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## Prandtl Number Effect on Assisted Convective Heat Transfer through a Solar Collector

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### ABSTRACT

Numerical study of the influence of Prandtl number on forced convective heat transfer through a riser pipe of a flat plate solar collector is done. The working fluid is  $Al_2O_3$ /water nanofluid. By Finite Element Method the governing partial differential equations are solved. The effect of the Prandtl number on the temperature and velocity field has been depicted. Comprehensive average Nusselt number, average bulk temperature, mean velocity, mid-height temperature inside the pipe, mean output temperature and collector efficiency are presented for the governing parameter mentioned above.  $Nu$  increases by 16% with the variation of  $Pr$  from 4.6 to 6.6 using nanofluid. Due to rising  $Pr$  heat transfer rate increases but collector efficiency devalues.

**KEYWORDS:** Assisted convection, finite element method, water- $Al_2O_3$  nanofluid, thermal efficiency

**MSC 2010 No.:** 65, 76

### 1. INTRODUCTION

Solar collectors are key elements in many applications, such as building heating systems, solar drying devices, etc. Solar energy has the greatest potential of all the sources of renewable energy especially when other sources in the country have depleted. The fluids with solid-sized nanoparticles suspended in them are called “nanofluids.” The suspended metallic or nonmetallic

nanoparticles change the transport properties and heat transfer characteristics of the base fluid. Nanofluids are expected to exhibit superior heat transfer properties compared with conventional heat transfer fluids. Forced convection occurs when a fluid flow is induced by an external force. In other words, it is a mechanism in which fluid motion is generated by an external source (like a pump, fan or mixer, suction device etc.). Significant amounts of heat energy can be transported very efficiently by this system and it is found very commonly in everyday life, including central heating, air conditioning, steam turbines and in many other machines.

Struckmann (2008) analyzed flat-plate solar collector where efforts had been made to combine a number of the most important factors into a single equation and thus formulate a mathematical model which would describe the thermal performance of the collector in a computationally efficient manner. Zambolin (2011) theoretically and experimentally performed solar thermal collector systems and components. Sandhu (2013) experimentally studied temperature field in flat-plate collector and heat transfer enhancement with the use of insert devices. Various new configurations of the conventional insert devices were tested over a wide range of Reynolds number (200-8000). Chabane et al. (2013) studied thermal performance optimization of a flat plate solar air heater. The received energy and useful energy rates of the solar air heaters were evaluated for various air flow rates were (0.0108, 0.0145, 0.0161, 0.0184 and 0.0203 kg.s<sup>-1</sup>) are investigated. Optimum values of air mass flow rates were suggested maximizing the performance of the solar collector. Mahian et al. (2013) performed a review of the applications of nanofluids in solar energy. The effects of nanofluids on the performance of solar collectors and solar water heaters from the efficiency, economic and environmental consideration viewpoints and the challenges of using nanofluids in solar energy devices were discussed.

Natarajan and Sathish (2009) studied role of nanofluids in solar water heater. Heat transfer enhancement in solar devices is one of the key issues of energy saving and compact designs. The aim of this paper was to analyze and compare the heat transfer properties of the nanofluids with the conventional fluids. Eismann and Prasser (2013) investigated correction for the absorber edge effect in analytical models of flat plate solar collectors. They derived a new correlation for the fin efficiency. Their correlation was easy to use for engineering purposes. The correlation allowed for more accurate efficiency/cost optimization. One of the major uncertainties of analytical models was eliminated. Wei et al. (2013) studied flat-plate solar heat collector with an integrated heat pipe. An improved structure of flat-plate solar heat collector applied in construction of the solar water heater system was proposed in this paper where the collector used one large integrated wickless heat pipe instead of side-by-side separate heat pipes. High stability and leak avoidance between the water cooling side and the solar heating side were the main advantages in their system.

Nasrin and Alim (2012, 2013) studied free and forced convective heat and mass transfer with entropy generation of nanofluid having single as well as double nanoparticles considering different geometry namely complicated cavity, direct absorption solar collector, prism shaped

solar collector, solar collector having wavy absorber, flat plate solar collector etc. They analyzed nanofluid flow with double nanoparticles using necessary modification of established nanoparticles effective properties. They showed how to enhance thermal performance of different types of solar collectors in their analyses.

In literature review, it is seen that there has been a good number of works in the field of heat transfer system through a flat plate solar collector. But there is some scope to work with temperature, heat transfer and enhancement of collector efficiency using nanofluid. In this paper, the forced convection flow through the riser pipe of a flat plate solar collector is studied numerically. The objective of this paper is to present temperature and velocity profiles as well as heat transfer system for the variation of Prandtl number.

## 2. PROBLEM FORMULATION

If  $I$  be the intensity of solar radiation, incident on the aperture plane of the solar collector having a collector surface area of  $A$ , then the amount of solar radiation received by the collector is:

$$Q_i = IA. \quad (1)$$

Basically, it is the product of the rate of transmission of the cover ( $\lambda$ ) and the absorption rate of the absorber ( $\kappa$ ). Thus,

$$Q_{recv} = I(\lambda\kappa)A. \quad (2)$$

As the collector absorbs heat its temperature is getting higher than that of the surrounding and heat is lost to the atmosphere by convection and radiation. The rate of heat loss ( $Q_{loss}$ ) depends on the collector overall heat transfer coefficient ( $U_l$ ), the collector temperature ( $T_{col}$ ) and ambient temperature ( $T_{amb}$ ). So,

$$Q_{loss} = U_l A (T_{col} - T_{amb}). \quad (3)$$

Thus, the rate of useful energy extracted by the collector ( $Q_{usfl}$ ), expressed as a rate of extraction under steady state conditions, proportional to the rate of useful energy absorbed by the collector, less the amount lost by the collector to its surroundings. This is expressed as:

$$Q_{usfl} = Q_{recv} - Q_{loss} = I(\lambda\kappa)A - U_l A (T_{col} - T_{amb}). \quad (4)$$

It is also known that the rate of extraction of heat from the collector may be measured by means of the amount of heat carried away in the fluid passed through it. Thus,

$$Q_{usfl} = mC_p (T_{out} - T_{in}), \quad (5)$$

where  $m$ ,  $C_p$ ,  $T_{in}$  and  $T_{out}$  mass flow rate, specific heat at constant pressure, inlet and outlet fluid temperature respectively.

Equation (4) may be inconvenient because of the difficulty in defining the collector average temperature. It is convenient to define a quantity that relates the actual useful energy gain of a collector to the useful gain if the whole collector surface were at the fluid inlet temperature. This quantity is known as “the collector heat removal factor ( $F_R$ )” and is expressed as:

$$F_R = \frac{mC_p(T_{out} - T_{in})}{A[I(\lambda\kappa) - U_l(T_{in} - T_{amb})]}. \quad (6)$$

The maximum possible useful energy gain in a solar collector occurs when the whole collector is at the inlet fluid temperature. The actual useful energy gain ( $Q_{usfl}$ ), is found by multiplying the collector heat removal factor ( $F_R$ ) by the maximum possible useful energy gain. This allows the rewriting of equation (4) as:

$$Q_{usfl} = F_R A [I(\lambda\kappa) - U_l(T_{in} - T_{amb})]. \quad (7)$$

The heat flux per unit area  $q$  is now denoted as

$$\frac{Q_{usfl}}{A} = q = I\lambda\kappa - U_l(T_{in} - T_{amb}). \quad (8)$$

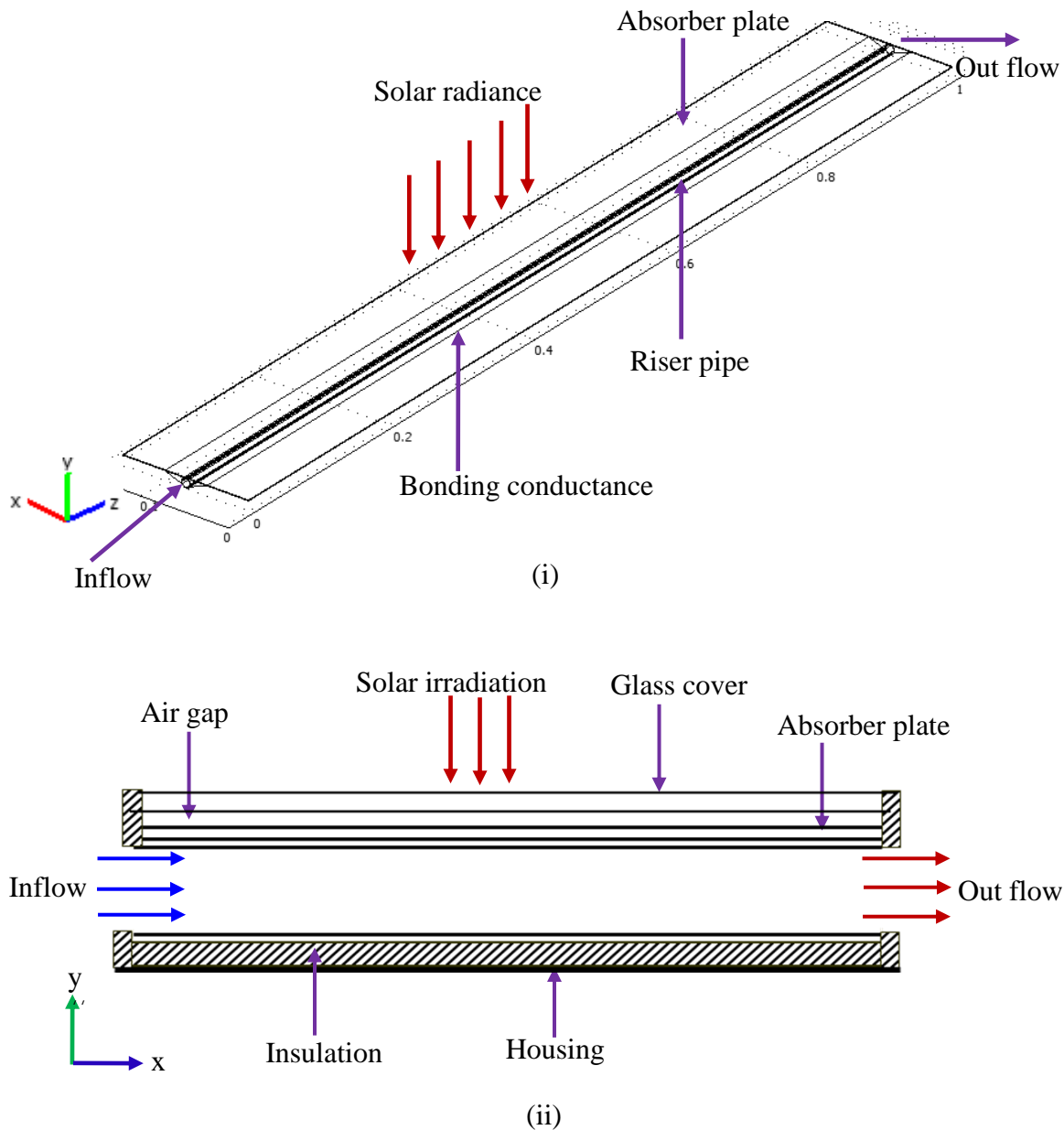
Equation (7) is a widely used relationship for measuring collector energy gain and is generally known as the “**Hottel-Whillier-Bliss equation**”.

The instantaneous thermal efficiency of the collector is:

$$\eta = \frac{Q_{usfl}}{AI} = \frac{F_R A [I(\lambda\kappa) - U_l(T_{in} - T_{amb})]}{AI} = F_R(\lambda\kappa) - F_R U_l \frac{(T_{in} - T_{amb})}{I}. \quad (9)$$

A schematic diagram of the system and its cross sectional views are shown in Figures 1 (i)-(ii). The numerical computation is carried on taking single riser pipe of FPSC. FPSC with single riser pipe gives the average heat transfer and fluid flow phenomena. Glass cover is at the top of the FPSC. It is made up of borosilicate which has thermal conductivity of 1.14 W/mK and refractive index of 1.47, specific heat of 750 J/kgK and coefficient of sunlight transmission of 95%. The wavelength of visible light is roughly 700 nm. Thickness of glass cover is 0.005m. There is an air gap of 0.005m between glass cover and absorber plate. Air density = 1.269 Kg/m<sup>3</sup>, specific heat = 287.058 J/kgK and thermal conductivity = 0.0243 W/mK. All these properties of air domain represent air of temperature at 298K. A dark colored copper absorber plate is under the air gap. Length, width and thickness of the absorber plate are 1m, 0.15m and 0.0005m

respectively. Coefficients of heat absorption and emission of copper absorber plate are 95% and 5% respectively.



**Figure 1.** Computational domain of the FPSC (i) 3D and (ii) 2D longitudinal views

The riser pipe has inner diameter 0.01 m and thickness 0.0005m. The riser tube is also made in copper metal. A trapezium shaped bonding conductance of copper metal is attached to the absorber and riser pipe. The fluid through the copper riser pipe is water-based nanofluid containing  $\text{Al}_2\text{O}_3$  nanoparticles whose thermo-physical properties are shown in Table 1.

