ,\ H_1\le y\le H\,, \tag{3}$$

$$\frac{\partial v_o}{\partial y}+\frac{\partial u_o}{\partial x}=0\,, \tag{4}$$

$$\begin{aligned}(1\text{-}C)\ \rho_f\left\{\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial x}+v_f\frac{\partial u_f}{\partial y}\right\}&=-\ (1\text{-}\ C\ )\ \frac{\partial p}{\partial x}+(1-C)\ \mu_s\ (C)\nabla^2 u_f\\ &\quad + CS\ (u_p-u_f)\,,\quad 0\le y\le H_1\,,\end{aligned} \tag{5}$$

$$\begin{aligned}(1\text{-}C)\ \rho_f\left\{\frac{\partial v_f}{\partial t}+u_f\frac{\partial v_f}{\partial x}+v_f\frac{\partial v_f}{\partial y}\right\}&=-\ (1\text{-}C)\ \frac{\partial p}{\partial y}+(1-C)\ \mu_s\ (C)\\ &\quad \mathrm{x}\nabla^2 v_f+CS\ (v_p-v_f)\,,\quad 0\le y\le H_1\,,\end{aligned} \tag{6}$$

$$\frac{\partial}{\partial y}\left[(1-C)v_f\right]+\frac{\partial}{\partial x}\left[(1-C)u_f\right]=0, \tag{7}$$

$$\rho_p\left\{\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right\}=-\ C\ \frac{\partial p}{\partial x}+\ CS\ (u_f-u_p),\quad 0\le y\le H_1, \tag{8}$$

$$\rho_p\left\{\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right\}=-\ C\ \frac{\partial p}{\partial y}+\ CS\ (v_f-v_p),\quad 0\le y\le H_1, \tag{9}$$

$$\frac{\partial}{\partial y}\ [C\ \ v_p]\ +\ \frac{\partial[C\ u_p]}{\partial x}\ =0, \tag{10}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is two-dimensional Laplacian operator with y as the vertical coordinate measured in the direction normal to the tube axis, (u_f, v_f) and (u_p, v_p) denote velocity components of the fluid and particle phases in (x, y) directions, respectively in the core region, $0\le y\le H_1$; (u_o, v_o) denotes velocity components of the fluid in the peripheral layer $H_1\le y\le H$; ρ_f and ρ_p be the actual densities of the material constituting fluid and particulate phases, respectively, (1-C)ρ_f is the fluid phase density, $C\rho_p$ the particulate phase density in the core region; p denotes pressure and C be the volume fraction of particulate phase; μ_o is the fluid viscosity in the peripheral region; $\mu_s(C) \simeq \mu_s$ is the suspension viscosity in the central layer and S being the drag coefficient of interaction for the force exerted by one phase on the other. In view of the argument stated earlier (Misra and Pandey, 2002), one may assume the form of interface (Shukla et al., 1980; Srivastava and Saxena, 1995) as: $H_1 = d_1(x) + b_1 \sin 2\pi/\lambda\,(z\text{-}ct)$ with $d_1(x) = d_{1o}+kx$, b_1 respectively as the central layer radius and interface wave amplitude and d_{1o} being the radius of the central layer at the inlet. The limitations of the present theoretical model are well described in Srivastava and Srivastava (1983) and Srivastava (2007b). It is worth to mention here that fluid density in the central and the peripheral regions has been assumed the same, ρ_f in view of the fact that the fluid in the peripheral layer is same as that of the suspending medium in the central core region.

Introducing the following dimensionless variables

$$x' = x/\lambda,\ y' = y/d_o, (u'_o, u_p', u_c') = (u_o, u_p, u_c)/c,\ \ t' = ct/\lambda,$$

$$(v'_p, v'_c) = \lambda\,(v_p, v_c)/cd_o,\ \ S' = Sd_o^2/\mu_o,\quad p' = d_o^2\,p/\lambda c\,\mu_c,$$

in to the equations (2) and (10), after dropping the primes, yields

$$\delta\,\mathrm{Re}\left\{\frac{\partial u_o}{\partial t} + u_o\frac{\partial u_o}{\partial x} + v_o\frac{\partial u_o}{\partial y}\right\} = -\frac{\partial p}{\partial x} + \delta^2\frac{\partial^2 u_o}{\partial x^2} + \frac{\partial^2 u_o}{\partial y^2},\ h_1 \le y \le h, \tag{11}$$

$$\delta^3\,\mathrm{Re}\left\{\frac{\partial v_o}{\partial t} + u_o\frac{\partial v_o}{\partial x} + v_o\frac{\partial v_o}{\partial y}\right\} = -\frac{\partial p}{\partial y} + \delta^2\left(\delta^2\frac{\partial^2 v_o}{\partial x^2} + \frac{\partial^2 v_o}{\partial y^2}\right),\ h_1 \le y \le h, \tag{12}$$

$$\frac{\partial v_o}{\partial y} + \frac{\partial u_o}{\partial x} = 0, \tag{14}$$

$$(1\text{-}C)\,\delta\,\mathrm{Re}\left\{\frac{\partial u_f}{\partial t} + u_f\frac{\partial u_f}{\partial x} + v_f\frac{\partial u_f}{\partial y}\right\} = -(1\text{-}C)\frac{\partial p}{\partial x} + (1-C)\,\mu\left(\delta^2\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2}\right)$$
$$+ CS\,(u_p - u_f),\quad 0 \le y \le h_1, \tag{15}$$

$$(1\text{-}C)\,\delta^3\,\mathrm{Re}\left\{\frac{\partial v_f}{\partial t} + u_f\frac{\partial v_f}{\partial x} + v_f\frac{\partial v_f}{\partial y}\right\} = -(1\text{-}C)\frac{\partial p}{\partial y} + (1-C)\,\mu\delta^2\left(\delta^2\frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2}\right)$$
$$+ CS\delta^2(v_p - v_f),\ 0 \le y \le h_1, \tag{16}$$

$$\frac{\partial}{\partial y}\left[(1-C)v_f\right] + \frac{\partial}{\partial x}\left[(1-C)u_f\right] = 0, \tag{17}$$

$$(\rho_p/\rho_f)\delta\,\mathrm{Re}\left\{\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right\} = -C\frac{\partial p}{\partial x} + \mathrm{CS}\,(u_f - u_p),\quad 0 \le y \le h_1, \tag{18}$$

$$(\rho_p/\rho_f)\delta^3 \mathrm{Re}\left\{\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y}\right\} = -C\frac{\partial p}{\partial y} + CS\delta^2(v_f - v_p),\ \ 0 \le y \le h_1, \tag{19}$$

$$\frac{\partial}{\partial y}[C\ v_p] + \frac{\partial [C\ u_p]}{\partial x} = 0, \tag{20}$$

where $Re = \rho_f\, cd_o/\mu_0$ and $\delta = d_o/\lambda$ are Reynolds number and wave number, respectively,

$$(\alpha, \phi, \phi_1) = (d_{1o}, b, b_1)/d_o,\quad \mu = \mu_p/\mu_c,\ \ (h, h_1) = (H, H_1)/d_o, = (1, \alpha) + \frac{k\lambda x}{d_o} + (\phi, \phi_1)\sin 2\pi(x-t).$$

Jaffirin and Shapiro (1971) observed that the Reynolds number is quite small when the wavelength is long and in such a case the inertial terms may be neglected. Thus, under the long wave approximation (i.e., $\delta << 1$), equations (11) – (20) reduce t

$$\frac{dp}{dx} = \frac{\partial^2 u_o}{\partial y^2},\ \ h_1 \le y \le h, \tag{21}$$

$$(1-C)\frac{dp}{dx} = \mu(1-C)\frac{\partial^2 u_f}{\partial y^2} + CS(u_p - u_f),\ 0 \le y \le h_1, \tag{22}$$

$$C\frac{dp}{dx} = CS(u_f - u_p),\ 0 \le y \le h_1, \tag{23}$$

The non-dimensional boundary conditions are

$$u_o = 0 \ \text{ at }\ y = h = 1 + \frac{k\lambda x}{d_0} + \phi \sin 2\pi(x-t), \ (24)$$

$$u_f = u_o \ \text{ and }\ \tau_f = \tau_o \ \text{ at }\ y = h_1 = \alpha + \frac{k\lambda x}{d_o} + \phi_1 \sin 2\pi(x-t), \tag{25}$$

$$\partial u_f/\partial y = 0 \ \ \text{at}\ y = 0, \tag{26}$$

with $\tau_f = (1-C)\mu_s \partial u_f/\partial y$ and $\tau_o = \mu_o \partial u_o/\partial y$; τ_f, τ_o are shearing stress of the core and peripheral regions, respectively.

The expression for the drag coefficient S and the empirical relation for the viscosity of the suspension are selected (Srivastava, 2007b; Srivastava et al., 2010) as

$$S = \frac{9}{2}\frac{\mu_o}{a_o^2}\frac{4 + 3(8C - 3C^2)^{1/2} + 3C}{(2 - 3C)^2}, \tag{27}$$

$$\mu_s = \frac{\mu_o}{(1 - mC)},$$

$$m = 0.07 \exp[2.49C + (1107/T)\exp(-1.69C)], \tag{28}$$

where T is measured in absolute temperature (^oK). The viscosity of suspension expressed by this formula is found to be reasonably accurate up to $C = 0.6$ (i.e., 60% particle concentration).

3. Analysis

The expression for the velocity profiles, u_o, u_f and u_p obtained as the solution of equations (21) – (23) subject to the boundary conditions (24)-(26), are given as

$$u_o = -\frac{1}{2}\frac{dp}{dx}(h^2 - y^2), \; 0 \le y \le h_1, \tag{29}$$

$$u_f = -\frac{1}{2(1-C)\mu}\frac{dp}{dx}\{h_1^2 - y^2 + \mu(1-C)(h^2 - h_1^2)\}, \;\; 0 \le y \le h_1, \tag{30}$$

$$u_p = -\frac{1}{2(1-C)\mu}\frac{dp}{dx}\left\{h_1^2 - y^2 + \mu(1-C)(h^2 - h_1^2) + \frac{2(1-C)\mu}{S}\right\}, \;\; 0 \le y \le h_1, \tag{31}$$

The flow flux (instantaneous volume flow rate), $q = (q'/2cd_o)$ is calculated as

$$\begin{aligned} q(x,t) &= \int_{h_1}^{h} u_o dy + \int_{0}^{h_1} [(1-C)u_f + Cu_p] dy \\ &= -\frac{1}{3(1-C)\mu}\frac{dp}{dx}\{(1-C)(h^3 - h_1^3) + h_1^3 + \delta h_1\}, \end{aligned} \tag{32}$$

where $\delta = 3C(1-C)\mu)\mu$, a non-dimensional suspension parameter.

