Ph.D. Preliminary Exam
Communications and Signal Processing

Department of Electrical and Computer Engineering
Prairie View A&M University
Spring 2014

Name:_____ **Date**_____

Question 1: (20 points)

(a) [5 points] Most signals encountered in real life such as voice and music have power spectral densities that are concentrated at lower frequencies with relatively little power in the higher frequency components. In such cases, it is expected (i.e., this can be proved mathematically) that phase modulation (PM) to be superior to frequency modulation (FM). It may seem logical for radio broadcast stations to use PM rather than FM. Can you explain why then FM (rather than PM) is used in radio broadcasting?

(b) Suppose you are interested in transmitting a baseband message signal of bandwidth 15 kHz via frequency modulation (FM). You are also given the following system parameters: the average-to-peak message power ratio is 0.1, the received signal power is 1 W, and the noise power spectral density is $N_o = 10^{-7}$ W/Hz.

(i) [10 points] If we require that the signal-to-noise ratio (SNR) at the output of the FM demodulator to be at least 30dB for sufficient fidelity, find the smallest possible transmission bandwidth of the FM modulated signal.

(ii) [5 points] If the maximum allowable transmission bandwidth is now constrained to be 100 kHz, find the highest possible SNR at output of the FM demodulator.

Question 2: (30 points)

(a) [7 points] Consider a test audio signal which is comprised of a single sinusoid $s(t) = 3\cos(500\pi t)$. If an uniform quantizer is used to quantize $s(t)$, how many bits of quantization are needed to achieve a signal-to-quantization-noise ratio (SQNR) of at least 40dB? What is the minimum transmission bandwidth of the resulting pulse code modulated digital signal?

(b) [23 points] Assume that a band-limited baseband signal $m(t) = \dfrac{\sin(20\pi t)}{\pi t}$ is sampled at 19 samples/second. The sampling function is a rectangular pulse train (i.e., flat-top sampling) with a unit amplitude and pulse width of 10^{-3} seconds. The sampled signal $m_s(t)$ is passed through an ideal low-pass reconstruction filter with a cut-off frequency of 10 Hz. Let us denote $m_f(t)$ to be the filtered waveform at the output of the ideal low-pass filter.

(i) [15 points] Sketch the magnitude of the frequency spectra for the signals $m(t)$, $m_s(t)$ and $m_f(t)$. Clearly label the axes and all critical points in your sketch.

(ii) [8 points] Write-down an analytical expression for the time-domain signal $m_f(t)$ and its Fourier transform. Compare this with the original signal $m(t)$.

Question 3: (25 points)

Determine all possible inverse z-transforms with corresponding regions of convergence of $X(z) = \dfrac{10 + z^{-1} - z^{-2}}{1 - 0.25z^{-2}}$. Which inverse z-transform is stable?

Question 4: (25 points)

(a) [15 points]

Suppose that $x[0] = 1,\ x[1] = 2,\ x[2] = 2,\ x[3] = 1$, and $x[n] = 0$ for all other integers n. Compute the 4-point Discrete Fourier Transform (DFT) of $x[n]$.

(b) [10 points]

A length-4 discrete-time signal $x[n]$ has Discrete Fourier Transform (DFT) X_k given by

$$X_k = \begin{cases} 2, & k = 0 \\ -2, & k = 1 \\ 4, & k = 2 \\ 6, & k = 3 \end{cases}.$$

Find the Discrete Fourier Transform (DFT) of $y[n] = (x[n])^2$.