Variance Minimization Stochastic Power Control in CDMA Systems

Lijun Qian, *Member, IEEE,* and Zoran Gajic, *Senior Member, IEEE*

***Abstract*— In this paper, the uplink power control problem is considered for CDMA cellular systems, where stochastic SIR measurements are performed at base stations. A distributed stochastic power control algorithm is proposed assuming SIR measurements contain white noise. The proposed scheme minimize the sum of variances of mobile's transmission power and signal-to-interference error. The algorithm derived is fully distributed in the sense that each user only needs to know its own signal-to-interference measurement and channel variation. Uncertainties of wireless channels are accomodated by using a robust estimator. Simulation results indicate that the proposed power control scheme has very fast convergence. In addition, it also works fine with 4-bit quantization.**

***Index Terms*— Stochastic power control, CDMA, Kalman filter, H_∞ filter.**

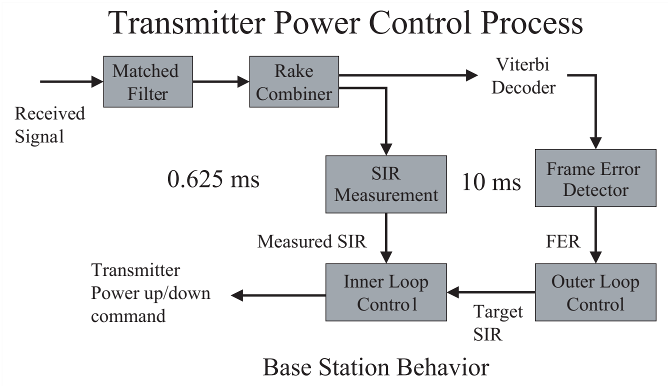


Fig. 1. Base station behavior in 3-G wireless systems.

I. Introduction

In a cellular wireless network, certain quality-of-service (QoS) should be maintained for all active users. A quantity that measures user's QoS is the signal-to-interference ratio (SIR). A general idea to achieve the desired SIR for all active users is to allocate network resources in a most efficient way. Resource allocation in a cellular wireless network includes channel allocation, power control, etc. The transmitter power is the most valuable resource in a wireless network. By proper transmitter power control, the interference can be minimized. At the same time, power control extends the battery life in the handset.

Due to unreliable radio links, it is challenging to ensure QoS in terms of the frame error rate (FER), in turn, the SIR in wireless networks. Since the third generation (3G) wireless systems use high data rate (multimedia) services, more accurate and fast responding power control is required. For real-time services such as IP voice and video, stringent delay requirements severely limits or even precludes retransmission of lost frames. Tight delay requirements translate into stringent requirements on SIR. As a result, in order to support real-time services, it is important to design accurate and fast responding power control algorithms such that the required QoS can be delivered to all users.

The initial work on deterministic SIR-based power control schemes for narrowband systems has been done by Zander [1], [2]. In [3], Wu extended Zander's problem formulation to CDMA systems by reordering the number of users in different cells. Power control schemes can be centralized [1], [4] or distributed [2], [5], [6]. A centralized controller has complete information about each user (e.g., all the link gains are known) and it decides control actions for all users. On the other hand, a distributed controller uses only local information to make a control decision for a user. For example, [2], [5], [6] only use information about user's own link gain and/or SIR to make a decision about their transmitted powers.

The algorithm convergence has to be studied carefully when a distributed power control scheme is used. The distributed power control algorithm by Foschini [6] was shown to converge either synchronously [6] or asynchronously [7]. The basic model in [6] is relaxed in [7] to allow asynchronism. Propagation delays are also considered in [7]. A framework for convergence of the generalized uplink power control was proposed by Yates [8]. Jantti and Kim [9] use a successive overrelaxation technique to derive second-order iterative algorithms called USOPC (unconstrained second-order power control) and CSOPC (constrained second-order power control), which have faster convergence speed than the first-order Jacobian iteration algorithms, [6].

It is well known that the power control problem in the uplink (mobile-to-base) is more challenging than the power control problem in the downlink (base-to-mobile). Hence, we only consider the uplink power control problem in CDMA systems.

In 3G CDMA systems, both open-loop and closed-loop power control is implemented in the uplink [11], [12], [13]. The open-loop power control adjusts initial access channel transmission power of the mobile station. Closed-loop power control is employed to combat fast channel fluctuations due to fading. The base station behavior in a closed-loop uplink power control architecture is shown in Fig. 1.

Manuscript received November 5, 2002; revised May 19, 2004; accepted January 30, 2005. The associate editor coordinating the review of this paper and approving it for publication was Y.-D. Yao. The work of L. Qian was completed during his doctoral study at Rutgers University. The work of Z. Gajic was supported in part by the National Science Foundation Grant ANIR 0106857.

L. Qian is with the Department of Electrical Engineering, Prairie View A&M University, Prairie View, TX 77095 (email: lijun_qian@pvamu.edu).

Z. Gajic is with WINLAB, Rutgers–The State University of New Jersey, Piscataway, NJ (email: gajic@winlab.rutgers.edu).

Digital Object Identifier 10.1109/TWC.2006.01013.

In this paper, we assume stochastic measurements of SIR at the base station according to the SIR measurement module, implemented in both IS-95 [10] and Wideband CDMA proposed for 3G wireless systems [11], [12]. Based on the received signal, the outer loop detects the Frame Error Rate (FER). Then the outer loop control maps the FER to target SIR value, see Fig. 1. The method for setting the target SIR is not addressed in this paper. The inner loop control decides the transmitter power by comparing the measurement of SIR with its target value. The proposed stochastic power control scheme will be applied for inner loop control. At the base station receiver, SIR measurements are performed after RAKE combining. The frame structure using time-multiplexed pilots supports SIR measurements [11], [14]. Both pilot and data symbols are used to measure instantaneous received signal power. Only the pilot symbols are used to measure instantaneous interference plus noise power, which is followed by averaging using a first-order filter [11], [14].

Most power control works have been done on CDMA systems assuming that the power control system is deterministic [1]- [9]. However, because of the stochastic nature of link gains, received power, interference, and Signal-to-Interference Ratio (SIR) are all random processes fluctuating in time. Hence, the development of a stochastic power control scheme is essential (mandatory).

Several papers were published on stochastic power control in wireless CDMA networks during the past several years using different problem formulations and assumptions. Ulukus and Yates [15] showed that it is realistic to assume that the average of the squared matched filter output in a L-bit interval is corrupted by additive white Gaussian noise (AWGN). They assume a very specific uplink implementation that uses binary phase shift keying (BPSK) and an L-bit averaging window, and devise a power control scheme that converges in the mean square sense to the optimal transmission power, with the optimal power meaning the optimal power of the corresponding deterministic case. In [16], the SIR estimation problem was studied based on a signal subspace method using the sample covariance matrix of the received signal. However, estimation of the SIR is not the focus of our research. Leung [17], [18] and Leung and Wang [19] used the Kalman filter to predict the interference assuming that the interference signal and its measurements are corrupted by AWGN in a TDMA system. Having obtained the predicted value for the interference, a nice simple scheme based on the definition formula of the signal to interference ratio (SIR) is used for mobile power updates [17], [18]. However, such a method neither optimize mobile powers nor SIR errors, and it is solely based on the accuracy of the predicted interference. Shoarinejad et al., [20], developed dynamic channel and power allocation algorithms using the Kalman filter to provide predicted measurements for both gains and the interference under the assumption that they are corrupted by AWGN. Jiang *et al.*, [21], developed a technique that uses the Kalman filter for power estimation in wireless networks. Qian [22], Qian and Gajic [23] used H-infinity filter to predict interference and proposed a power adjustment scheme based on the SIR error optimization that theoretically converges in one iteration. Varanasi and Das [24] studied stochastic power control for nonlinear multiuser receivers, assuming a much more complex form for the receiver than the one based on the matched filter–currently used in wireless CDMA networks. Kandukuri and Boyd [25] used a nonlinear convex optimization technique to minimize the mobile powers and the outage probability subject to some statistical assumptions on gains that include Rayleigh fading induced outage probability for every link. There are also several works on stochastic power control assuming specific channel models. For example, a flat fading channel model is assumed and the evolution of the channel is described by stochastic differential equations in [26]. The performance index is the total transmission power. A linear programming approach is proposed in [26], however, no iterative or distributed algorithms or simulation results are given. In [27], only a 2-user case is considered analytically where link gains are assumed to be varying with shadow fading effect. The multiuser case is only studied through simulation.

In [15], the average of the squared matched filter output (total received power) was assumed to have white Gaussian noise. The power evolution was based on the total received power at the base station. In this research work, we assume that the measurements of SIR from the SIR measurement module at the base station contain Gaussian white noise. We propose a SIR-based stochastic power control scheme using stochastic control theory. Usually, the instantaneous received signal power is assumed to contain white noise. The average interference plus noise power from the low-pass filter could be treated as a fixed quantity during one Power Control Group (PCG). Thus the measurements of SIR can be assumed to contain white noise. The stochastic power control scheme proposed in this paper will make use of the SIR measurement mechanism within the 3G wireless proposal and formulate the stochastic power control problem as a linear stochastic discrete-time system driven by white noise.

We study the variance minimization problem for the weighted sum of variances of SIR error and transmission power when the SIR is corrupted by AWGN. We first propose to use additive power updates with increments proportional to the SIR error and then exploit the fact that the corresponding difference equation for the variance is represented by a linear discrete-time system driven by white noise whose variance satisfies the difference Lyapunov equation. We devise a power controller by minimizing the corresponding components in the solution matrix of the difference Lyapunov equation. Our approach is pretty much different than all other approaches used so far in the study of stochastic power control for wireless CDMA networks. In this research work, we do not assume any specific channel model, since any particular model will only fit specific wireless environment. Instead, we use an estimator to estimate the channel variation. In addition, a robust estimator (e.g., the H_∞ filter) provides accurate estimates regardless of the statistics of channel disturbances/uncertainties.

This paper is organized as follows. A stochastic power control scheme is derived in Section 2 by formulating the power control system as a linear time-varying system driven by white noise. In Section 3, the Kalman filter and robust estimators are used to predict channel variations. The quantization effect is considered in Section 4 through simulation. Section 5 gives conclusions and comments.

II. Power Control Formulation as a Minimum Variance Problem

In this section, the mobile power assignment power problem is formulated as a linear stochastic control problem, where the time-varying linear system is driven by white noise.

The SIR of an active link from mobile station i to base station n in a wireless system is defined as

$$\gamma_{ni}(k) = \frac{Lh_{ni}p_i(k)}{\sum_{j\neq i} h_{nj}p_j(k) + \sigma^2} \tag{1}$$

where h_{ni} and h_{nj} are the link gains from mobile station i and j to base station n, respectively, a k denotes discrete time.. $p_i(k)$ and $p_j(k)$ are the transmission power from mobile i and j, respectively. σ^2 is the background noise. We assume that L is the processing gain in spread spectrum wireless systems, e.g., in CDMA 2000, $L = 64$ or 128 or 256. Let us denote the denominator by I_i, which represents the received interference, then the SIR can be rewritten as

$$\gamma_{ni}(k) = \frac{Lh_{ni}p_i(k)}{I_i(k)} \tag{2}$$

Assuming that a mobile only transmits to one base station during the time of power control, then $\gamma_{ni}(k)$ can be simplified as $\gamma_i(k)$. Let $\delta_i(k) = \frac{Lh_{ni}}{I_i(k)}$ denote the channel variation (introduced in [28]). $\delta_i(k)$ will be estimated and predicted in the proposed power control scheme.

Define the performance criterion as a sum of variance of mobile's transmission power and variance of SIR error

$$J_i(k) = [\mathrm{Var}(p_i(k+1)) + \mathrm{Var}(e_i(k+1))] \tag{3}$$

The objective function defined above represents the goal of execising stochastic power control: reduce the variance of SIR error to guarantee good service quality of mobile users, while reducing the volatile change in transmission power which in turn makes the interference fluctuates smoothly, which is crucial for voice users.

Assume that stochastic measurements of SIR of user i at time instant k at the base station are modeled as

$$y_i(k) = \gamma_i(k) + w_i(k) \tag{4}$$

where $y_i(k)$ is the measurement of the SIR, $\gamma_i(k)$ is the actual value of the SIR, and $w_i(k)$ is a zero-mean, mutually uncorrelated white noise stochastic process. Using the idea that the mobile's transmission power change is proportional to the SIR error, we suggest the following dynamics for power updates

$$p_i(k+1) = p_i(k) + \alpha_i(k)[\gamma_i^{tar} - y_i(k)] \tag{5}$$

where $\alpha_i(k)$ is the controller gain to be determined.

The optimal stochastic power control problem is to minimize $J_i(k)$ for every user i at every time step k by choosing the appropriate controller gain $\alpha_i(k)$

$$\min_{\alpha_i(k)} [\mathrm{Var}(p_i(k+1)) + \mathrm{Var}(e_i(k+1))] \tag{6}$$

with initial conditions given by

$$\mathrm{E}\{\mathbf{x}_i(0)\} = \bar{\mathbf{x}}_i(0) = \begin{bmatrix} \bar{p}_i(0) \\ \bar{e}_i(0) \end{bmatrix} \tag{7}$$

and

$$\mathrm{Var}\{\mathbf{x}_i(0)\} = Q_i(0) \tag{8}$$

Using (4) and (5), we obtain that

$$p_i(k+1) = p_i(k) + \alpha_i(k)e_i(k) - \alpha_i(k)w_i(k) \tag{9}$$

where $e_i(k) = \gamma_i^{tar} - \gamma_i(k)$. From the definition of the channel variation $\delta_i(k)$, we have $\gamma_i(k) = \delta_i(k)p_i(k)$. Using this expression while considering the SIR error evolution, we have (10). Combining equations (9) and (10), we obtain the following second-order dynamic system driven by white noise for the mobile's transmission power and the SIR error

$$\begin{bmatrix} p_i(k+1) \\ e_i(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_i(k) \\ -\delta_i(k+1) & -\delta_i(k+1)\alpha_i(k) \end{bmatrix} \begin{bmatrix} p_i(k) \\ e_i(k) \end{bmatrix} + \begin{bmatrix} -\alpha_i(k) \\ \delta_i(k+1)\alpha_i(k) \end{bmatrix} w_i(k) + \begin{bmatrix} 0 \\ \gamma_i^{tar} \end{bmatrix} \tag{11}$$

Introducing $A_i(k)$, $G_i(k)$, and Γ_i as the notation for the matrices in the above equation, we have

$$A_i(k) = \begin{bmatrix} 1 & \alpha_i(k) \\ -\delta_i(k+1) & -\delta_i(k+1)\alpha_i(k) \end{bmatrix} \tag{12}$$

$$G_i(k) = \begin{bmatrix} -\alpha_i(k) \\ \delta_i(k+1)\alpha_i(k) \end{bmatrix} \tag{13}$$

$$\Gamma_i = \begin{bmatrix} 0 \\ \gamma_i^{tar} \end{bmatrix} \tag{14}$$

Defining the state vector as

$$\mathbf{x}_i(k) = \begin{bmatrix} p_i(k) \\ e_i(k) \end{bmatrix} \tag{15}$$

equation (11) becomes

$$\mathbf{x}_i(k+1) = A_i(k)\mathbf{x}_i(k) + G_i(k)w_i(k) + \Gamma_i \tag{16}$$

where $w_i(k)$ is white noise with $E\{w_i(k)\} = 0$, with the white noise covariance matrix given by $E\{w_i(k)w_i(n)\} = W\Delta(k-n)$, where W is the power spectrum density of w_i. $\Delta(k-n)$ is the Kronecker delta function defined by

$$\Delta(k-n) = \begin{cases} 1\ , & \text{k=n} \\ 0\ , & \text{otherwise} \end{cases} \tag{17}$$

Let the mean of $\mathbf{x}_i(k)$ be $\bar{\mathbf{x}}_i(k)$ with the initial value equal to $[\bar{p}_i(0) \quad \bar{e}_i(0)]^T$. It can be shown that the mean is given by [29]- [30]

$$\mathrm{E}\{\mathbf{x}_i(k)\} = \bar{\mathbf{x}}_i(k) = \prod_{j=0}^{k-1} A_i(j)\bar{\mathbf{x}}_i(0) + \sum_{l=1}^{k-1}\prod_{j=1}^{l} A_i(k-j)\Gamma_i + \Gamma_i \tag{18}$$

The variance matrix of $\mathbf{x}_i(k)$, denoted by $Q_i(k) = Var\{\mathbf{x}_i(k)\}$, is a 2×2 matrix. The variance of linear system (16) driven by white noise satisfies the Laypunov difference equation [29], [30], [31] assuming that the system state and white noise are independent (uncorrelated), as shown in (19).

$$\begin{aligned} e_i(k+1) &= \gamma_i^{tar} - \gamma_i(k+1) \\ &= \gamma_i^{tar} - \delta_i(k+1)p_i(k+1) \\ &= \gamma_i^{tar} - \delta_i(k+1)p_i(k) - \delta_i(k+1)\alpha_i(k)e_i(k) + \delta_i(k+1)\alpha_i(k)w_i(k) \end{aligned} \tag{10}$$

$$\begin{aligned} Q_i(k+1) &= \mathrm{E}\{(\mathbf{x}_i(k+1) - \bar{\mathbf{x}}_i(k+1))(\mathbf{x}_i(k+1) - \bar{\mathbf{x}}_i(k+1))^T\} \\ &= A_i(k)Q_i(k)A_i^T(k) + G_i(k)WG_i^T(k) \end{aligned} \tag{19}$$

$$Q_{11}(k+1) = Q_{11}(k) + 2\alpha_i(k)Q_{12}(k) + (\alpha_i(k))^2Q_{22}(k) + (\alpha_i(k))^2W \tag{21}$$

$$Q_{12}(k+1) = -\delta_i(k+1)[Q_{11}(k) + 2\alpha_i(k)Q_{12}(k) + (\alpha_i(k))^2Q_{22}(k) + (\alpha_i(k))^2W] \tag{22}$$

Note that $Q_i(k) = Q_i^T(k) \geq 0$ (symmetric positive semidefinite for every k). Let us partition the 2×2 matrix Q (index i is dropped for simplicity) as

$$\begin{aligned} Q = Q^T &= \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} \\ &= \begin{bmatrix} \mathrm{Var}(p(k)) & \mathrm{Cov}(p(k), e(k)) \\ \mathrm{Cov}(p(k), e(k)) & \mathrm{Var}(e(k)) \end{bmatrix} \end{aligned} \tag{20}$$

In view of (9), (10), and (20), the Lyapunov difference equation (19) is partitioned as shown in (21), (22), and (23). From (21), (22), and (23) we observe that for every user i

$$Q_{12}(k) = -\delta_i(k)Q_{11}(k) \ , \quad \forall k \tag{24}$$

$$Q_{22}(k) = (\delta_i(k))^2Q_{11}(k) \ , \quad \forall k \tag{25}$$

It follows from (20), (24) and (25) that

$$\begin{aligned} &\min_{\alpha_i(k)} [\mathrm{Var}(p_i(k+1)) + \mathrm{Var}(e_i(k+1))] \\ &\Leftrightarrow \min_{\alpha_i(k)} [Q_{11}(k+1) + Q_{22}(k+1)] \\ &\Leftrightarrow \min_{\alpha_i(k)} [(1 - \alpha_i(k)\delta_i(k))^2Q_{11}(k) + (\alpha_i(k))^2W] \end{aligned} \tag{26}$$

Using the fact that $Q_{11}(k) \geq 0$, and $W > 0$, formula (26) represents a convex function of $\alpha_i(k)$. It is straight forward to derive the optimal value of $\alpha_i(k)$ as

$$\frac{\partial[(1 - \alpha_i(k)\delta_i(k))^2Q_{11}(k) + (\alpha_i(k))^2W]}{\partial\alpha_i(k)} = 0 \tag{27}$$

which yields

$$\alpha_i^{opt}(k) = \frac{\delta_i(k)Q_{11}(k)}{W + (\delta_i(k))^2Q_{11}(k)} \tag{28}$$

that achieves the minimum of the criterion function.

In a practical situation, it is usually difficult to measure/calculate the variance of the transmission power ($Q_{11}(k)$). As a result, it would be difficult to calculate the optimal gain ($\alpha_i^{opt}(k)$). However, since the power of the SIR measurement noise is usually much smaller comparing with the signal power, we could use the following *sub-optimal solution* of the controller gain

$$\alpha_i^{subopt}(k) = \frac{1}{\delta_i(k)} \tag{29}$$

After the sub-optimal gain $\alpha_i^{subopt}(k)$ is employed, the system matrix $A_i(k)$ becomes

$$A_i^{subopt}(k) = \begin{bmatrix} 1 & \frac{1}{\delta_i(k)} \\ -\delta_i(k+1) & -\frac{\delta_i(k+1)}{\delta_i(k)} \end{bmatrix} \tag{30}$$

To examine the stability of this discrete time-varying system, we use the well known Lyapunov stability theory for time-varying systems, which can be found in [8, Chapter 23, pp. 438-439].

Theorem 1: The system matrix under the sub-optimal control $A_i^{subopt}(k)$ is stable for all k.

The proof of the above theorem is given in Appendix, and can be found in [22]. The proposed power control algorithm using sub-optimal controller gain is

$$p_i(k+1) = p_i(k) + \frac{1}{\delta_i(k)}[\gamma_i^{tar} - y_i(k)] \tag{31}$$

It is a fully distributed algorithm. Each mobile user only needs to know its own stochastic SIR measurement $y_i(k)$ and channel variation $\delta_i(k)$ to decide the power control command. The stability of the discrete time-varying power control system driven by white noise guarantees the convergence of the proposed distributed power control algorithm.

Let us examine the mean of the state variables under the sub-optimal controller gain $\alpha_i^{subopt}(k)$. Replace $A_i(j)$ and $A_i(k-j)$ by $A_i^{subopt}(j)$ and $A_i^{subopt}(k-j)$ in equation (18), respectively. Note that $A_i^{subopt}(j)A_i^{subopt}(j-1) = 0 \ , \ \forall j > 1$. Using this property, we obtain (32). We observe that when channels are slowly changing, i.e., when $\delta_i(k) \approx \delta_i(k-1)$, we have $\mathrm{E}\{\gamma_i^{subopt}(k)\} \approx \gamma_i^{tar}$.

The variances of the transmission power and SIR error under sub-optimal control are

$$\begin{bmatrix} \mathrm{Var}\{p_i^{subopt}(k)\} \\ \mathrm{Var}\{e_i^{subopt}(k)\} \end{bmatrix} = \begin{bmatrix} \frac{W}{\delta_i^2(k-1)} \\ \delta_i^2(k)\frac{W}{\delta_i^2(k-1)} \end{bmatrix} \tag{33}$$

The minimal values of the performance criteria become

$$J_i^{subopt}(k) = \frac{1 + \delta_i^2(k+1)}{\delta_i^2(k)}W \tag{34}$$

The block diagram of the proposed power control scheme is shown in Fig. 2. The estimator in the feedback loop and

$$Q_{22}(k+1) = (\delta_i(k+1))^2[Q_{11}(k) + 2\alpha_i(k)Q_{12}(k) + (\alpha_i(k))^2Q_{22}(k) + (\alpha_i(k))^2W] \quad (23)$$

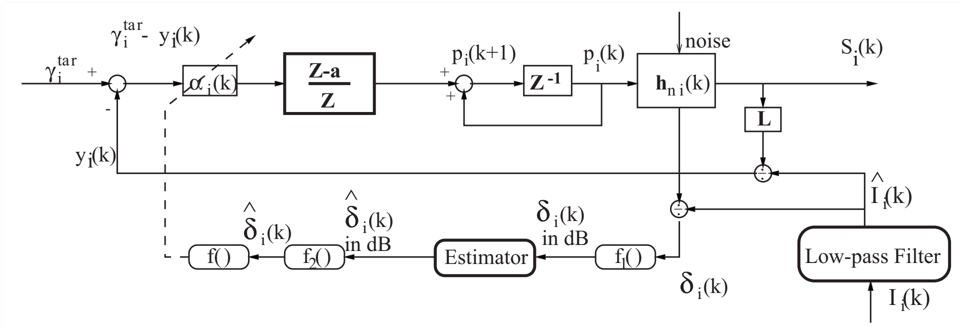


Fig. 2. The estimator aided stochastic power control system with the proportional derivative controller.

$$\begin{aligned}
\mathrm{E}\{\mathbf{x}_i^{subopt}(k)\} &= \bar{\mathbf{x}}_i^{subopt}(k) = \prod_{j=0}^{k-1} A_i^{subopt}(j)\bar{\mathbf{x}}_i(0) + \sum_{l=1}^{k-1}\prod_{j=1}^{l} A_i^{subopt}(k-j)\Gamma_i + \Gamma_i \\
&= A_i^{subopt}(k-1)\Gamma_i + \Gamma_i \\
&= \begin{bmatrix} \frac{\gamma_i^{tar}}{\delta_i(k-1)} \\ (1 - \frac{\delta_i(k)}{\delta_i(k-1)})\gamma_i^{tar} \end{bmatrix}
\end{aligned} \quad (32)$$

the $\frac{Z-a}{Z}$ block in the forward loop will be explained later in Section 3A and Section 3C, respectively.

Comment:
When the SIR measurement noise is Gaussian white noise, the above stochastic power control scheme with gain $\alpha_i^{opt}(k)$ is optimal because the state variables, SIR error and mobile's transmission power, are Gaussian too. Hence, their statistics is completely determined by their means and variances. Note that the output of a linear system driven by Gaussian white noise is also Gaussian white noise stochastic process [29].

When the SIR measurements contain white Gaussian noise, the sub-optimal controller gain is equal to the reciprocal of the channel variation. We assume that the sum of a large number of interferers is relatively constant during one slot (0.625 ms). As a consequence, the channel variation remains constant during one slot.

A. Simulation Example

The performance of the proposed optimal stochastic power control scheme is tested through simulation. The simulation environment is described as follows. A CDMA system with 7 hexagonal cells and 16 users per cell is considered. The operating frequency is 1.9 GHz and the bandwidth of each channel is assumed to be 1.23 MHz, which is in accordance with the CDMA 2000 standard [13]. The data rate is set at 9600 bps and the processing gain is set at 128 (21 dB). The target SIR is 7 dB, which corresponds to the bit error rate (BER) being less than 10^{-3}. Note that the SIR defined in this research work includes the processing gain, which is denoted by $\frac{E_b}{N_0}$ in the standard IS-95B [38]/CDMA 2000 [13], where E_b is the energy per information bit and N_0 is the interference power spectral density.

In the simulation, we further make the following assumptions:

1) The effects of antenna directivity, voice activity factor, and soft handoff are ignored.
2) The minimum and maximum power can be transmitted by a mobile (IS-95B [38]) are p_{min} = 8 dBm (6.3 mW) and p_{max} = 33 dBm (2 W), respectively.
3) The background noise power is 0.05 mW.
4) The transmitted power is updated periodically, every 0.625 msec, which corresponds to 1,600 Hz fast closed-loop power control frequency proposed in IMT-2000.
5) The location of the mobiles are assumed to be uniformly distributed in a cell.
6) It is assumed that the link gains have the following form

$$h_{ni}(k) = d_{ni}^{-4}(k)S_{ni}(k) \quad (35)$$

where $d_{ni}(k)$ is the distance from the ith mobile to the nth base station at time instant k, S_{ni} is a log-normal distributed stochastic process.
7) It is assumed that the cell diameter is 2 km. $d_{ni}(k)$ is a 2-D uniformly distributed random variable.
8) It is assumed that the standard deviation of S_{ni} is 8 dB, [33].

The proposed stochastic power control scheme is applied. The average SIR is plotted in Fig. 3, which is calculated as

$$\bar{\gamma}(k) = \frac{1}{N}\sum_{i=1}^{N}\gamma_i(k) \quad (36)$$

We observe that the SIRs converge to the target SIR rather quickly.

The variance of transmission power is plotted as a function of time steps in Fig. 4. We observe that the variance of transmit

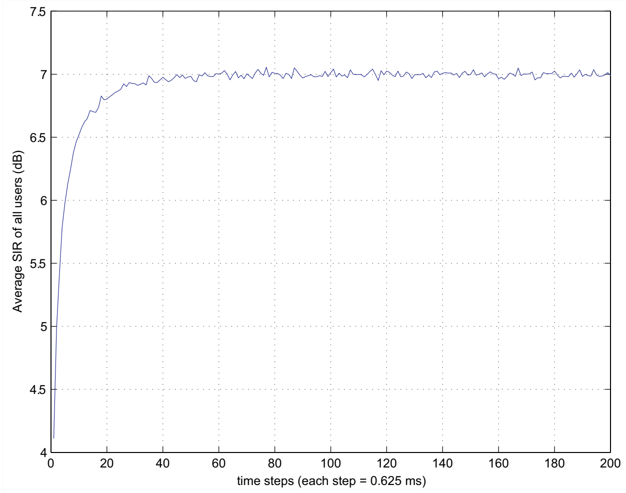


Fig. 3. The average SIR of all users under sub-optimal stochastic power control.

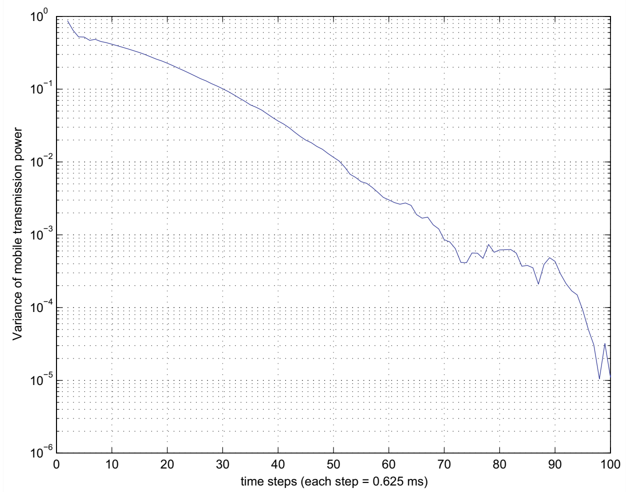


Fig. 4. The variance of mobile's transmission power under proposed sub-optimal stochastic power control.

power decreases very fast with time, to about 10^{-5} within 100 steps (62.5 ms).

III. Extended Results and Analysis

The link gains in a wireless system are subject to uncertainties caused by the random nature of channel fading. In the proposed stochastic power control system, all the uncertainties of link gains are incorporated into the uncertainties of channel variations. In practical system implementation, in order to perform the proposed stochastic power control scheme, an estimate of the channel variation is needed. Previous work using the Kalman filter to estimate/predict interference can be found in [18]. As described in Section 1, the instantaneous interference plus noise power at the base station receiver could be measured using pilot symbols and then averaged through a low-pass filter. Here, we use the averaged interference plus noise power to calculate the channel variation.

A. Estimation of Channel Variations

The dynamics of the channel variation (in dB) can be modeled as

$$\delta_i(k) = \delta_i(k-1) + w_i(k-1) \tag{37}$$

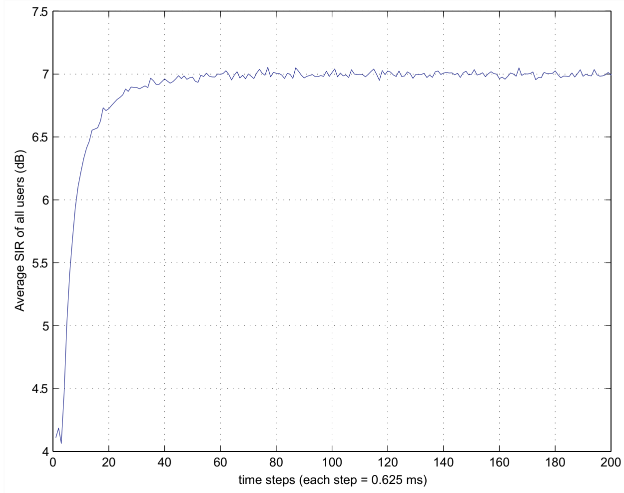


Fig. 5. Average SIR of all the users using the Kalman filter.

where $w_i(k)$ represents the process noise (disturbance) introduced by the link gain. Let $z_i(k)$ be the measurement of $\delta_i(k)$.

$$z_i(k) = \delta_i(k) + v_i(k) \tag{38}$$

where $v_i(k)$ is the measurement noise at the base station receiver. If we assume that $w_i(k)$ and $v_i(k)$ are Gaussian, the Kalman filter may be employed to get the estimates of the channel variation. Let $w_i(k) \sim N(0, Q_i(k))$ and $v_i(k) \sim N(0, R_i(k))$, and assume that they are uncorrelated. The proposed Kalman filter is given by [34]

$$\hat{\delta}_i(k) = \hat{\delta}_i(k-1) + K^{\delta}(k)[z_i(k) - \hat{\delta}_i(k-1)] \tag{39}$$

where $K^{\delta}(k)$ is the Kalman filter gain, which is computed by [34]

$$K^{\delta}(k) = \frac{P^{\delta}(k-1) + Q_i(k)}{P^{\delta}(k-1) + Q_i(k) + R_i(k)} \tag{40}$$

where $P^{\delta}(k)$ can be calculated iteratively as [34]

$$P^{\delta}(k) = \frac{R_i(k)[P^{\delta}(k-1) + Q_i(k)]}{P^{\delta}(k-1) + Q_i(k) + R_i(k)} \tag{41}$$

The statistics of the initial conditions are defined by $E[\delta_i(0)] = \hat{\delta}_i(0)$, $E[\delta_i(0)^2] = P_0^{\delta}$.

The block diagram of the proposed estimator is shown in Fig. 2 in the feedback loop. Functions $f_1()$ and $f_2()$ are used for converting to dB scale and the other way around.

The estimator in the feedback loop could be the Kalman filter if the fluctuation of channel variation and the measurement noise are Gaussian distributed. In general, the fluctuation of channel variation is not Gaussian distributed. In that case, a more appropriate choice is the H_∞ filter (instead of Kalman filter) [22], [39].

By using the filtering technique, we obtain an estimator based controller

$$p_i(k+1) = p_i(k) + \hat{\alpha}_i(k)[\gamma_i^{tar} - y_i(k)] \tag{42}$$

where $\hat{\alpha}_i^{subopt}(k) = 1/\hat{\delta}_i(k)$.

To show the convergence of the users' SIRs to the target value, we ran the stochastic power control algorithm by using the Kalman filter and the H_∞ filter as estimators, respectively.

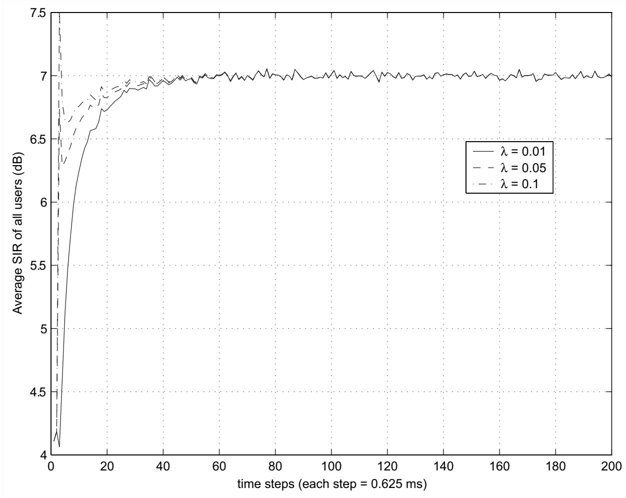


Fig. 6. Average SIR of all the users using the H_∞ filter with different values for λ.

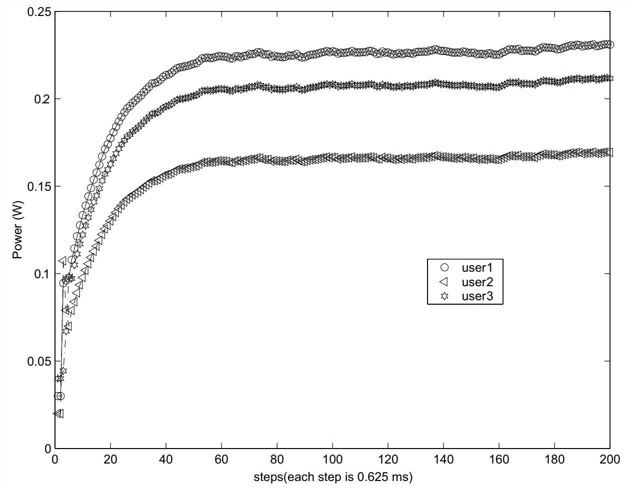


Fig. 7. Transmission power of users.

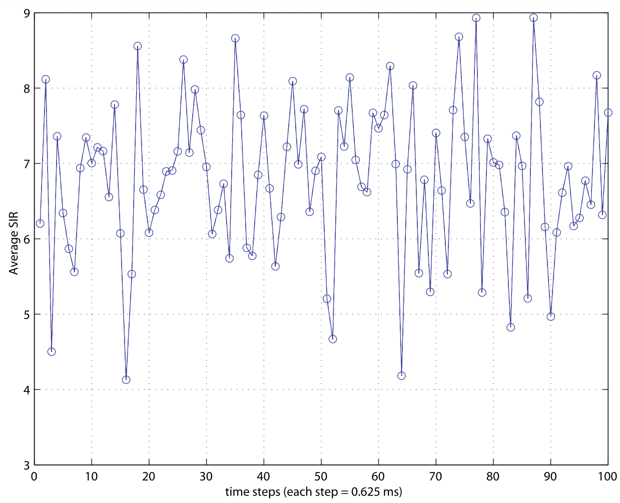


Fig. 8. SIR performance when channel fading is severe.

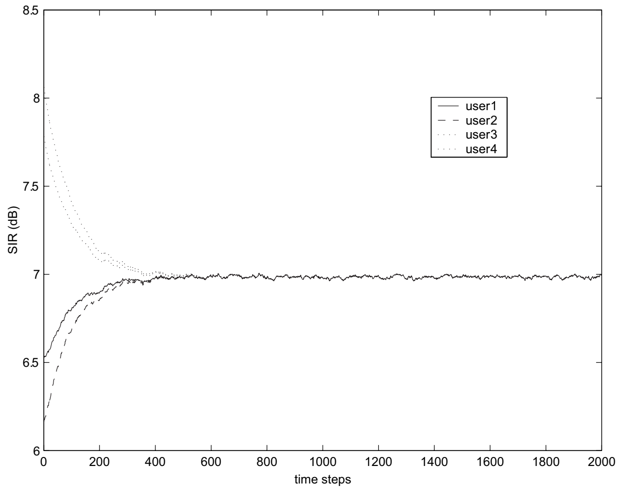


Fig. 9. SIR performance by adding a low-pass filter when channel fading is severe.

The simulation platform is the same as the one described in Section IIA. The average SIR is plotted in Figs. 5 and 6, which is defined in (36). We observe that the SIRs converge to the target SIR as expected in both cases. Note that the H_∞ filter has a parameter λ as an additional degree of freedom that can be chosen to achieve satisfactory estimation [39].

The transmission powers of users (only 3 of them are plotted) are shown in Fig. 7. They converge to the deterministic solutions within 60 steps. Fig. 8 show the actual SIR performance of users under severe fading conditions. Applying a low-pass filter to the SIR measurements, we observe in Fig. 9 that the SIR performance of users (only 4 users randomly chosen are plotted) is improved due to averaging. However, the response time is long (about 400 ms).

B. Performance Comparison between Proposed Minimum Variance Power Control and the Stochastic Power Control of [15]

In order to compare the performance between proposed Minimum Variance Power Control and the Stochastic Power Control developed in [15], we ran the minimum variance power coontrol algorithm using H_∞ filter as channel estimator, side-by-side with the Stochastic Power Control algorithm (equation (28) in [15]). The simulation platform is the same as the one described in Section IIA, where white Gaussian noise is added to the received power at each base station.

The parameter of the H_∞ filter is $\lambda = 0.1$. Both $a_n = 0.01$ and $a_n = \frac{1}{n}$ (as suggested in [15]) have been tested with $L = 1$ for the Stochastic Power Control algorithm proposed in [15]. Note that perfect knowledge of channel gain is assumed to be available for the Stochastic Power Control algorithm.

The average SIR of all users in the system, as defined in (36), has been plotted in Fig. 10. Even without knowledge of channel gains, the proposed minimum variance power coontrol algorithm outperforms the Stochastic Power Control algorithm for both $a_n = 0.01$ and $a_n = \frac{1}{n}$. The proposed algorithm converges to the SIR target in 20 steps, while the Stochastic Power Control algorithm needs much longer time to converge. The advantage of robust estimator, namely, the H_∞ filter, is demonstrated in this performance comparison study, Fig. 10. Note that the results show a similar pattern of convergence of the Stochastic Power Control algorithm, as observed in Fig. 6 in [15], for both $a_n = 0.01$ and $a_n = \frac{1}{n}$, as expected.

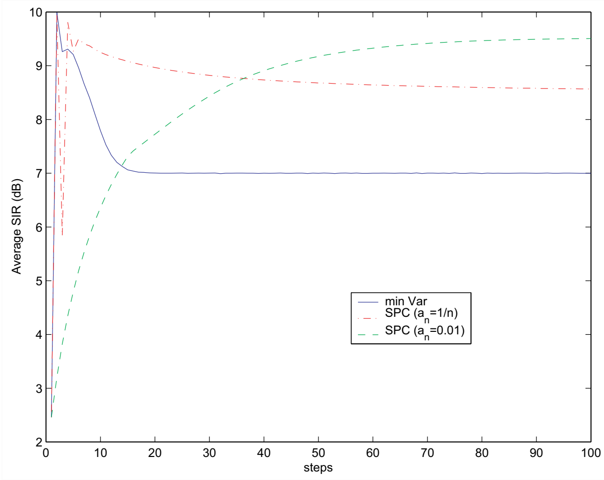


Fig. 10. Comparison of SIR performances for the proposed method and the method of [15].

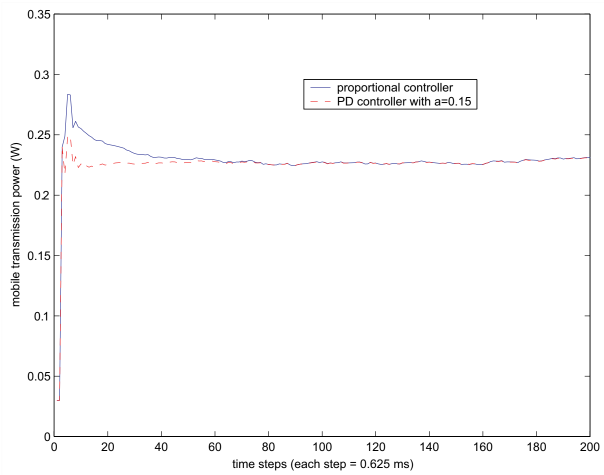


Fig. 11. Transmission power of a typical user with the added proportional derivative controller.

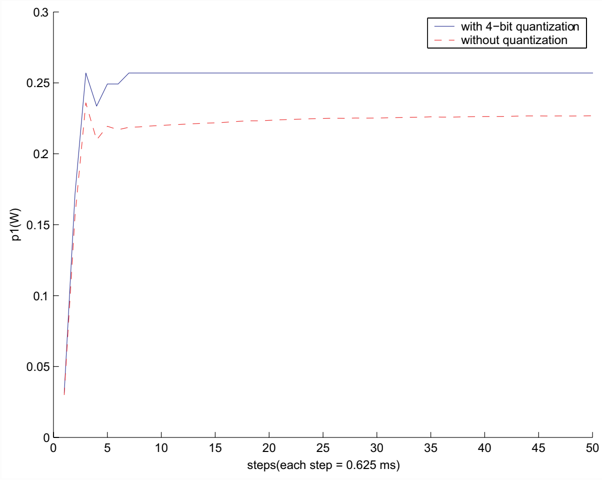


Fig. 12. Transmission power of a typical user with/without quantization.

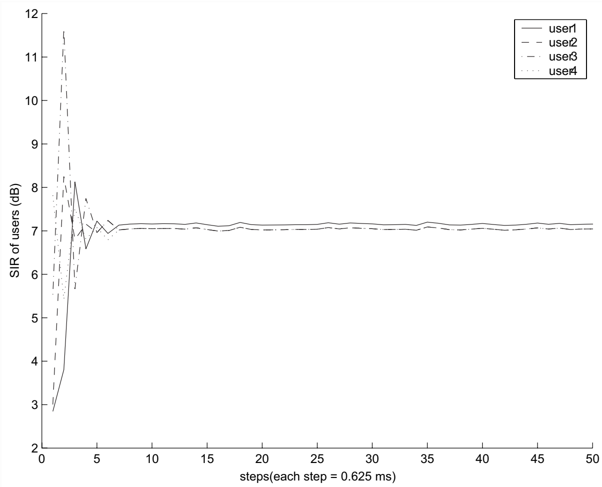


Fig. 13. The SIR of users with quantization.

C. Improvement by adding Proportional Derivative (PD) Controller

In order to reduce the overshoot of the proposed controller, we introduce a PD controller. It is well-known that PD controller will decrease the overshoot and improve the transient response. The block of the proposed PD controller in the forward loop is shown in Fig. 2 as the block with the transfer function $\frac{Z-a}{Z}$.

After applying the PD controller, we have an additional controller parameter a to tune. It is observed from Fig. 11 that with $a = 0.15$ we get a very good performance.

IV. Effect of Quantization

In current IS-95 and IS-2000 systems, only up/down bang-bang power control with fixed step size (0.5 dB or 1 dB) is implemented. It is well known that the fixed-step power control algorithm can not be optimized for the time-varying channel, and will introduce constant power control error. In [35], the continuous value of mobile's transmission power is rounded to corresponding discrete levels by either "ceiling" or "flooring". The fixed step size constraints (0.5 dB or 1 dB) are not taken into consideration in [35]. A multi-step approach, called pulse-code-modulation (PCM) realization, is proposed in [36]. Simulations indicated that the multi-step approach achieves a better performance than the fixed-step power control algorithm. Applying quantization on the proposed stochastic power control command, we get a multi-step-like power control algorithm. If the number of the transmit power control (TPC) bits is n, then the number of the different power level is 2^n.

The performance of the proposed control scheme with 4-bit quantization is shown in Figs. 12 and 13. Again, users are randomly chosen. The transmission power is rounded to the nearest discrete power level. The performance of the proposed power control scheme will not be affected too much with 4-bit quantization. The SIRs converge pretty fast to the desired target value.

In the current IS-95 system and CDMA-2000 system, power control bits (1 bit) is transmitted every slot with a fixed power control step size (1 dB or 0.5 dB). In the proposed scheme, power control bits (4 bit) could be transmitted every 4 slots but with multiple step size (16 different step sizes). The power control performance will be improved due to the fast convergence of the proposed scheme and the ratio of information bits to power control bits remains unchanged.

V. Conclusions and Discussions

The stochastic power control problem considered in [15] assumes that the matched filter output contains white Gaussian noise. A stochastic power control scheme is derived by using a L-bit window averaged square of the matched filter output and link gain. In this research work, based on the SIR measurement mechanism for time-multiplexed pilot signal (reported in 3G wireless systems proposals), the SIR measurement is assumed to contain white Gaussian noise. The stochastic power control scheme is obtained by minimizing the sum of variances of mobile's transmission power and SIR error. Even though both schemes converge, the scheme proposed in this paper has much faster convergence speed than the one in [15]. In terms of complexity, the scheme proposed in this work needs an estimator to estimate the channel variation, but it does not require the knowledge of the link gain.

In practical situations, the noise within the SIR measurement may not be modeled as white Gaussian noise [16], [18]. Hence, the stochastic power control problem may be formulated as a linear quadratic optimal control problem with the system driven by non-Gaussian noise, and the solution of such systems can be found in [37]. In such a case, the stochastic power control scheme proposed in this paper will be sub-optimal if the higher-order non-Gaussian white noise moments are small so that they do not affect the system performance much.

It may be desirable to get an estimate of the SIR rather than using the SIR measurement directly. However, since the dynamics of the SIR is not linear, the extended Kalman filter has to be used because the filtering problem becomes nonlinear [34].

The proposed power control system could be easily generalized by introducing weighting factors into the optimization criterion

$$J_i^{new}(k) = [\beta_1 \mathrm{Var}(p_i(k+1)) + \beta_2 \mathrm{Var}(e_i(k+1))] \quad (43)$$

where β_1 and β_2 are positive constants and will be decided by the system designer, depending on which term (the variance of the transmission power, or the variance of the SIR error) is more important ($\beta_1 > \beta_2$ indicates that minimization of the power variance is more important than minimizarion of the SIR error variance, and vice versa).

In this paper, in order to obtain the optimal solution of the stochastic power control problem analytically, only the SIR measurements are assumed to be stochastic. The link gains are assumed to be fixed during each Power Control Group (PCG) in both [15] and this work. In practice, however, because the propagation conditions change from time to time, the link gains are random processes. Hence the received power, interference and SIR are all random processes. It is very difficult to get the analytical results of power control algorithms in such cases. In [27], only a 2-user case is considered analytically where link gains are assumed to be varying with shadow fading effect. Multiuser case is only studied through simulation. In our extension of the sub-optimal solution, an estimator is proposed to deal with the changing of the link gains. It estimates the channel quality in terms of channel variation at each Power Control Group (PCG) in real-time. Simulation results indicate the effectiveness of this scheme.

Appendix
Proof of Theorem 1.

Proof: The linear state equation

$$x(k+1) = A(k)x(k) \quad , \qquad x(k_0) = x_0 \quad (44)$$

is uniformly stable if there exists an $n \times n$ matrix sequence $R(k)$ that for all k is symmetric and

$$\eta I \leq R(k) \leq \rho I \quad (45)$$

$$A^T(k)R(k+1)A(k) - R(k) \leq 0 \quad (46)$$

where η and ρ are finite positive constants [32]. It is easy to verify that the proposed optimal power control system is stable using the above stability criterion by taking

$$R(k) = \begin{bmatrix} \rho - \eta & \delta_i(k) \\ \delta_i(k) & 2 \end{bmatrix} \quad (47)$$

where η and ρ are finite positive constants satisfying

$$\rho \geq 2\eta \quad (48)$$

$$(\rho - 2\eta)(2 - \eta) \geq \max_k \delta_i^2(k) \quad (49)$$

$$-\eta(2 - \rho) \leq \min_k \delta_i^2(k) \quad (50)$$

$R(k)$ is a 2×2 symmetric matrix for all k. Firstly, let us check condition (45).

$$R(k) - \eta I = \begin{bmatrix} \rho - 2\eta & \delta_i(k) \\ \delta_i(k) & 2 - \eta \end{bmatrix} \quad (51)$$

The above matrix is positive semidefinite if (48) and (49) are satisfied.

$$R(k) - \rho I = \begin{bmatrix} -\eta & \delta_i(k) \\ \delta_i(k) & 2 - \rho \end{bmatrix} \quad (52)$$

The above matrix is negative semidefinite if (50) is satisfied. Secondly, check condition (46). It is obvious that the above matrix is negative semidefinite. Furthermore, conditions (48), (49) and (50) can be easily satisfied. Just choose η to be a very small positive number and choose ρ to be a very large positive number. Note that adding a zero mean white noise stochastic process ($G_i(k)w_i(k)$) and a constant term (Γ_i) as system inputs will not affect the internal stability of the system, since it does not depend on the input signals [32]. ■

References

[1] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 57-62, Feb. 1992.

[2] J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 305-311, Aug. 1992.

[3] Q. Wu, "Performance of optimum transmitter power control in CDMA cellular mobile systems," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 571-575, Mar. 1999.

[4] S. Grandhi, R. Vijayan, D. Goodman, and J. Zander, "Centralized power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 466-468, Nov. 1993.

