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Effect of Rising Temperature Due to Ozone Depletion on the Dynamics of a Prey-Predator System: A Mathematical Model

O.P. Misra and Preety Kalra

School of Mathematics and Allied Sciences
Jiwaji University
Gwalior (M.P.), India
misra_op@rediffmail.com; kalra.preety@gmail.com

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Abstract

It is well recognized that the greenhouse gas such as Chlorofluoro Carbon (*CFC*) is responsible directly or indirectly for the increase in the average global temperature of the Earth. The presence of *CFC* is responsible for the depletion of ozone concentration in the atmosphere due to which the heat accompanied with the sun rays are less absorbed causing increase in the atmospheric temperature of the Earth. The increase in the temperature level directly or indirectly affects the dynamics of interacting species systems. Therefore, in this paper a mathematical model is proposed and analyzed using stability theory to asses the effects of increasing temperature due to the greenhouse gas *CFC* on the survival or extinction of populations in a prey-predator system. A threshold value in terms of a stress parameter is obtained which determines the extinction or existence of populations in the underlying system.

Keywords: Model, Equilibria and Stability, Prey-Predator Populations, Greenhouse gases, Temperature

MSC (2010)No.: 92, 92B05

1. Introduction

The atmospheric concentration of greenhouse gas such as chlorofluoro carbon (*CFC*) is increasing due to rapid industrialization, extensive use of automobiles, burning of fossil fuels, construction of power plants and other anthropogenic activities. The excess of *CFC* depletes the

ozone layer in the atmosphere and therefore the heat accompanied with the sun rays are less absorbed, raising the atmospheric temperature (Stordal (1986), Singer (1989), Misra et al. (2012), Misra et al. (2011)). There is a growing body of evidence that the change in the environmental temperature has impacts on the existence of interacting species systems. Increasing evidence suggests that recent climate warming has already affected various aspects of ecological communities, including organism phenology, species abundance and distribution, population dynamics and community- and ecosystem-level properties (reviewed in (Hughes (2000), McCarty (2001), Stenseth et al. (2002), Walther et al. (2002)); meta-analyses in (Parmesan and Yohe (2003), Root et al. (2003), Jing and Morin (2004)).Both theoretical (Ives and Gilchrist (1993), Ives (1995), Abrams (2001)) and empirical studies (Jing and Morin (2004), Brown et al. (2001),Davis (1998a), Davis (1998b), Post (1999))have suggested that interspecific interactions could alter species' responses to climate change substantially. Species interactions may themselves be affected by climate change. Both the sign and intensity of interactions may be influenced by climate. For example, climate warming can affect the strength of predator-prey interactions (Post (1999), Sanford (1999), Jing and Morin (2004)).

Petchey et al. (2010) provide an intriguing example of how one of the fundamental food-web properties that is connectance-changes with increasing temperature. [Yvon-Durocher et al. (2010), Sarmento et al. (2010)] focus on how climatic warming affects the metabolic rate of organisms, that is, the power required to sustain them and how these changes in metabolism scale up to ecosystem processes. In both the papers the metabolic theory of ecology (Brown et al. (2004)) is used to predict the changes in the process rates under increasing temperature. It has been shown in the papers [Brown et al. (2004), West et al. (1997)] that the increasing temperature changes the processes at different levels of biological organization. Species higher in the food web like top predators tend to be more sensitive to temperature change [Voigt et al. (2003)]. Top predators moving towards cooler climates may trigger trophic cascades and coextinctions may also occur [Schmitz (2003)].

A persistent warming trend, driven largely by anthropogenic production of greenhouse gases, is projected to cause the global surface temperature to rise between 1.4^{O} and $5.8^{O}C$ by the end of the 21st century (compared with $0{\cdot}6^{O}C$ in the 20th century, IPCC 2001). How to predict accurately the responses of species and communities to rapid climate warming in the 21st century thus, emerges as an important question [Jing and Morin (2004)]. The increase in temperature level may directly or indirectly affect the dynamics of interacting species systems. Therefore, it is essential to assess mathematically the effects of increasing *CFC* on populations in order to take necessary measures to avoid any adverse impact of rising temperature on an ecosystem. It is suggested that for understanding the consequences of the greenhouse effects on ecosystem an investigation of the interspecific interactions within biotic communities is required. A very few models to study temperature dependent interacting species systems exist [Misra et al. (2012), Misra et al. (2011), Wollkind and Logan (1978), Wollkind et al. (1988), Wollkind et al. (1991), Collings et al. (1990), Collings (1992), Collings (1995), Logan et al. (2006), Logan and Wolesensky (2007), Norberg and Deangelts (1997), Zhang and Kreis (2008)].

In view of the above, therefore in this paper, a mathematical model has been proposed and analyzed to study the effects of increasing temperature due to greenhouse gas *CFC* on the

survival or extinction of populations in a prey-predator system. In the model it is assumed that the temperature increases due to greenhouse gas *CFC* because of the depletion of the ozone concentration in the atmosphere. In the model it is further assumed that the rise in temperature negatively effects the intrinsic growth rate of the prey and adversely effects the prey-predator interaction phenomenon.

2. Mathematical Model

Let N_1 denotes the density of a prey population which is growing logistically and N_2 denotes the density of a predator population. C denotes the concentration of CFC (Chlorofluoro carbon). Z denotes ozone concentration. We consider here that T is elevated temperature or average increased temperature of the surrounding environment where the species live.

For a predator let its searching capacity per unit prey is d_1. Hence, searching capacity of the predator population for prey density N_1 is $d_1 N_1$. If the handling capacity per unit prey by a predator is d_2 then, the handling capacity for prey density N_1 is $d_2 N_1$.

It is assumed in the model construction that the searching capacity is adversely affected by the increasing temperature of the environment and therefore d_1 is taken to be $\frac{d_1}{1+B(T-T_0)}$.

Similarly, it is also assumed in the model formulation that the handling capacity is also adversely affected by the increasing temperature of the environment and therefore, d_2 is considered to be equal to $\frac{d_2}{1+B(T-T_0)}$. Thus, the total searching and handling capacity by a predator for prey density N_1 is given by

$$\frac{d_1 N_1}{1+B(T-T_0)} + \frac{d_2 N_1}{1+B(T-T_0)} = \frac{(d_1+d_2)N_1}{1+B(T-T_0)} = \frac{a_1 N_1}{1+B(T-T_0)}, \tag{1}$$

where, $a_1 = d_1 + d_2$.

From the expression (1), we note that when the environment is at the normal temperature; T_0, that is, at $T = T_0$, the predator behaves naturally and there is no change in their searching and handling capacity. We also notice from (1) that the predation rate will only be affected when temperature T exceeds T_0. With the above notations and assumptions, the mathematical model of the system under consideration is given by the following system of nonlinear differential equations:

$$\frac{dN_1}{dt} = r_1(T)N_1 - \frac{a_1 N_1 N_2}{1+B(T-T_0)} - \frac{r_{10} N_1^2}{K_{10}}, \tag{2}$$

$$\frac{dN_2}{dt} = -r_{20}N_2 + \frac{a_2 N_1 N_2}{1 + B(T - T_0)}, \tag{3}$$

$$\frac{dC}{dt} = P - \frac{C}{\tau} - \beta CZ, \tag{4}$$

$$\frac{dZ}{dt} = Q_0 - \alpha_2 Z - \beta ZC, \tag{5}$$

$$\frac{dT}{dt} = \frac{K_1}{K_2 + Z} - \alpha_1 (T - T_0), \tag{6}$$

with the initial conditions as:

$$N_1(0) > 0,\ N_2 0) > 0, C(0) > 0,\ Z(0) > 0, T(0) > 0.$$

In the present analysis we assume the following form of $r_1(T)$:

$$r_1(T) = r_{10} - r_{11}(T - T_0),\ \ r_1(T) > 0\ \forall T,\ r_1(T_0) = r_{10} \tag{7}$$

and $a_2 = \gamma a_1$ where γ is conversion coefficient. It may be noted here that at the normal temperature T_0, the growth rate of prey population is r_{10} which is its intrinsic growth rate. The system parameters are defined as follows:

r_{20} is natural death rate of predator population. B is a constant which measures the stress of temperature on both searching and handling capacity. P is input rate of C. τ is average atmospheric residence time of *CFC*. β is the depletion rate of ozone due to *CFC*. Q_0 is the natural formation rate of ozone concentration in the atmosphere. T_0 is average normal temperature of earth surface of the area occupied by the populations under study. $r_1(T)$ is growth rate of population N_1. K_{10} is carrying capacity of prey population N_1. α_1 is coefficient of surface heat transfer and α_2 is natural depletion rate of ozone concentration. Here, all the parameters $K_1, K_2, r_{10}, K_{10}, r_{11}, B, a_1, a_2, Q_0, \beta, \alpha_1, \alpha_2$ and r_{20} are all taken to be positive constants.

For the case when the effects of *CFC*, temperature and ozone on the prey-predator system are not considered then we have the following basic model for the system of prey-predator populations considering carrying capacity in prey population:

$$\frac{dN_1}{dt} = r_{10}N_1 - a_1 N_1 N_2 - \frac{r_{10}N_1^2}{K_{10}}, \tag{8}$$

$$\frac{dN_2}{dt} = -r_{20}N_2 + a_2 N_1 N_2\,. \tag{9}$$

3. Boundedness and Dynamical Behaviour

In this section we will establish that the solutions of the model given by the set of equations (2) to (6) with equation (7) are bounded in R^5_+. The boundedness of solutions is given by the following lemma.

Lemma 3.1.

All the solutions of the model will lie in the region

$$V_1 = \Big\{(N_1, N_2, C, Z, T) \in R^5_+ : 0 < N_1 \le K_{10}, 0 < N_2 \le {r_{10}K_{10}}\Big/{\eta} = N_M, 0 < C \le C_M, 0 < Z_m \le Z \le Z_M,\ 0 < T_m \le T \le T_M\Big\},$$

as $t \to \infty$, for all positive initial values $(N_1(0),\ N_2(0), C(0),\ Z(0), T(0)) \in R^5_+$, where,

$$C_M = P\tau,\ T_M = \frac{K_1}{\alpha_1(K_2 + Z_m)} + T_0,\ T_m = \frac{K_1}{\alpha_1(K_2 + Z_M)} + T_0,\ Z_M = \frac{Q_0}{\alpha_2} \text{ and } Z_m = \frac{Q_0}{\alpha_2 + \beta P\tau}.$$

Proof:

From Equations (2) and (3) we get,

$$\begin{aligned}\frac{d(N_1 + N_2)}{dt} &\le (r_{10} - r_{11}(T - T_0)N_1) - r_{20}N_2 - \frac{N_1N_2}{1 + B(T - T_0)}(a_1 - a_2)\\ &\le r_{10}K_{10} - r_{11}(T_m - T_0)N_1 - r_{20}N_2 - \frac{N_1N_2}{1 + B(T - T_0)}(a_1 - a_2)\\ &\le r_{10}K_{10} - \eta(N_1 + N_2),\end{aligned}$$

if $a_2 < a_1$, i.e. $\gamma < 1$, where, $\eta = \min(r_{11}(T_m - T_0), r_{20})$.

Then by the usual comparison theorem we get as $t \to \infty$:

$$(N_1 + N_2) \le \frac{r_{10}K_{10}}{\eta}$$

and hence,

$$N_2 \le \frac{r_{10}K_{10}}{\eta}$$

From Equation (6) we get,

$$\frac{dT}{dt} \le A - \alpha_1 T ,$$

where

$$A = \frac{K_1}{K_2 + Z_m} + \alpha_1 T_0 .$$

Then by the usual comparison theorem we get as $t \to \infty$:

$$T \le \frac{A}{\alpha_1} ,$$

i.e.,

$$T \le \frac{K_1}{\alpha_1 (K_2 + Z_m)} + T_0 = T_M .$$

Again from Equation (6) we get ,

$$\frac{dT}{dt} \ge \frac{K_1}{(K_2 + Z_m)} + \alpha_1 T_0 - \alpha_1 T$$

Then by the usual comparison theorem we get as $t \to \infty$:

$$T \ge \frac{K_1}{\alpha_1 (K_2 + Z_m)} + T_0 = T_m$$

Similarly from Equations. (4) and (5), we get as $t \to \infty$:

$$C \le P\tau = C_M , \; Z \le \frac{Q_0}{\alpha_2} = Z_M$$

Again from Equation (5), we get

$$\frac{dZ}{dt} \ge Q_0 - \alpha_2 Z - \beta Z P \tau$$

By the usual comparison theorem we get as $t \to \infty$:

$$Z \geq \frac{Q_0}{\alpha_2 + \beta P \tau} = Z_m .$$

This completes the proof of the Lemma 3.1.

We now find all the feasible equilibria of the system (2) - (6). The system of equations (2) - (6) has three feasible equilibria $E_i (i = 1,2,3)$ as given below:

1. $E_1(N_1^*, N_2^*, C^*, Z^*, T^*)$, where, $N_1^* = 0$, $N_2^* = 0$,

$$C^* = \frac{P\tau}{1 + \beta \tau Z^*}, \tag{10}$$

$$Z^* = \frac{-a_2 + \sqrt{a_2^2 - 4a_1 a_3}}{2a_1}, \tag{11}$$

$$a_1 = \alpha_2 \beta \tau,\ a_2 = \alpha_2 + \beta\tau(P - Q_0),\ a_3 = -Q_0,$$

$$T^* = \frac{1}{\alpha_1}\left(\frac{K_1}{K_2 + Z^*} + \alpha_1 T_0\right), \tag{12}$$

2. $E_2(N_1^*, N_2^*, C^*, Z^*, T^*)$, where, $N_1^* = \frac{r_1(T^*)K_{10}}{r_{10}}$, $N_2^* = 0$, and C^*, Z^*, T^* are given by (10)-(12), respectively. The equilibrium E_2 exists if $r_1(T^*) > 0$.

3. $E_3(N_1^*, N_2^*, C^*, Z^*, T^*)$, where, $N_1^* = \frac{r_{20}}{\gamma a_1}(1 + B(T^* - T_0))$,

$$N_2^* = \frac{1 + B(T^* - T_0)}{K_{10}\gamma a_1^2}\left[r_1(T^*)K_{10}\gamma a_1 - r_{10}r_{20}(1 + B(T^* - T_0))\right],$$

and C^*, Z^*, T^* are given by (10) - (12) respectively.

The equilibrium E_3 exists if $T^* > T_0$ and $r_1(T^*)K_{10}\gamma a_1 > r_{10}r_{20}(1 + B(T^* - T_0))$ or

$$\frac{r_{10}}{K_{10}} > \frac{\gamma a_1 r_{11}(T^* - T_0)}{\gamma a_1 K_{10} - r_{20}(1 + B(T^* - T_0))}. \tag{13}$$

Remark 1.

The analytic threshold value of B can be determined from the expression given in the existence of equilibrium E_3 that is,

$$r_1(T^*)K_{10}\gamma > r_{10}r_{20}\frac{(1+B(T^*-T_0))}{a_1}$$

which implies

$$B < \frac{1}{(T^*-T_0)}\left(\frac{\gamma a_1 K_{10} r_1(T^*)}{r_{10}r_{20}} - 1\right) = B_0 .$$

It is concluded that Equilibrium E_3 exists, i.e., the prey and predator populations would co-exist if the parameter B is less than its threshold value B_0. Further, it is shown that if the value of the parameter B is more than its threshold value B_0 then the predator will die out. This condition can be biologically stated as the product of carrying capacity, conversion coefficient and growth rate of prey population with temperature is greater than the product of growth rate of prey population, natural death rate of predator population and inverse of searching and handling capacities.

Remark 2.

From the equilibrium value it is noted that the environmental temperature increases on account of decreasing ozone concentration in the atmosphere (see Figure 1). Further, it may be noted that the equilibrium ozone concentration decreases due to the increase in the equilibrium concentration of Chlorofluoro-Carbon (see Figure 2).

Remark 3.

The system of equations (8) - (9) has three feasible equilibria $E_1'(0,0)$,

$$E_2'(K_{10},0) \text{ and } E_3'(N_1'^*, N_2'^*), \text{ where, } N_1'^* = \frac{r_{20}}{\gamma a_1},\ N_2'^* = \frac{1}{K_{10}\gamma a_1^2}\left[r_{10}K_{10}\gamma a_1 - r_{10}r_{20}\right].$$

The equilibrium E_3' exists if $K_{10}\gamma a_1 > r_{20}$ is satisfied.

Here, we see that the equilibrium value E_3' and its existence criteria do not depend on temperature.

Stability Results of Basic Model

From the stability analysis of the system (8) - (9), it is found that

(i) $E_1^{'}$ is unstable.
(ii) $E_2^{'}$ is linearly asymptotically stable if the condition $K_{10}\gamma a_1 < r_{20}$ is satisfied.
(iii) $E_3^{'}$ is linearly asymptotically stable.

Further, from the stability analysis it is noted that $E_2^{'}$ is linearly asymptotically stable, only when $E_3^{'}$ does not exist and $E_3^{'}$ is linearly asymptotically stable only when $E_2^{'}$ is unstable.

Now we discuss the dynamical behaviour of the model (2) - (6).

3.1. Local Stability

The characteristic equation associated with the variational matrix about equilibrium E_1 is given by

$$(J_1-\lambda)(J_2-\lambda)(J_7-\lambda)\{(J_6-\lambda)(J_3-\lambda)-J_4J_5\}=0, \tag{14}$$

where,

$$J_1=r_1(T^*);J_2=-r_{20};J_3=-\frac{1}{\tau}-\beta Z^*;J_4=-\beta C^*;$$
$$J_5=-\beta Z^*;J_6=-(\alpha_2+\beta C^*);J_7=-\alpha_1.$$

From the nature of the roots of the characteristic equation (14) we observe that the equilibrium point E_1 is locally unstable provided $r_1(T^*)>0$.

Remark 3.