The two dimensional governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (10)$$

$$\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (11)$$

$$\rho_{nf} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (12)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (13)$$

$$\left( \frac{\partial^2 T_a}{\partial x^2} + \frac{\partial^2 T_a}{\partial y^2} \right) = 0, \quad (14)$$

where

$$\alpha_{nf} = k_{nf} / (\rho C_p)_{nf}$$

is the thermal diffusivity,

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s$$

is the density,

$$(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_s$$

is the heat capacitance,

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

is the viscosity of Brinkman model (1952),

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$$

is the thermal conductivity of Maxwell Garnett (MG) model (1904),

$$Pr = \frac{\nu_f}{\alpha_f}$$

is the Prandtl number and

$$Re = \frac{u_{in}L}{\nu_f}$$

is the Reynolds number.

The boundary conditions of the riser pipe are:

at all solid boundaries:

$$u = v = 0,$$

at the solid-fluid interface:

$$k_{nf} \left( \frac{\partial T}{\partial y} \right)_{nf} = k_{solid} \left( \frac{\partial T_a}{\partial y} \right)_{solid},$$

at the inlet boundary:

$$T = T_{in}, u = u_{in},$$

at the outlet boundary: convective boundary condition  $p = 0$ ,  
at the top surface of absorber: heat flux

$$-k_a \frac{\partial T_a}{\partial y} = q = I\tau\kappa - U_l(T_{in} - T_{amb}), \text{ and}$$

at outer surface of riser pipe:

$$\frac{\partial T}{\partial y} = 0.$$

**Table 1.** Thermo physical properties of fluid and nanoparticles at 300K

Physical Properties	Fluid phase (Water)	Al <sub>2</sub> O <sub>3</sub>
$C_p$ (J/kgK)	4179	765
$\rho$ (kg/m <sup>3</sup> )	997.1	3970
$k$ (W/mK)	0.613	40
$\alpha \times 10^7$ (m <sup>2</sup> /s)	1.47	131.7

For water based nanofluid flow the expression of local Nusselt number is as follows:

$$\overline{Nu} = \frac{-k_{nf} \left( \frac{\partial T}{\partial y} \right) \left( \frac{L}{k_f} \right)}{\left( \frac{qL}{k_f} \right)} = - \left( \frac{k_{nf}}{k_f} \right) \left( \frac{k_f}{q} \right) \left( \frac{\partial T}{\partial y} \right). \quad (15)$$

The non-dimensional form of local heat transfer rate at the riser pipe solid surface is

$$\overline{Nu} = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial Y}.$$

The above equations are non-dimensionalized by using the following dimensionless, quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}, \theta = \frac{(T - T_{in})k_f}{qL}.$$

By integrating the local Nusselt number over the riser pipe, the average heat transfer rate of the collector can be written as

$$Nu = \frac{1}{L} \int_0^L \overline{Nu} dX.$$

The mean bulk temperature and average sub domain velocity of the fluid inside the collector may be written as

$$T_{av} = \frac{\iint_A T d\bar{A}}{\iint_A d\bar{A}} = \frac{\iint_A T d\bar{A}}{HL} \quad \text{and} \quad V_{av} = \frac{\iint_A V d\bar{A}}{\iint_A d\bar{A}} = \frac{\iint_A V d\bar{A}}{HL}, \quad (16)$$

where  $\bar{A}$ ,  $L$ ,  $H$  and  $V$  are the area, length and height of the absorber tube, magnitude of subdomain velocity, respectively.

The overall heat transfer loss from the collector is the summation of three separate components, the top loss coefficient, the bottom loss coefficient and the edge loss coefficient. The empirical relations for these coefficients are mentioned by Duffie and Beckman (1991) and Al-Ajlan et al. (2003) as follows:

$$U_l = U_t + U_b + U_e, \quad (17)$$

where  $U_t$  is the heat loss coefficient from the top,  $U_b$  is the heat loss coefficient from the bottom, and  $U_e$  is the heat loss coefficient from the edges of collector. These can be calculated from the following relations:



$$U_t = \left\{ \frac{N}{\frac{C}{T_a} \left[ \frac{T_a - T_{amb}}{N + f} \right]^c} + \frac{1}{h_{amb}} \right\}^{-1} + \frac{\sigma(T_a + T_{amb})(T_a^2 + T_{amb}^2)}{\frac{1}{(\varepsilon_a + 0.00591Nh_{amb})} + \frac{2N + f - 1 + 0.13\varepsilon_a - N}{\varepsilon_c}},$$

$$U_b = \frac{k_b}{x_b}, \quad U_e = \frac{U_e A_e}{A} = \frac{k_e}{x_e} \left[ \frac{2(L+W)H}{LW} \right],$$

$$C = 520 * (1 - 0.000051\beta^2), \text{ and}$$

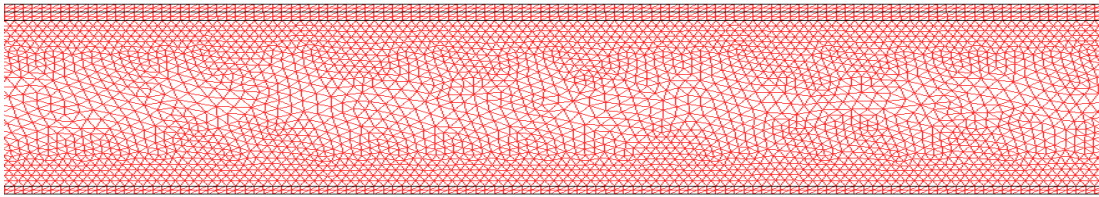
$$f = (1 + 0.089h_a - 0.1166h_{amb}\varepsilon_a) * (1 + 0.07866N).$$

### 3. NUMERICAL IMPLEMENTATION

The Galerkin finite element method Taylor and Hood (1973) and Dechaumphai (1999) is used to solve the non-dimensional governing equations along with boundary conditions. The equation of continuity has been used as a constraint due to mass conservation and this restriction may be used to find the pressure distribution. The finite element method of Reddy (1994) is used to solve the equations (10) - (14), where the pressure  $P$  is eliminated by a constraint. The continuity equation is automatically fulfilled for large values of this constraint. Then the velocity components ( $u$ ,  $v$ ) and temperature ( $T$ ) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. Three points Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions.

#### 3.1. Mesh Generation

In the finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volume, etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Figure 2 displays the finite element mesh of the present physical domain.



**Figure 2.** Mesh generation of the computational domain

### 3.2. Grid Independent Test

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for  $Re = 480$  and  $Pr = 5.8$  in a solar collector. In the present work, we examine five different non-uniform grid systems with the following number of elements within the resolution field: 42,010, 99,832, 1,40,472, 1,68,040 and 1,92,548. The numerical scheme is carried out for highly precise key in the average Nusselt number for water-alumina nanofluid ( $\phi = 2\%$ ) as well as base fluid ( $\phi = 0\%$ ) for the aforesaid elements to develop an understanding of the grid fineness as shown in Table 2. The scale of the average Nusselt numbers for nanofluid and clear water for 1,68,040 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 1,68,040 elements is preferred for the computation.

**Table 2.** Grid test at  $Pr = 5.8$ ,  $\phi = 2\%$ ,  $I = 215\text{W/m}^2$  and  $Re = 480$

Elements	42,010	99,832	1,40,472	1,68,040	1,92,548
$Nu$ (Nanofluid)	1.87872	1.99127	2.10934	2.14351	2.14378
$Nu$ (Base fluid)	1.59326	1.69225	1.81524	1.84333	1.84341
Time (s)	127.52	308.75	581.11	897.23	1295.31