[5] S. Grandhi, R. Vijayan and D. Goodman, "Distributed power control in cellular radio systems," *IEEE Trans. Commun.*, vol. 42, pp. 226-228, Feb/Mar/Apr. 1994.

$$(A_i^{opt}(k))^T R(k+1) A_i^{opt}(k) - R(k) = \begin{bmatrix} 0 & \frac{\rho-\eta}{\delta_i(k)} - \frac{\delta_i(k)}{2} \\ \frac{\rho-\eta}{\delta_i(k)} - \frac{\delta_i(k)}{2} & \frac{\rho-\eta}{\delta_i^2(k)} - 2 \end{bmatrix} \tag{53}$$

[6] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 641-646, Nov. 1993.

[7] D. Mitra, "An asynchronous distributed algorithm for power control in cellular radio systems," in *Proc. 4th WINLAB Workshop of 3rd Generation wireless Information Networks*, 177-186, Piscataway, NJ, Oct. 1993.

[8] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341-1347, Sept. 1995.

[9] R. Jantti and S. Kim, "Second-order power control with asymptotically fast convergence," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 447-457, 2000.

[10] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*, Addison-Wesley, 1995.

[11] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next-generation mobile communications systems," *IEEE Commun. Mag.*, vol. 36, pp. 56-69, Sept. 1998.

[12] E. Dahlman, B. Gudmundson, M. Nilsson, and J. Skold, "UMTS/IMT-2000 based on wideband CDMA," *IEEE Commun. Mag.*, vol. 36, no. 9, pp. 70-80, Sept. 1998.

[13] ITU, RTT 45.5, CDMA 2000 standard.

[14] S. Seo, T. Dohi, and F. Adachi, "SIR-based transmit power control of reverse link for coherent DS-CDMA mobile radio," *IEICE Trans. Commun.*, vol. E81-B, pp. 1508-1516, July 1998.

[15] S. Ulukus and R.D. Yates, "Stochastic power control for cellular radio systems," *IEEE Trans. Commun.*, vol. 46, pp. 784-798, June 1998.

[16] D. Ramakrishna, N. Mandayam, and R. Yates, "Subspace based estimation of the signal-to-interference ratio for CDMA cellular systems," *IEEE Trans. Veh. Technol.*, vol. 49, 1732-1742, 2000.

[17] K. K. Leung, "Power control by interference prediction for broadband wireless packet networks" *IEEE Trans. Wireless Commun.*, vol. 1, pp. 256-265, Apr. 2002.

[18] K. Leung, "A Kalman-filter method for power control in broadband wireless networks," *IEEE INFOCOM*, vol. 2, pp. 948-956, Mar. 1999.

[19] K. Leung and L. Wang, "Integrated link adaptation to improve error troughput performance in broadband wireless packet networks," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 619-629, 2002.

[20] K. Shoarinejad, J. Speyer, and G. Pottie, "Integrated predictive power control and dynamic channel assignment in mobile radio systems," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 976-988, Sept. 2003.

[21] T. Jiang, N. Sidiropoulos, and G. Giannakis "Kalman filtering for power estimation in mobile communications," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 151-161, Jan. 2003.

[22] L. Qian, *Optimal Power Control in Cellular Wireless Systems*, Doctoral Dissertation, Rutgers–The state University of New Jersey, Piscataway, NJ, May, 2001.

[23] L. Qian and Z. Gajic, "Optimal distributed power control in cellular wireless systems," *Dynamics of Continuous Discrete and Impulsive Systems*, vol. 10, pp. 537-559, 2003.

[24] M. Varanasi and D. Das, "Fast stochastic power control algorithms for nonlinear multiuser receivers," *IEEE Trans. Commun.*, vol. 50, pp. 1817-1827, 2002.

[25] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 46-55, Jan. 2002.

[26] C. Charalambous, S. Denic, S. Djouadi, and N. Menemenlis, "Stochastic Power Control for Short-term Flat Fading Wireless Networks: Almost sure QoS measures," *Proc. 40th IEEE Conference on Decision and Control*, pp. 1049-1052, Orlando, FL, Dec 2001.

[27] C. W. Sung and W. S. Wong, "Performance of a cooperative algorithm for power control in cellular systems with a time-varying link gain matrix," *Wireless Networks*, vol. 6, pp. 429-439, 2000.

[28] S. Ariyavisitakul and L. F. Chang, "Signal and interference statistics of a CDMA system with feedback power control," *IEEE Trans. Commun.*, vol. 41, pp. 1626-1634, Nov. 1993.

[29] H. Kwakernaak and R. Sivan, *Linear Optimal Control Systems*. Wiley, 1972.

[30] P. Kumar and P. Varaiya, *Stochastic Systems: Estimation, Identification, and Adaptive Control*. Prentice Hall, 1986.

[31] Z. Gajic and M. Qureshi, *Lyapunov Matrix Equation in System Stability and Control*. Academic Press, San Diego, 1995.

[32] W. J. Rugh, *Linear System Theory*. Prentice-Hall, 1996.

[33] G. Stuber, *Principles of Mobile Communication*. Kluwer Academic Publishers, 1996.

[34] M. Grewal and A. Andrews, *Kalman Filtering - Theory and Practice*. Prentice-Hall, 1993.

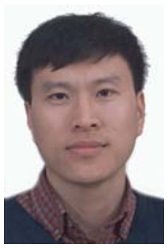
[35] M. Andersin, Z. Rosberg, and J. Zander, "Distributed discrete power control in cellular PCS," *Wireless Personal Commun.*, vol. 6, pp. 211-231, 1998.

[36] C. Chang, J. Lee, and F. Ren, "Design of power control mechanisms with PCM realization for the uplink of a DS-CDMA cellular mobile radio system," *IEEE Trans. Veh. Technol.*, vol. 45, pp. 522-530, Aug. 1996.

[37] A. Germani and G. Mavelli, "Optimal quadratic solution for the non-Gaussian finite-horizon regulator problem," *Systems & Control Letters*, vol. 38, pp. 321-331, 1999.

[38] TIA/EIA Interim Standard-95B: Mobile station-base station compatibility standard for dual-mode wideband spread spectrum cellular system, TIA, 1998.

[39] K. Zhou and J. Doyle, *Essentials of Robust Control*. Upper Saddle River, NJ: Prentice Hall, 1997.



Lijun Qian is an assistant professor in the Department of Electrical Engineering at Prairie View AM University. He received his B.E. from Tsinghua University in Beijing, M.S. from Technion-Israel Institute of Technology, and Ph.D. from WINLAB, Rutgers - The State University of New Jersey. Before joining PVAMU, he was a researcher at Networks and Systems Research Department of Bell Labs in Murray Hill, NJ. His major research interests are in wireless communications and mobile networks, network security and intrusion detection, control theory and game theory.

Zoran Gajic received Dipl. Ing. (5 year program) and Mgr. Sci. (2 year program) degrees in Electrical Engineering from the University of Belgrade, and an M.S. degree in Applied Mathematics and Ph. D. in Systems Science Engineering from Michigan State University. Dr. Gajic is a Professor of Electrical and Computer Engineering at Rutgers University. He has been teaching electrical circuits, linear systems and signals, controls, and networking courses at the same school since 1984. Dr. Gajic's research interests are in controls systems, wireless communications, and networking. He is the author or coauthor of more than sixty journal papers, primarily published in IEEE Transactions on Automatic Control and IFAC Automatica journals, and seven books in the fields of linear systems and linear and bilinear control systems published by Academic Press, Prentice Hall, Marcel Dekker, and Springer Verlag. His textbook Linear Dynamic Systems and Signals, Prentice Hall, 2003 has been translated into the Chinese Simplified language. Professor Gajic has delivered two plenary lectures at international conferences and presented more than one hundred conference papers. He serves on the editorial board of the journal Dynamics of Continuous, Discrete, and Impulsive Systems, and was a guest editor of a special issues of that journal, on Singularly Perturbed Dynamic Systems in Control Technology.