

**Department of Electrical & computer Engineering  
Prairie View A&M University  
Ph.D. Preliminary Examination  
Engineering Mathematics  
Spring, 2011**

**Note: Write legibly and show clearly the steps to justify your answers.**

Problem 1

Let  $X$  be a continuous random variable with probability density function

$$f_X(x) = \frac{1}{2} e^{-x} = \frac{1}{2} e^{-x} = \frac{1}{2} e^{-x}$$

- (a) Find  $P(|X| \geq 4)$   
(b) Find  $P(X^2 + X > 0)$

~~$P(X \geq 4)$~~

~~$P(X \geq 4) = 1 - P(X < 4)$~~

~~$P(X \geq 4) = 1 - \int_0^4 \frac{1}{2} e^{-x} dx$~~

Problem 2:

Suppose the length of life of a certain kind of light bulb, after it is installed, is exponentially distributed with a mean length of 10 days. As soon as one light bulb burns out, a similar one is installed in its place. Find the probability that more than 50 bulbs will be required during a one-year (365 days) period.

Problem 3

Let  $X$  be a continuous random variable with probability density function  $f_X(x)$ .

a) For any  $a > 0$ , show that  $P(|X - \mu_X| \geq a) \leq \frac{\sigma_X^2}{a^2}$  where  $\mu_X$  and  $\sigma_X^2$  are the mean and variance of  $X$  respectively.

b) If  $f_X(x) = 0$  for  $x < 0$ , show that, for any  $a > 0$ ,  $P(X \geq a) \leq \frac{\mu_X}{a}$

Problem 4

Let A be a 2x2 matrix:  $A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$

- Find the eigenvalues and corresponding eigenvectors of A.
- Find a matrix P that diagonalizes A
- Compute  $P^{-1}AP$ .

$$\det(A - \lambda I_2)$$

Problem 5

Solve the following differential equation

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - 3y = \cos x$$

subject to  $y(0) = 1$ ,  $\frac{dy(0)}{dx} = -2$ .

TABLE STANDARDIZED NORMAL DISTRIBUTION FUNCTION

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t \exp(-x^2/2) dx \quad \Phi(-t) = 1 - \Phi(t)$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7643	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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Name of the Student:  
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Signature of the Student: