Department of Electrical & Computer Engineering
Prairie View A&M University

Ph.D. Preliminary Examination
in
Mathematics
Spring, 2014

Write legibly.
No points will be given for answers that show no work.

Do not use cell phone during the examination.
(Calculator will be provided upon request)

Note: Each problem is worth 20 points.

Name: _____ Date: March 21, 2014

1. Suppose a rectangular coordinate system is located in space such that the temperature T at the point P(x, y, z) is given by the formula $T = \frac{100}{x^2 + y^2 + z^2}$.

   (a) Find the rate of change of T with respect to distance at the point P(1, 3, -2) in the direction of the vector **a** = **i** - **j** + **k**.

   (b) In what direction from P does T increase most rapidly? What is the maximum rate of change of T at P?

2. The manager of a movie theater determines that the average time moviegoers wait in line to buy a ticker for this week's film is 10 minutes and the average time they wait to buy popcorn is 5 minutes. Assuming that the waiting times are independent, find the probability that a moviegoer waits a total of less than 20 minutes before taking his or her seat. Assume that both of the waiting times are modeled by exponential density functions.

3. Find the Laplace transform F(s) and sketch the pole-zero plot with the ROC (region of convergence) for the following signals f(t);

(a) $f(t) = e^{-3t}u(t) + e^{-2t}u(-t)$

(b) $f(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(c) $f(t) = e^{2t}u(t) + e^{-3t}u(-t)$

4. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

5. State with **reasons** whether the following signals are periodic or non-periodic. For periodic signals, find the period and what harmonics are present in the signals.

   (a) $f_1(t) = 3 + 5\cos(\frac{1}{2}t + \theta_1) + 2\cos(\frac{2}{3}t + \theta_2) + 8\cos(\frac{7}{8}t + \theta_3)$

   (b) $f_2(t) = 4\cos(2t + \theta_1) + 5\sin(\pi t + \theta_2)$

   (c) $f_3(t) = 7\sin(3\sqrt{2}t + \theta_1) + 3\cos(6\sqrt{2}t + \theta_2)$