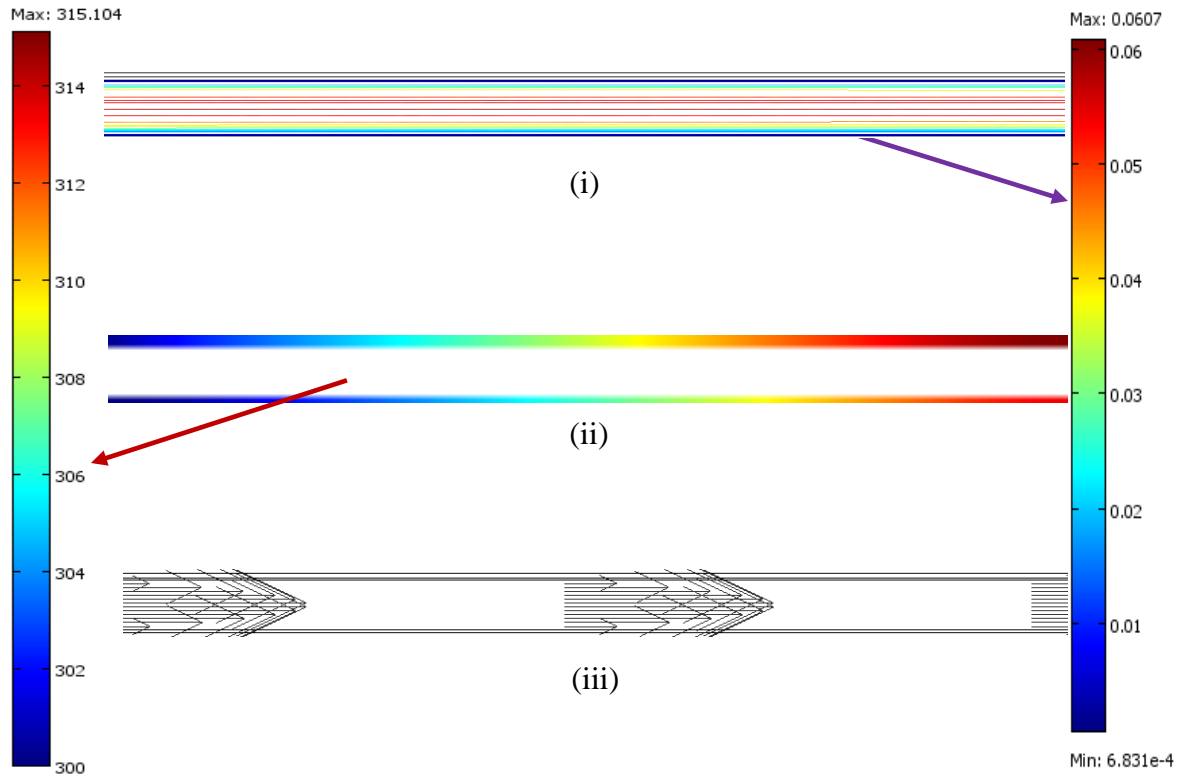
## 4. RESULTS AND DISCUSSION

Finite element simulation is applied to perform the analysis of laminar forced convection temperature and fluid flow through a riser pipe of a flat plate solar collector filled with water/alumina nanofluid. Effect of the Prandtl number ( $Pr$ ) on heat transfer and collector efficiency has been studied. The range of  $Pr$  for this investigation vary from 4.6 to 6.6 where solar irradiation ( $I$ ), Reynolds number ( $Re$ ), and solid volume fraction nanofluid ( $\phi$ ) remain fixed at  $215\text{ W/m}^2$ , 480 and 2% respectively. The mass flow rate per unit area ( $m$ ) is  $0.0248\text{ Kg/s}$ , overall heat transfer coefficient ( $U_l$ ) is  $8\text{ W/m}^2\text{K}$ , aperture area of the collector ( $A$ ) is  $1.8\text{ m}^2$ , number of glass ( $N$ ) is 1, the tilt angle of collector is  $0^\circ$  and nanoparticle size is 10 nm.

Effect of  $Pr$  is shown in the Figures 3 (i)-(iii). The streamlines in the Figure 3(i) occupying the whole riser pipe is found at the Prandtl number ( $Pr = 5.8$ ). Color of streamlines at the solid surfaces of riser pipe is deep blue and at the middle is red. This is due to the fact that no-slip condition is maintained at the solid surfaces of riser pipe. Thus maximum velocity is obtained at the middle of the pipe. Also the boundary plot represents that fluid passes through the pipe and takes heat from top surface of riser pipe. That's why boundary temperature becomes low near the inlet and gradually becomes high near the exit boundary. The temperature of top surface of riser pipe is higher than the bottom surface. Arrow plot indicates direction of fluid flow. From Figure

3(iii) it is observed that nanofluid flow enters at the left inlet opening and exits from the right outlet opening.

The effect of Prandtl number ( $Pr$ ) on average Nusselt number ( $Nu$ ) at the top of riser pipe, average temperature ( $T_{av}$ ), magnitude of average velocity ( $V_{av}$ ), mid-height temperature ( $T$ ) of water-copper nanofluid, mean outlet temperature ( $T_{out}$ ) and percentage of collector efficiency with  $Re = 480$ ,  $I = 215\text{W/m}^2$ ,  $\phi = 2\%$  are shown in Figures 4(i)-(vi). The values of  $Pr$  are chosen as 4.6, 5.2, 5.8 and 6.6.