If $r_1(T^*)<0$, then E_1 is locally asymptotically stable and obviously both prey and predator populations would die out eventually.

The characteristic equation related to the equilibrium point E_2 is obtained as

$$(G_2-\lambda)(G_1-\lambda)(G_7-\lambda)\{(G_3-\lambda)(G_6-\lambda)-G_4G_5\}=0, \tag{15}$$

where

$$G_1=r_1(T^*)-\frac{2r_{10}N_1^*}{K_{10}}=-r_1(T^*);G_2=-r_{20}+\frac{a_2N_1^*}{1+B(T^*-T_0)};$$

$$G_3=-\frac{1}{\tau}-\beta Z^*;G_4=-\beta C^*;G_5=-\beta Z^*;$$

$$G_6=-(\alpha_2+\beta C^*);G_7=-\alpha_1.$$

From the characterstic equation (15) we find that the equilibrium point E_2 is linearly asymptotically stable under the condition given by:

$$\frac{r_{10}}{K_{10}}<\frac{\gamma a_1 r_{11}(T^*-T_0)}{\gamma a_1 K_{10}-r_{20}(1+B(T^*-T_0))},\ T^*>T_0. \tag{16}$$

The characteristic equation associated with the variational matrix about equilibrium E_3 is given by

$$(\alpha_1+\lambda)\{P_6P_7-(P_5-\lambda)(P_8-\lambda)\}\{(P_1-\lambda)(P_4-\lambda)-P_2P_3\}=0, \tag{17}$$

where,

$$P_1=r_1(T^*)-\frac{a_1N_2^*}{1+B(T^*-T_0)}-\frac{2r_{10}N_1^*}{K_{10}}=-\frac{r_{10}r_{20}}{K_{10}\gamma a_1}(1+B(T^*-T_0));$$

$$P_2=-\frac{a_1N_1^*}{1+B(T^*-T_0)};P_3=\frac{\gamma a_1N_2^*}{1+B(T^*-T_0)};P_4=-r_{20}+\frac{\gamma a_1N_1^*}{1+B(T^*-T_0)};$$

$$P_5=-\frac{1}{\tau}-\beta Z^*;P_6=-\beta C^*;P_7=-\beta Z^*;P_8=-(\alpha_2+\beta C^*).$$

From the nature of the roots of characteristic equation (17) we find that the equilibrium point E_3 is linearly asymptotically stable provided $T^*>T_0$.

Further, from the above analysis it is noted that E_2 is linearly stable only when E_3 does not exist and E_3 is linearly stable only if E_2 is unstable.

It is observed from the analysis that the stability conditions are depending upon the equilibrium temperature level and average normal temperature.

3.2. Global Stability

Next, we discuss the global stability of the interior equilibrium point E_3.

Theorem 3.1.

The box V_1 is a compact positively invariant set in space (N_1, N_2, C, Z, T).

Proof:

Consider the system given by Equations. (2)-(6). To prove the theorem, we consider the point $X' = (N_1', N_2', C'. Z', T')$ out side the box V_1, with $N_1' > K_{10}$, $N_2' > r_{10}K_{10}/\eta$, $C' > P\tau, Z' > \frac{Q_0}{\alpha_2}$ and $T' > \frac{A}{\alpha_1}$ and take the box V_1 in the phase space (N_1, N_2, C, Z, T) with one vertex located at the origin and the other at X'.

Now, let us compute the angle that the flow makes with each one of the faces of V_1 not lying on the coordinate planes. Consider the planes $\pi_{N_1} : N_1 = N_1'$, $\pi_{N_2} : N_2 = N_2'$, $\pi_C : C = C'$, $\pi_Z : Z = Z'$ and $\pi_T : T = T'$ and let $n_{N_1}, n_{N_2}, n_C, n_Z$ and n_T are outward unit normal vectors (with respect to box V_1), respectively, to each plane. Then,

$$n_{N_1} \frac{dX}{dt}\Big|_{\pi_{N_1}} = N_1' \left(r_1(T') - \frac{a_1 N_2'}{1 + B(T' - T_0)} - \frac{r_{10} N_1'}{K_{10}} \right),$$

and we get

$$n_{N_1} \frac{dX}{dt}\Big|_{\pi_{N_1}} \le N_1' \left(-r_{11} T' - \frac{a_1 N_2'}{1 + B(T' - T_0)} \right),$$

hence,

$$n_{N_1} \frac{dX}{dt}\Big|_{\pi_{N_1}} \le 0.$$

Similarly we can show that

$$n_{N_2} \frac{dX}{dt}\Big|_{\pi_{N_2}} \le 0,\ n_C \frac{dX}{dt}\Big|_{\pi_C} \le 0,\ n_Z \frac{dX}{dt}\Big|_{\pi_Z} \le 0,\ n_T \frac{dX}{dt}\Big|_{\pi_T} \le 0,$$

where $\frac{dX}{dt}=\left(\frac{dN_1}{dt},\frac{dN_2}{dt},\frac{dC}{dt},\frac{dZ}{dt},\frac{dT}{dt}\right)$. Thus, the flow along the normals to each of the plane is again moving towards the box. Clearly we can say that box V_1 is compact positively invariant box. This completes the proof of the Theorem 3.1.

Now it is clear by the above theorem that the trajectories of the system cannot cross V_1 once they enter inside. It is also observed that the interior equilibrium E_3 lies inside V_1. Moreover, E_3 is only attractor inside V_1, which is established in the following theorem.

Theorem 3.2.

The equilibrium E_3 is non-linearly asymptotically stable with respect to solution initiating in the interior of V_1 if the following inequalities hold:

$$A_1 r_{10}\left(\frac{\gamma a_1 N_M}{1+B(T_m-T_0)}+r_{20}\right)>\frac{K_{10}{a_1}^2}{(1+B(T_m-T_0))^2}(1-\gamma {N_2}^* A_1)^2 \tag{18}$$

and

$$2A_3A_2(\alpha_2+\beta C^*)\left(\frac{1}{\tau}+\beta Z_m\right)>\beta^2(A_3C^*+A_2Z_m)^2. \tag{19}$$

Proof:

Taking the perturbations about the equilibrium value as follows:

$$N_1={N_1}^*+u_1(t), N_2={N_2}^*+u_2(t), C=C^*+v_1(t),$$

$$Z=Z^*+x(t), T=T^*+t_1(t).$$

The non-linearised system of equations from (2) to (6) about equilibrium point E_3 is given by

$$\frac{du_1}{dt}=(N_1^*+u_1)\left(\begin{array}{l}-r_{11}t_1-\dfrac{a_1N_2^*Bt_1}{(1+B(T^*-T_0))(1+B(T-T_0))}-\dfrac{r_{10}u_1}{K_{10}}\\ -\dfrac{a_1u_2}{1+B(T-T_0)}\end{array}\right), \tag{20}$$

$$\frac{du_2}{dt} = -r_{20}u_2 - \frac{\gamma a_1 N_1^* N_2^* B t_1}{(1+B(T^*-T_0))(1+B(T-T_0))} + \frac{\gamma a_1}{1+B(T-T_0)}(N_1 u_2 + N_2^* u_1), \tag{21}$$

$$\frac{dv_1}{dt} = -\frac{v_1}{\tau} - \beta C^* x - \beta(Z^*+x)v_1 \tag{22}$$

$$\frac{dx}{dt} = -\alpha_2 x - \beta C^* x - \beta(Z^*+x)v_1 \tag{23}$$

$$\frac{dt_1}{dt} = \frac{-K_1}{(K_2+Z^*)(K_2+Z^*+x)} + hw_1 - \alpha_1 t_1. \tag{24}$$

Consider,

$$G(t) = \left[u_1 - N_1^* \log\left(1+\frac{u_1}{N_1^*}\right) \right] + \frac{1}{2}A_1 u_2^2 + \frac{1}{2}A_2 v_1^2 + \frac{1}{2}A_3 x^2 + \frac{1}{2}A_4 t_1^2,$$

where, A_i(i=1 to 4) are arbitrary positive constants.

The time derivative of $G(t)$ is given by

$$\frac{dG}{dt} = \frac{u_1}{N_1^* + u_1}\frac{du_1}{dt} + A_1 u_2 \frac{du_2}{dt} + A_2 v_1 \frac{dv_1}{dt} + A_3 x_1 \frac{dx_1}{dt} + A_4 t_1 \frac{dt_1}{dt}.$$

Now, using the system of equations (20)-(24) in $\frac{dG}{dt}$ in the region V_1, we get

$$\begin{aligned}
\frac{dG}{dt} \le -\Bigg\{ & \frac{r_{10}}{2K_{10}}u_1^2 + \frac{a_1(1-\gamma_1 N_2^* A_1)}{1+B(T_m-T_0)}u_1 u_2 + \frac{A_1}{2}\left(\frac{\gamma a_1 N_M}{1+B(T_m-T_0)} + r_{20}\right)u_2^2 + \frac{r_{10}}{2K_{10}}u_1^2 \\
& + \left(r_{11} + \frac{a_1 N_2^* B}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right)u_1 t_1 + \frac{A_4}{3}\alpha_1 t_1^2 + \frac{A_1}{2}\left(\frac{\gamma a_1 N_M}{1+B(T_m-T_0)} + r_{20}\right)u_2^2 \\
& + \frac{A_1 B\gamma a_1 N_1^* N_2^*}{(1+B(T^*-T_0))(1+B(T_m-T_0))}u_2 t_1 + \frac{A_4}{3}\alpha_1 t_1^2 + \frac{A_3}{2}(\alpha_2 + \beta C^*)x^2 + \beta(A_3 C^* + A_2 Z_m)x v_1 \\
& + A_2\left(\frac{1}{\tau} + \beta Z_m\right)v_1^2 + \frac{A_3}{2}(\alpha_2 + \beta C^*)x^2 + \frac{A_4 K_1}{(K_2+Z^*)(K_2+Z_M)}x t_1 + \frac{A_4}{3}\alpha_1 t_1^2 \Bigg\}.
\end{aligned}$$

Using the Sylvester's criteria in the right hand side of the above expression and then choosing A_1, A_3, A_4 as follow

$$2A_4 > 3\frac{K_{10}}{\alpha_1 r_{10}}\left[r_{11}+\frac{a_1 N_2^* B}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right]^2,$$

$$3A_1\left[\frac{B\gamma a_1 N_1^* N_2^*}{(1+B(T^*-T_0))(1+B(T_m-T_0))}\right]^2 < 2A_4\left[r_{20}+\frac{\gamma a_1 N_M}{1+B(T_m-T_0)}\right]\alpha_1,$$

$$2A_3(\alpha_2+\beta C^*) > 3A_4\left[\frac{K_1}{(K_2+Z)(K_2+Z_m)}\right]^2.$$

It may be shown that $\frac{dG}{dt}$ is negative definite if the conditions (18) and (19) are being satisfied. Thus, it is proved that E_3 is globally (non-linearly) asymptotically stable in the region V_1.

4. Numerical Example

For the model, consider the following values of parameters-

r_{10}=5, r_{20}=2, K_{10}=20.0, r_{11}=0.01, a_1=0.4,a_2=0.2,

γ=0.5, P=0.8, T_0=10, τ=20.0, β=0.02,

Q_0=2, α_1=0.1, α_2=1.0, K_1=5, K_2=1.

For the above set of values of parameters and B=0.01<B_0=0.0497, we obtain the following value of interior equilibrium point $E_3(N_1^*, N_2^*, C^*, Z^*, T^*)$

N_1^*=11.86, N_2^*=5.48, C^*=9.54, Z^*=1.68, T^*=28.62.

It is noted that for the above set of parametric values, the stability conditions (13), (18) and (19) are satisfied. Hence, E_3^* is globally asymptotically stable (see Figures 3 and 5).

For the above parametric values, when B=0.052>B_0=0.0497 is considered then the conditions for the existence of the interior equilibrium point E_3 is violated and in this case the equilibrium point E_2 exists with the following equilibrium values:

N_1^*=19.25, N_2^*=0.00, C^*=9.54, Z^*=1.68, T^*=28.62.

The linear stability condition given by (16) for E_2 is satisfied for the above set of parametric values with B=0.0497 and the stability behaviour of the model for $B>B_0$ is shown in Figure 4.

$N_1'^*$=10.00 and $N_2'^*$=6.25 (when effect of temperature is not considered).

5. Conclusion

From the linear stability analysis of the equilibrium point E_2 it is concluded that the population with density N_2 would tend to extinction and population with density N_1 would survive but at lower equilibrium value due to the decrease in its growth rate on account of elevated temperature. The non-trivial positive equilibrium point E_3 exists only when the equilibrium point E_2 is not stable. Hence, from the linear as well as non-linear stability analysis of the non-trivial positive equilibrium E_3 it is concluded that the prey and predator populations would co-exist if the parameter B; which measures the stress of temperature is less than its threshold value B_0 (see Figures 3 and 6).

Further, it is shown that if the value of the parameter B is more than its threshold value B_0 then the predator population tends to extinction (see Figures 4 and 7). It is concluded from the analytical and numerical equilibrium value of E_3 and E_3' that the density of prey population with temperature is more than the density of prey population when effect of temperature is not considered (see Figure. 8). It is also shown from the equilibrium value of E_3 and E_3' that the density of predator population with temperature is less than the density of predator population when effect of temperature is not taken (see Figure 9).

It is further concluded from the numerical example that the equilibrium density of prey population is more in the absence of predator under the adverse effect of temperature. However, in the presence of predator population there is a significant decrease in the equilibrium density of prey population as compared to the case when there is no predation under the stress of temperature. Numerical example has been supplemented to validate the analytical results. The graphs of all the variables have been plotted with respect to time and from these graphs the stability behaviour is illustrated (see Figures 3, 4 and 5).

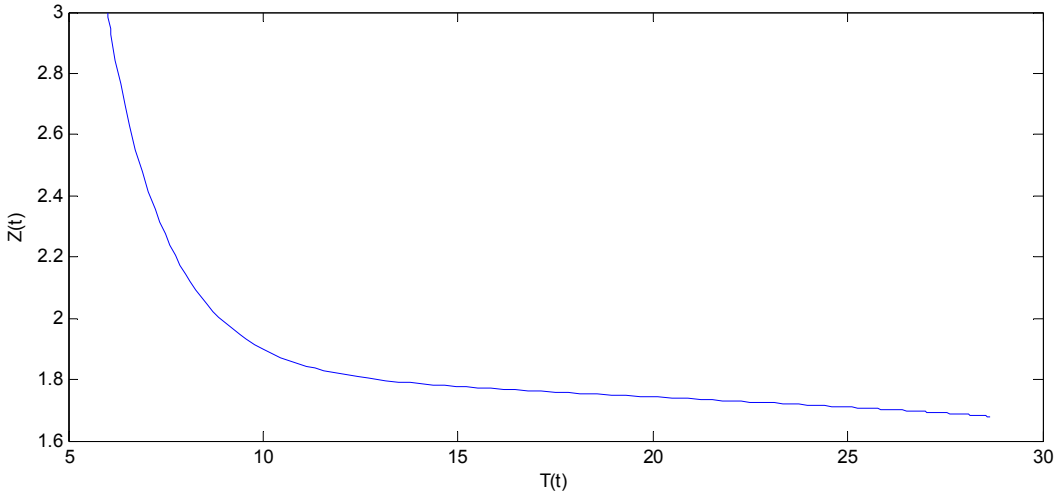

Figure 1. Phase space diagram for temperature $T(t)$ and ozone concentration $Z(t)$.

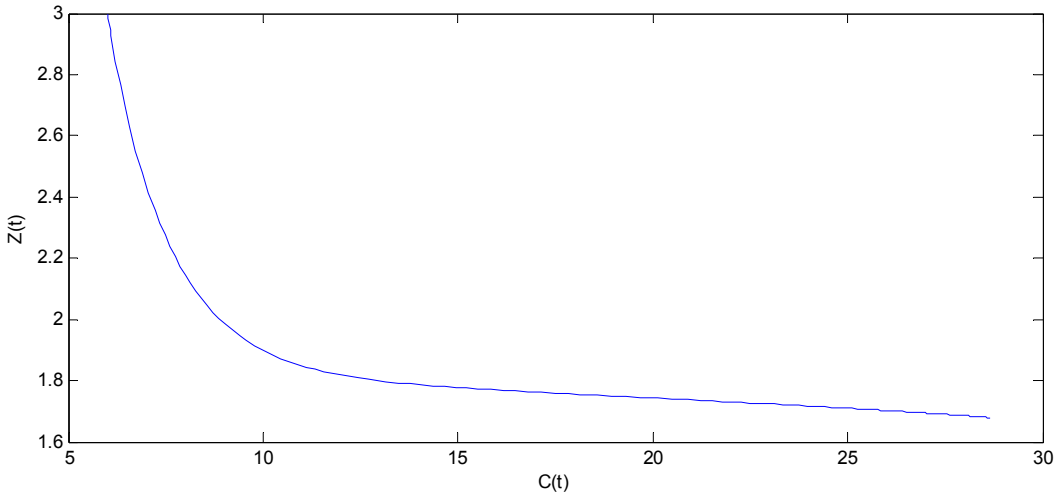

Figure 2. Phase space diagram for concentration of chlorofluoro carbon $C(t)$ and ozone concentration $Z(t)$.