Using now the fact that the total sum is equal to the sum of fluxes across the two regions : $0 \le y \le h_1$ and $h_1 \le y \le h$, one arrives at the relations (Medhavi, 2009; Shukla et al., 1980), $\phi_1 = \alpha\phi$ and $h_1 = \alpha h.$ Substitution of these relations into equation (32), yields

$$-\frac{dp}{dx} = \frac{3\mu(1-C)\,q(x,t)}{\eta h^3 + \delta\alpha h}, \tag{33}$$

with

$$\eta = (1-C)(1-\alpha^3)\mu + \alpha^3.$$

The pressure rise, $\Delta p_L(t)$ and the friction force at the walls, $F_L(t)$ in the channel of length, L in their non-dimensional form are obtained as

$$\Delta p_L(t) = \int_0^{L/\lambda} \frac{dp}{dx} dx = -3(1-C)\mu \int_0^{L/\lambda} \frac{q(x,t)}{\eta h^3 + \delta\alpha h} dx, \tag{34}$$

$$F_L(t) = \int_0^{L/\lambda} h\left(-\frac{dp}{dx}\right) dx = 3(1-C)\mu \int_0^{L/\lambda} \frac{q(x,t)}{\eta h^2 + \delta\alpha} dx. \tag{35}$$

Setting k = 0 in equations (34) and (35), one derives the expressions for the pressure rise and friction force for a two-layered particulate suspension in a uniform channel with $\alpha = 1$ in equations (34) and (35), the results for a single-layered particle-fluid suspension is derived. It is interesting to note that when C = 0, the core mixture reduces to the same fluid as in the peripheral layer and consequently the role of the interface automatically disappears and the results obtained above reduce to the corresponding result of single-phase Newtonian viscous fluid in a non-uniform channel.

4. Numerical Results and Discussion

In order to discuss the results quantitatively, computer codes are now developed for the numerical evaluations of the analytical results obtained in equations (34) and (35) for various parameter values at the temperature of 37^0C in a channel of half width 0.01 cm. In view of the fact that the peripheral layer thickness strongly depends on the core mixture viscosity besides other factors (Bugliarello and Sevilla, 1970; Srivastava, 2007b), we choose $2a_0$ (diameter of a suspended particle)=8μm, the peripheral layer thickness $\varepsilon(\mu m) \approx \varepsilon(C) = 6.18,\ 4.67,\ 3.60,\ 3.12,\ 2.58,\ 2.18,$ corresponding to the particle concentrations, C =

0.1, 0.2, 0.3 0.4, 0.5, 0.6, respectively from Srivastava (2007b). The value of α is obtained from the relation: $\alpha = 1 - \varepsilon/d_o$. We further assume the form of

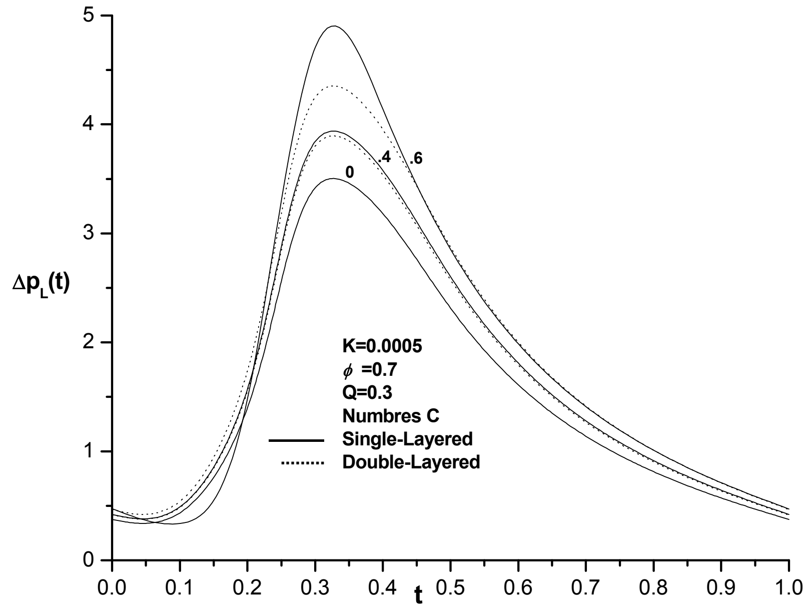

Fig. 2 Variation of pressure rise, $\Delta p_L(t)$ in a non-uniform channel with time, t for different C.

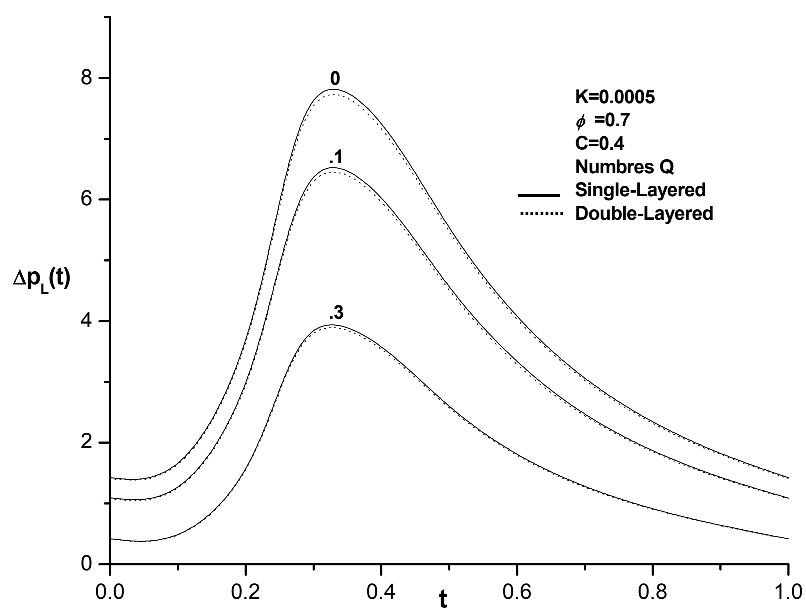

Fig. 3 Variation of pressure rise, $\Delta p_L(t)$ in a non-uniform channel with time, t for different Q.

instantaneous flow rate $q(x,t)$, periodic in (x-t) as (Gupta and Seshadri, 1976; Srivastava and Srivastava, 1988; Mekheimer, 2002)

$$q(x,t) = Q + \phi \sin 2\pi(x - t), \tag{36}$$

where Q is the time average of the flow over one period of the wave. The above form of $q(x,t)$ has been assumed in view of the fact that the constant value gives $\Delta P_L(t)$ always negative and hence there would be no pumping action.

The dimensionless pressure rise, $\Delta p_L(t)$ and friction force, $F_L(t)$ over the channels length, L for various values of the flow rate, Q amplitude ratio, ϕ and the particle concentration. C is computed using equation (36). The average pressure rise Δp_L and the friction force, F_L are then evaluated by averaging $\Delta p_L(t)$ and $F_L(t)$, respectively over one period of the wave. Other parameters are selected (Srivastava and Srivastava, 1984; Mekheimer, 2002) as

$$L = \lambda = 10cm, \; k = \frac{0.5d_0}{L} = 0.0005.$$

The integrals involved in equations (34) and (35) are evaluated using Simpson rule and some of the critical results are displayed in Figs. 2 – 15 graphically.

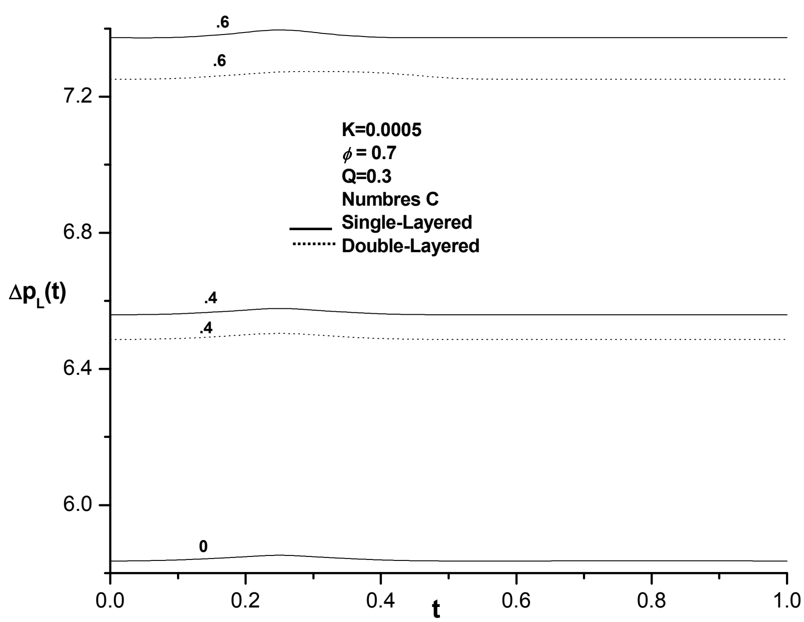

Fig. 4 Variation of pressure rise, $\Delta p_L(t)$ in a uniform channel with time, t for different C.

In both the single and double-layered analyses the pressure rise $\Delta p_L(t)$ increases with particle concentration, C. Depending on the particle concentration, C, the peak value of $\Delta p_L(t)$ occurs between $0.3 \le t \le 0.4$. (Fig. 2). The pressure rise $\Delta p_L(t)$ decreases with the increasing flow rate for any given set of other parameters (Fig. 3). The flow characteristic, $\Delta p_L(t)$ possesses similar characteristics with respect to any parameter in both the uniform and non-uniform channel. However, $\Delta p_L(t)$ assumes much smaller magnitude in non-uniform channel than its corresponding value in the uniform channel (Figs. 2-5). It is further to note that for any given vale of t, $\Delta p_L(t)$ assumes significantly lower magnitude in two-layered analysis than its corresponding value in single-layered analysis (Figs. 3 and 5).

The average pressure rise, Δp_L versus time average flow rate, Q has been plotted in Figs. 6 and 7 which indicate a linear relationship between Δp_L and Q and thus the maximum flow

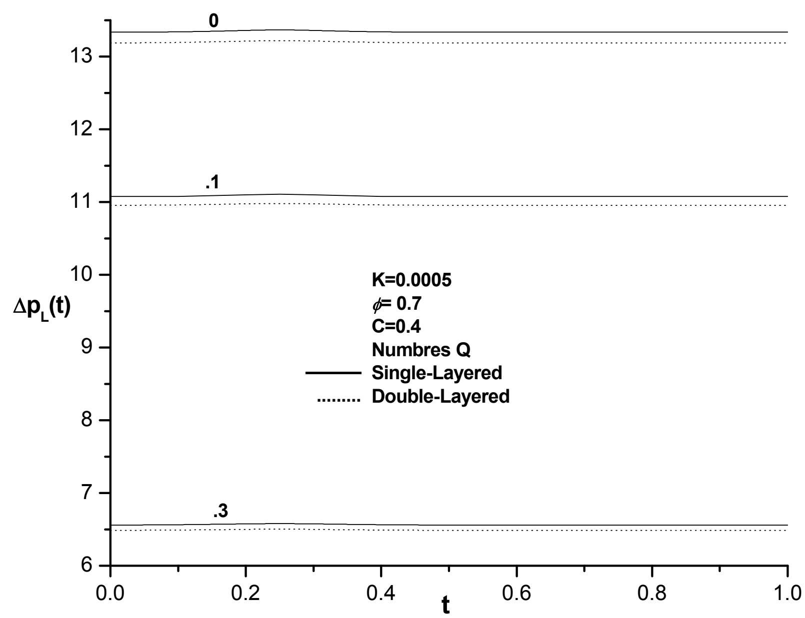

Fig. 5 Variation of pressure rise, $\Delta p_L(t)$ in a uniform channel with time, t for different Q.

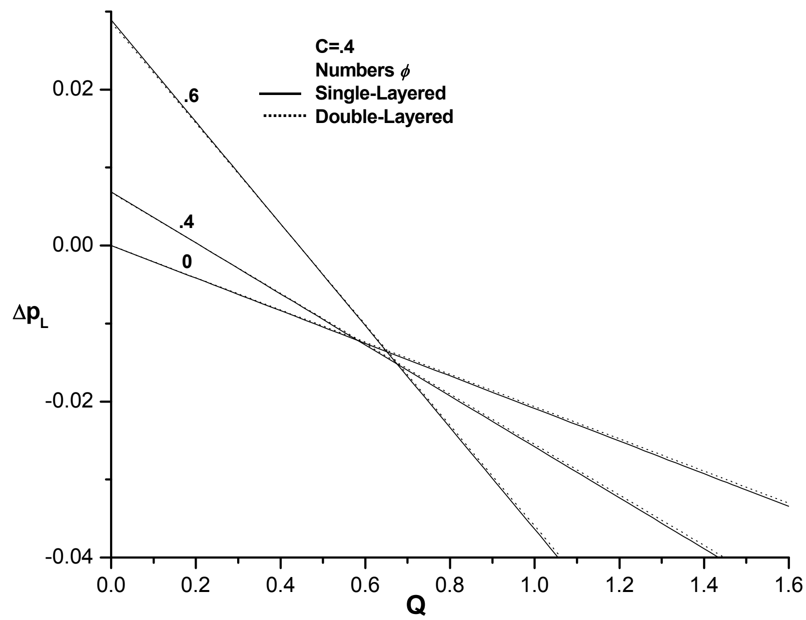

Fig. 6 Pressure-flow rate relationship in a non-uniform channel for different ϕ.

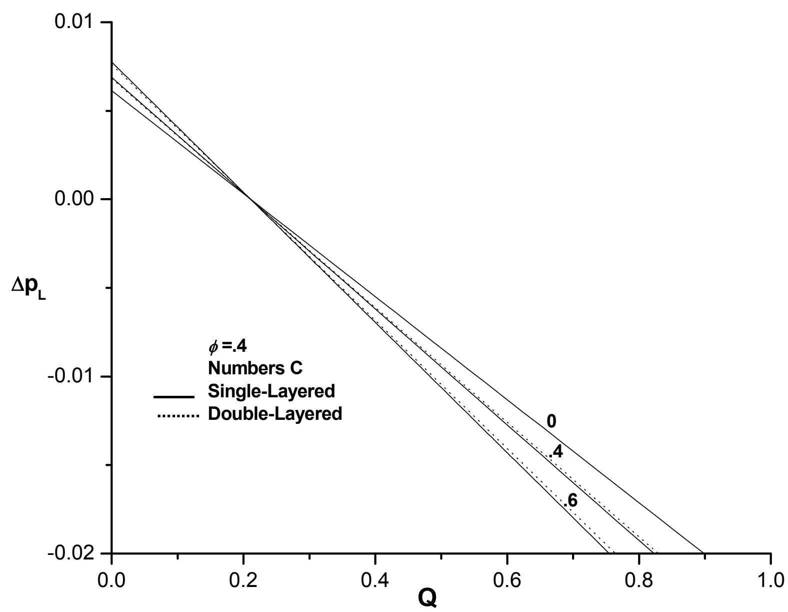

Fig. 7 Pressure-flow rate relationship in a non-uniform channel for different C.

rate is achieved at zero pressure rise and maximum pressure occurs at zero flow rate. The flow characteristic, Δp_L is found to be indefinitely increasing with the amplitude ratio, ϕ for any given flow rate, Q and the particle concentration, C in both the single and double-layered analyses and assume a very