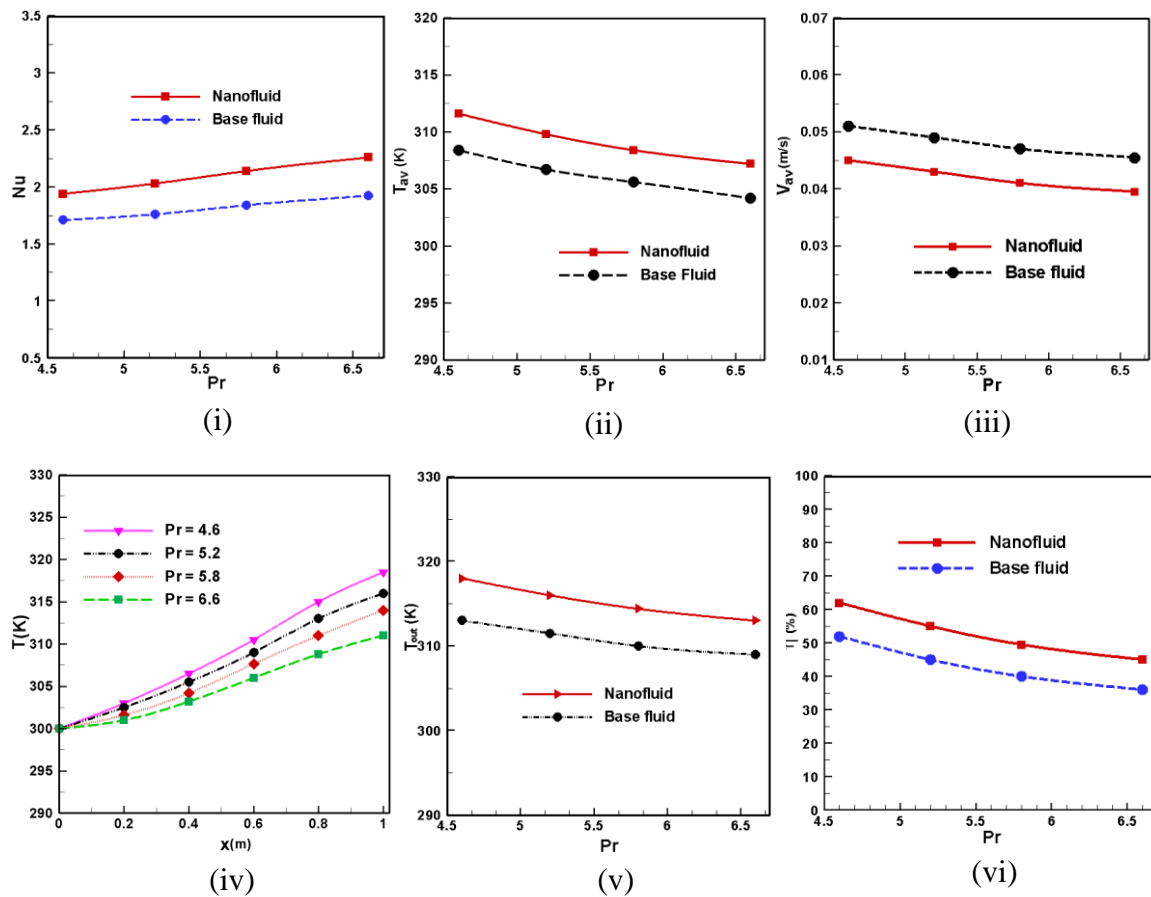
**Figure 3:** Effect of  $Pr$  at 5.8 on (i) streamlines, (ii) boundary plot and (iii) arrow plot

All these values of Prandtl number represent water at different temperatures. From the Figure 4(i) it is clearly observed that as  $Pr$  increases, heat transfer rate enhances for both fluids.  $Nu$  increases by 16% and 12% with the variation of  $Pr$  from 4.6 to 6.6 for water-copper nanofluid and clear water respectively.

As  $Pr$  increases average bulk temperature ( $T_{av}$ ) of the fluids decreases gradually for both fluids. It is well known that increasing Prandtl number devalues fluid temperature. Nanofluid has higher temperature than base fluid.

Mean velocity ( $V_{av}$ ) devalues for higher Prandtl number because fluid with higher viscosity can't move freely like lower viscous fluid. Water- $Al_2O_3$  nanofluid has lower mean velocity than clear water.

Temperature ( $T$ ) of water-Cu nanofluid at the middle of riser pipe is seen in Figure 4(iv). Nanofluid temperature increases when it passes through the riser pipe. But temperature of nanofluid diminishes with growing  $Pr$ .