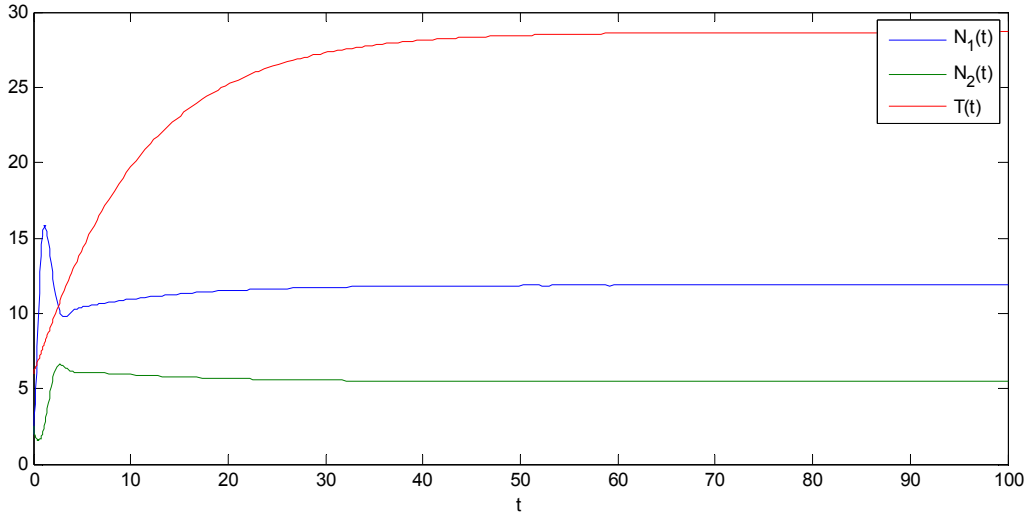

Figure 3. Time series graph for prey population $N_1(t)$, predator population $N_2(t)$ and temperature $T(t)$ when $B<B_0$.

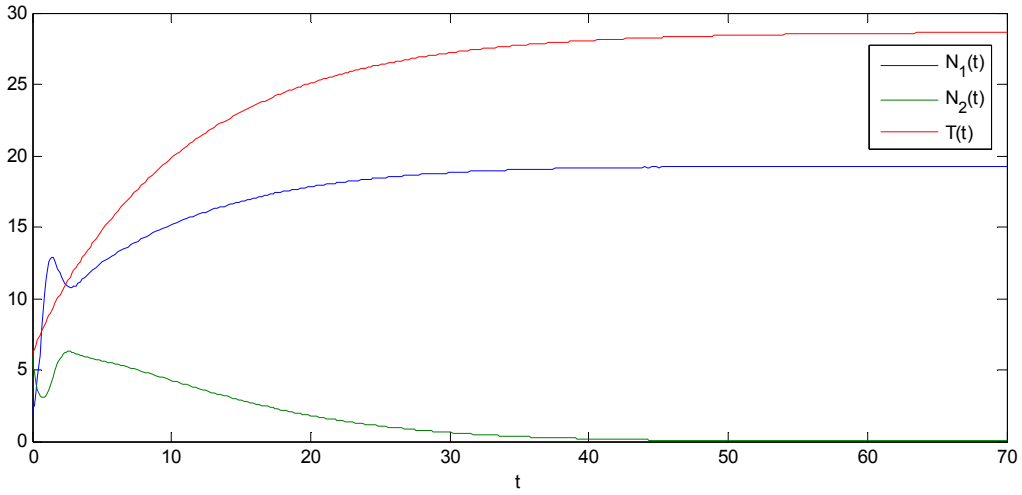

Figure 4. Time series graph for prey population $N_1(t)$, predator population $N_2(t)$ and temperature $T(t)$ when $B>B_0$.

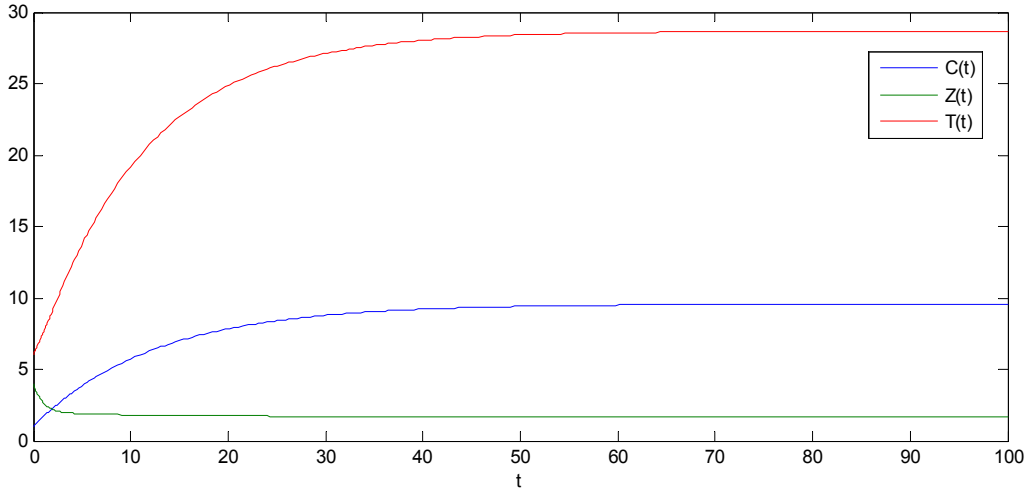

Figure 5. Time series graph for concentration of chlorofluoro carbon $C(t)$, ozone concentration $Z(t)$ and temperature $T(t)$.

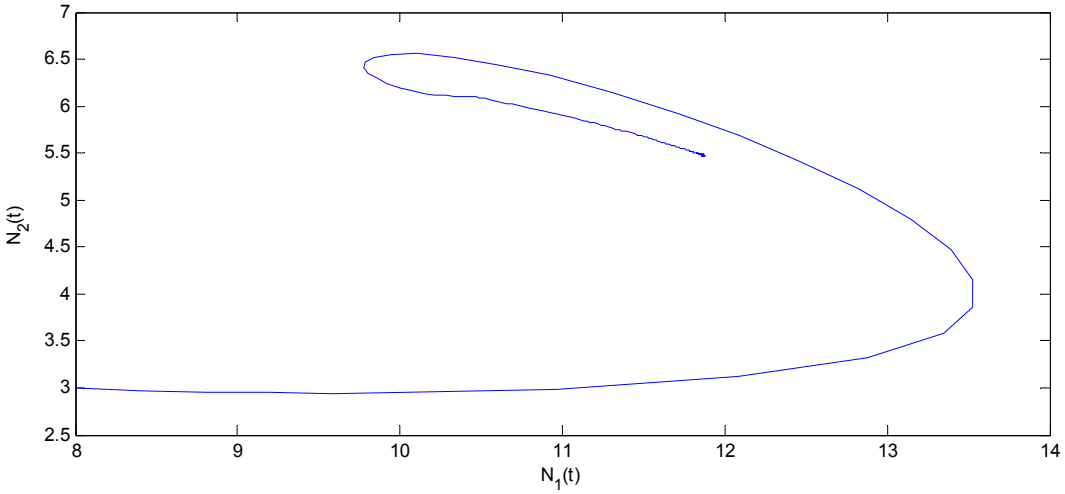

Figure 6. Phase space diagram for prey population $N_1(t)$ and predator population $N_2(t)$ when $B<B_0$.

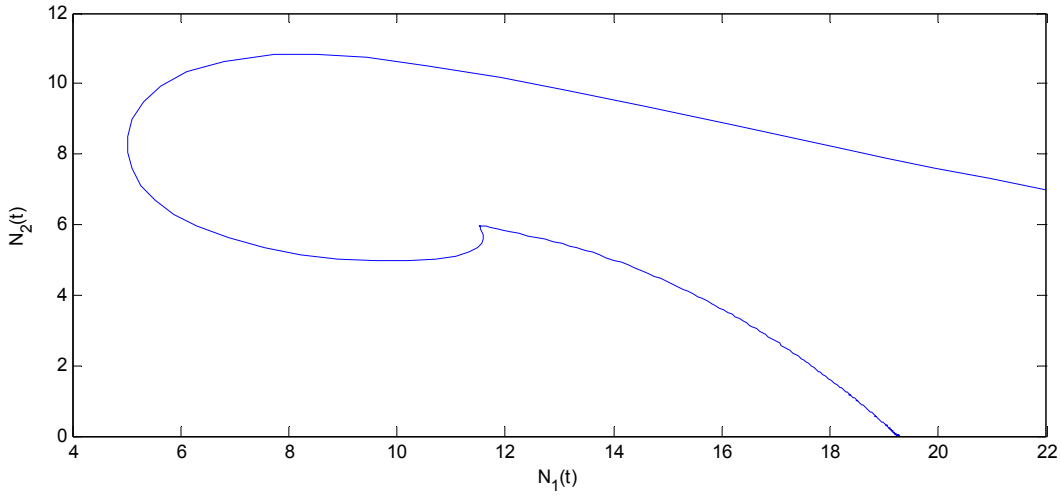

Figure 7. Phase space diagram for prey population $N_1(t)$ and predator population $N_2(t)$ when $B>B_0$.

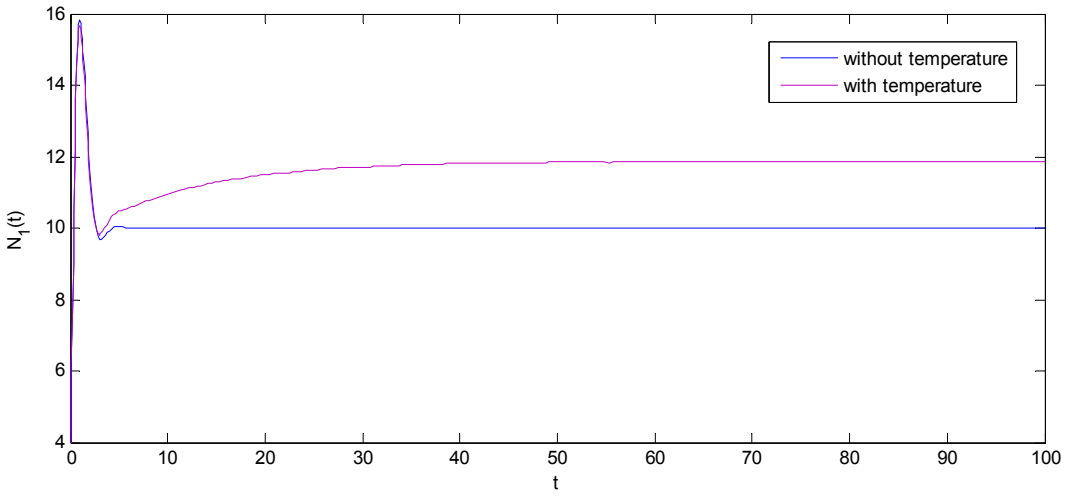

Figure 8. Graph for prey population $N_1(t)$ and time t with and without the effect of temperature when $B>B_0$.

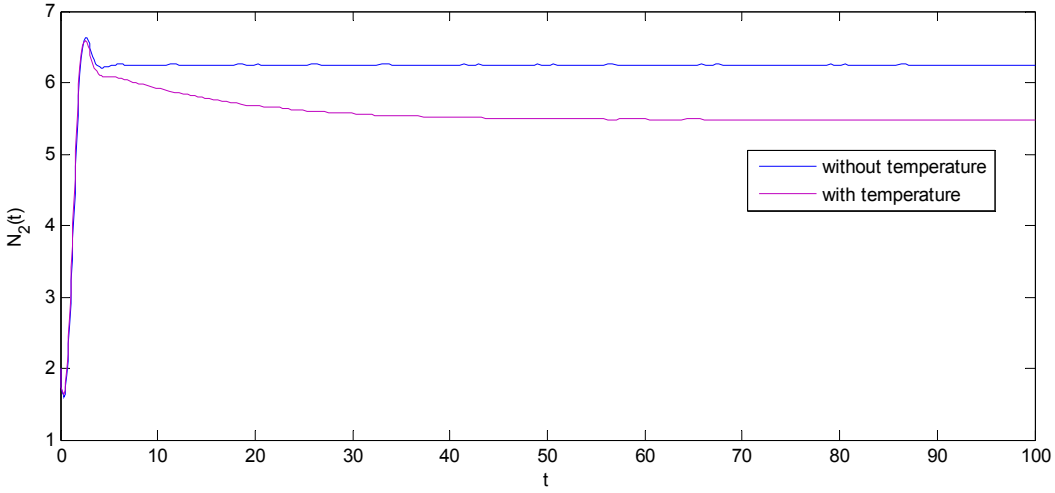

Figure 9. Graph for predator population $N_2(t)$ and time t with and without the effect of temperature when $B>B_0$.

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