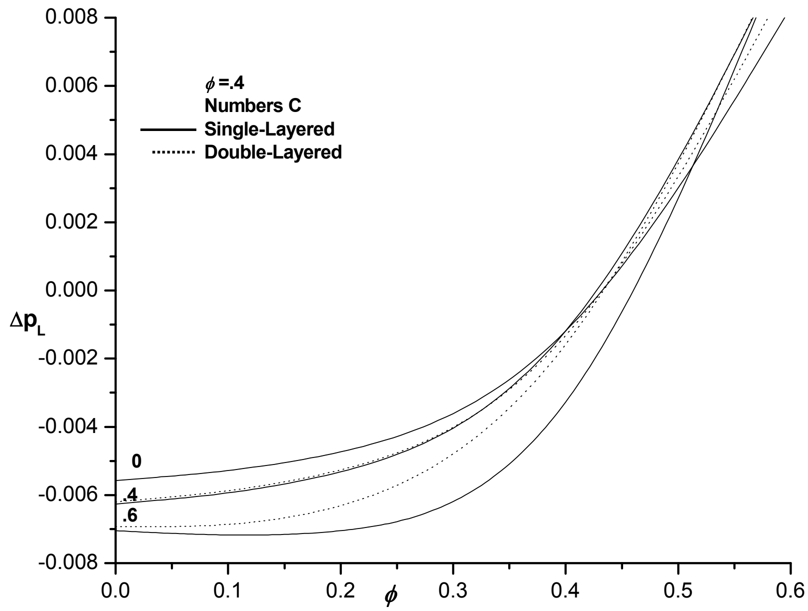

Fig. 8 Variation of average pressure rise, Δp_L with ϕ in a non-uniform channel.

high asymptotic magnitude as $\phi \to 0.6$ (Figs. 8 and 9). The average pressure rise, Δp_L seems to be steeply increasing with the particle concentration, C for small values of the flow

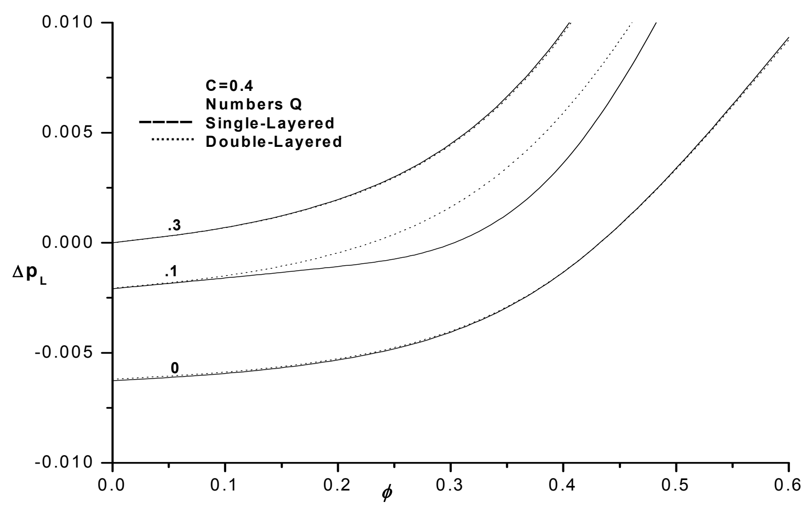

Fig. 9 Variation of average pressure rise, Δp_L with ϕ in a non-uniform channelfor different Q.

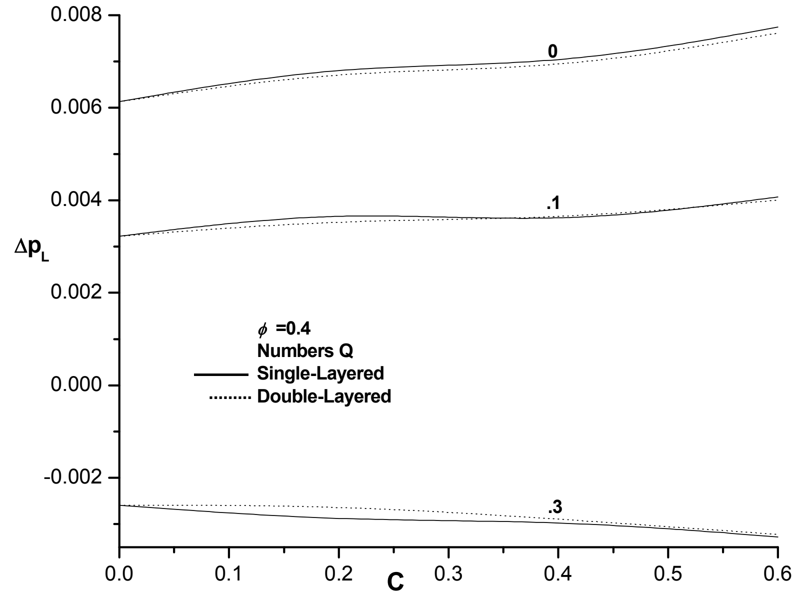

Fig. 10 Variation of average pressure rise, Δp_L with C in a non-uniform channel for different Q.

rate $Q(\leq 0.2)$, however, the flow characteristic, Δp_L is found to be decreasing with increasing flow rate, Q for higher values of Q (Fig.10). For any given amplitude ratio, Q, Δp_L decreases with increasing flow rate, Q (Fig.11).

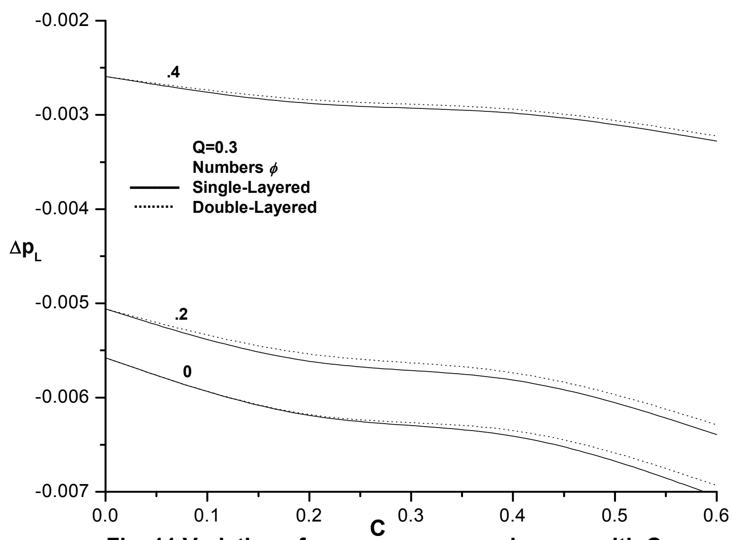

Fig. 11 Variation of average pressure rise, Δp_L with C in a non-uniform channel for different ϕ.

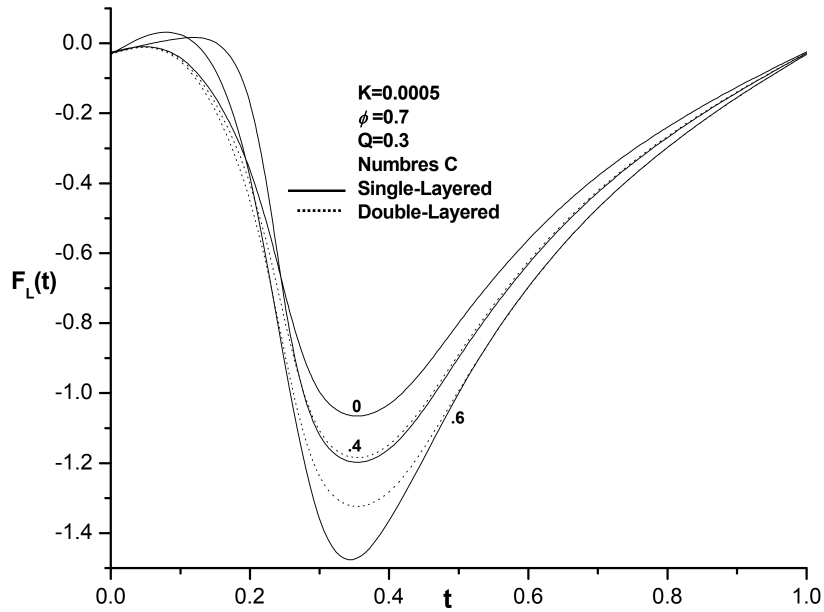

Fig. 12 Variation of friction force, $F_L(t)$ in a non-uniform channel with time, t for different C.

The non-dimensional friction, $F_L(t)$ decreases with increasing particle concentration, C in both the single and double-layered analyses (Fig. 12). $F_L(t)$ is found to be decreasing with decreasing the flow rate, Q. The lowest magnitude of $F_L(t)$ occurs for $0.3 \le t \le 0.4$. The flow characteristic, $F_L(t)$ assumes much lower magnitude in non-uniform channel than its corresponding value in uniform channel (Figs. 14 and 15). Numerical results further reveals that there exists a linear relationship between the average friction force, F_L and the average flow rate, Q. An inspection of the illustrations also reveal that the friction force $F_L(t)$ and its averaged value, F_L possesses characteristics similar to that of $\Delta p_L(t)$ and Δp_L , respectively with respect to any given parameter.

Present investigation has been carried out under various approximations and assumptions. Comments needs to made here regarding the same and use of the some of the parameters involved in the study. It is noted that the peripheral layer thickness, ε decreases with increasing particle concentration, C in the core region, consequently the parameter α increases with particle concentration, C. The explanation regarding the shape of the interface needs to be given here. It is known from the published works (Brasseur et.al., 1987; Rao and Usha, 1995) that the interface shape depends on the viscosity ratio of the fluids in the two regions (central and peripheral) and it does not bear a constant ratio of radii of the central and peripheral layers during the course of peristaltic action. However, the shape of interface is not significantly affected when the viscosity

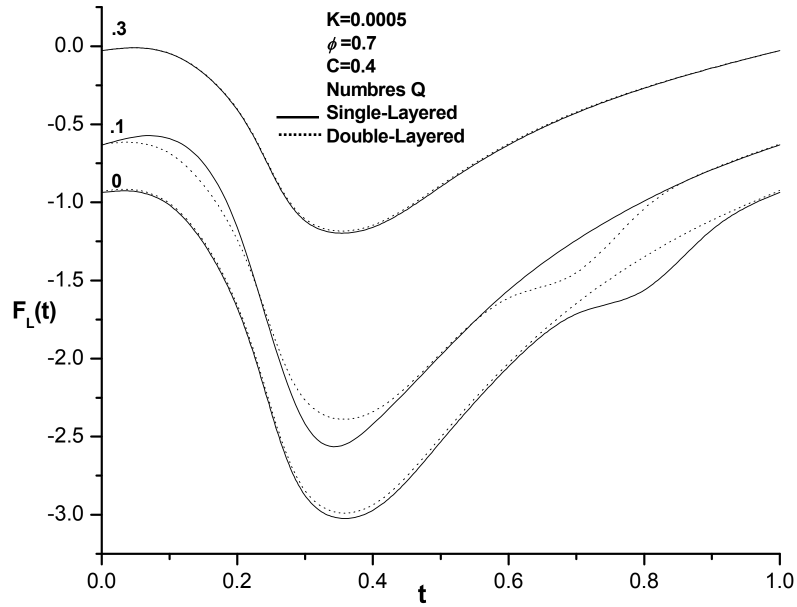

Fig. 13 Variation of friction force, $F_L(t)$ in a non-uniform channel with time, t for different Q.

of the fluid in one of the layers is varied with respect to the fluid viscosity in the other layer and consequently the radii ratio remains approximately constant (Misra and Pandey, 2002). The peripheral layer fluid viscosity μ_o remains always constant and it is suspension viscosity, μ_s varies with particle concentration, C in the central core layer. The condition stated in Misra and Pandey (2002) is obviously satisfied which allows the use of constant value of α for a given particle concentration, *C*. Other limitations of the study are well addressed by earlier investigators including Shapiro et al. (1969), Shukla et al (1980), Srivastava and Saxena (1995), Medhavi (2010), etc.

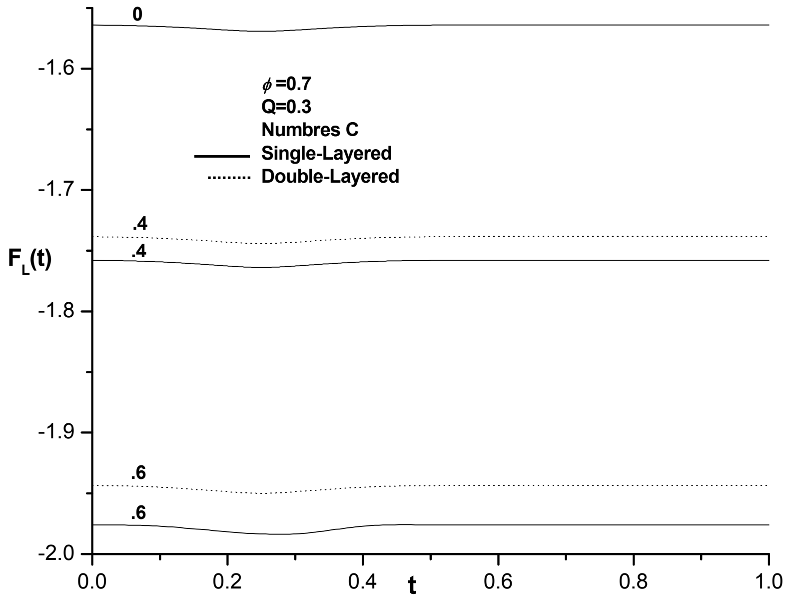

Fig. 14 Variation of friction force, $F_L(t)$ in a uniform channel with time, t for different C.

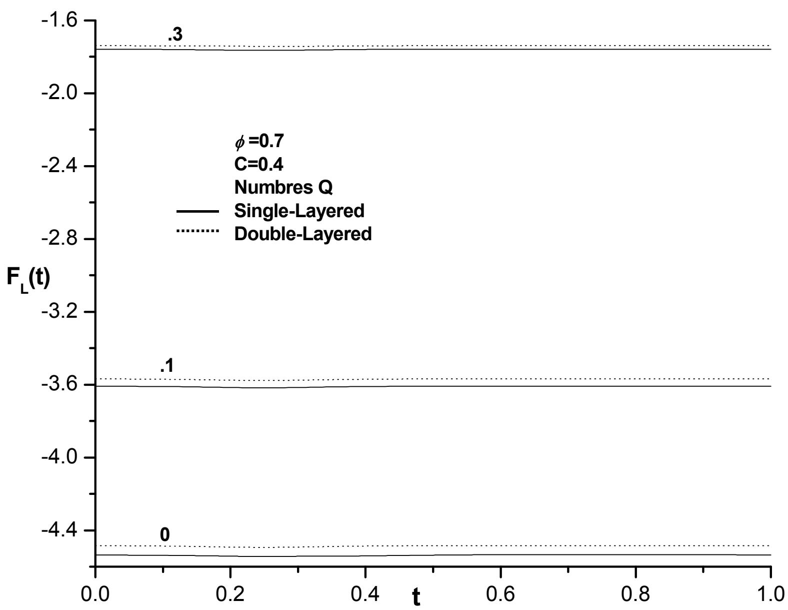

Fig. 15 Variation of friction force, $F_L(t)$ in a uniform channel with time, t for different Q.

5. Conclusions

To investigate the simultaneous effects of the peripheral layer and the particle concentration on the peristaltic pumping, the flow induced by peristaltic waves of a two-layered particulate suspension has been studied. It has been found that the pressure rise increases with the particle concentration in the core region and assumes a significantly lower magnitude in two-layered analysis than its corresponding value in one-fluid model. A linear relationship between the pressure and the flow is exhibited. The friction force possesses characteristics opposite to those of the pressure rise with respect to any given parameter. As evident from the published literature (Misra and Pandey, 2002; Medhavi and Singh, 2009) peristalsis does play an important role in vasomotion of small blood vessels in addition to the pulsatile flow, the findings of the present theoretical work (Srivastava, 2007b), may be applied to explain the flow behavior of blood in small vessels with varying cross-section.

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