**Figure 4.** Effect of  $Pr$  on (i) mean Nusselt number, (ii) mean bulk temperature, (iii) mean velocity, (iv) mid-height temperature, (v) mean output temperature and (vi) collector efficiency

Figure 4(v) shows that mean outlet temperature ( $T_{out}$ ) decreases with increasing Prandtl number. This describes in this way that increasing Prandtl number decreases fluid temperature. As a result lower tempered fluid can't become so hot at outlet exit like higher tempered fluid. Average outlet temperature becomes 318K, 316K, 314K, 313K for nanofluid and 313K, 312K, 310K, 309K for water respectively with the variation of  $Pr = 4.6, 5.2, 5.8$  and  $6.6$ .

Using higher values of Prandtl number the collector efficiency decreases. In this scheme water/aluminium oxide nanofluid ( $\phi = 2\%$ ) performs better than clear water ( $\phi = 0\%$ ). Thermal

efficiency devalues from 66%-45% for nanofluid and 52%-36% for water due to the variation of Prandtl number  $Pr$  from 4.6 to 6.6.

## 5. CONCLUSION

The results of the numerical analysis lead to the following conclusions:

The structure of the fluid streamline, boundary plot and arrow plot through the solar collector is found to significantly depend upon the Prandtl number. The  $Al_2O_3$  nanoparticle with the highest  $Pr$  is established to be most effective in enhancing performance of heat transfer rate than base fluid. Collector efficiency is obtained lower for growing  $Pr$ . Mean bulk temperature diminishes for both fluids with rising  $Pr$ . Mean velocity is obtained higher for base fluid with falling  $Pr$ .

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## NOMENCLATURES

$A$	aperture area of solar collector ( $m^2$ )
$A_e$	area base on the perimeter of collector ( $m^2$ )
$\bar{A}$	area of 2D cross section of riser pipe ( $m^2$ )
$C$	constant defined in section 2
$C_p$	specific heat at constant pressure ( $Jkg^{-1}K^{-1}$ )
$f$	friction factor
$h$	convective heat transfer coefficient ( $Wm^{-2}K^{-1}$ )
$h_a$	convective heat transfer coefficient between glass and ambient air ( $Wm^{-2}K^{-1}$ )
$H$	height of the collector ( $m$ )
$I$	solar irradiation ( $Wm^{-2}$ )
$k$	thermal conductivity of fluid ( $Wm^{-1}K^{-1}$ )
$k_b$	back insulation conductivity ( $Wm^{-1}K^{-1}$ )
$k_e$	edge insulation conductivity ( $Wm^{-1}K^{-1}$ )
$L$	length of the solar collector ( $m$ )
$N$	number of glass
$Nu$	average Nusselt number
$p$	pressure ( $kgms^{-2}$ )
$Pr$	Prandtl number
$q$	heat flux ( $Wm^{-2}$ )
$Re$	Reynolds number
$T$	fluid temperature ( $K$ )
$u, v, w$	velocity components along x, y, z direction ( $ms^{-1}$ )

$U_l$	overall heat transfer coefficient ( $Wm^{-2}K^{-1}$ )
$V$	magnitude of velocity ( $ms^{-1}$ )
$\bar{V}$	volume of riser pipe ( $m^3$ )
$x_b$	back insulation thickness ( $m$ )
$x_e$	edge insulation thickness ( $m$ )
$x, y$	Cartesian coordinates ( $m$ )
$X, Y$	dimensionless Cartesian coordinates
$W$	width of collector ( $m$ )

### Greek symbols

$\alpha$	thermal diffusivity ( $m^2s^{-1}$ )
$\beta$	tilt angle ( $^\circ$ )
$\lambda$	transmitivity
$\varepsilon$	emissivity
$\eta$	thermal efficiency
$\phi$	nanoparticles volume fraction
$\theta$	dimensionless fluid temperature
$\mu$	dynamic viscosity of the fluid ( $m^2s^{-1}$ )
$\nu$	kinematic viscosity of the fluid ( $m^2s^{-1}$ )
$\rho$	density of the fluid ( $kgm^{-3}$ )
$\kappa$	absorption coefficient

### Subscripts

$a$	absorber
$amb$	ambient
$av$	average
$b$	back
$c$	cover
$col$	collector
$e$	edge
$f$	fluid
$in$	input
$loss$	lost
$nf$	nanofluid
$out$	output
$recv$	received
$s$	solid particle
$t$	top
$usfl$	useful