Energy Efficient Adaptive Modulation in Wireless Cognitive Radio Ad Hoc Networks

Song Gao, Lijun Qian*, Dhadesugoor. R. Vaman
ARO/ARL Center for Battlefield Communications Research
Prairie View A&M University, Texas A&M University System
Prairie View, Texas 77446, USA

***Abstract*—In this paper, an energy efficient adaptive modulation scheme is proposed for a wireless cognitive radio ad hoc network, where each node is equipped with cognitive radio and the network is an OFDMA system operating on time slots. In each slot, the users with new traffic demand will sense the spectrum and locate the available subcarrier set. Then they choose subcarriers with favorable channel condition individually while avoid introducing harmful interference to the existing users. Given the delay requirement, an adaptive modulation strategy is proposed for each individual user to minimize the energy consumption per bit over the available subcarrier set by selecting the optimal constellation size. The optimal solution of energy efficient adaptive modulation is derived by using Dinkelback-type algorithm, and a sub-gradient search algorithm is proposed to locate the optimal constellation size. Furthermore, a sub-optimal constant bit allocation strategy is presented to address the delay constraints and a distributed power control is performed to manage the co-channel interference among new users when needed. Simulation results demonstrate the effectiveness of our approach.**

I. Introduction

Mission Critical Networking (MCN) is under intensive research recently due to its wide-spread applications such as in military operations, disaster relief, etc. Usually MCN requires fast deployment as well as without infrastructure support. This makes wireless ad hoc network a promising candidate for MCN. Furthermore, efficient operation of MCN plays a pivotal role because such networks are typically resource (such as energy and spectrum) constraint. In this work, distributed resource allocation problem is considered through joint design of adaptive modulation and cognitive radio for an energy and spectrum constraint wireless ad hoc network to maximize energy efficiency and spectrum utilization.

Cognitive Radio (CR) [1] provides the capability of accessing the spectrum opportunistically and greatly improves spectrum utilization. Challenges arise with such dynamic and hierarchical means of accessing the spectrum, especially for the dynamic resource allocation of CR users by adapting their transmission parameters to the varying spectrum condition while adhering to quality of service (QoS) requirements [3]. In this paper, an energy constrained wireless CR ad hoc network is considered, where each node is equipped with CR and has limited battery energy. One of the critical performance measures of such networks is network lifetime. Hence energy efficient resource management schemes are desired. In this context, the present paper provides a framework for energy efficient adaptive modulation techniques in wireless CR ad hoc networks that employ orthogonal frequency division multiple access (OFDMA) [2]. OFDMA is well suited for cognitive radio because it is agile in selecting and allocating subcarriers dynamically and facilitates decoding at the receiving end of each subcarrier.

The CR OFDMA network operates on time slots. Existing users transmit a pilot signal periodically on occupied subcarriers. By detecting the presence of such a pilot signal, emerging CR users can determine the available subcarrier set in a target spectral range [4], and then tune transmission parameters to map the information bits efficiently over the available subcarrier set.

Adaptive modulation, also called bit loading, has been extensively explored in previous works. The basic idea behind adaptive modulation algorithm is to ensure that the most efficient mode is always employed over varying channel conditions. Compared with non-adaptive methods which require a fixed margin to maintain acceptable performance when the channel quality is poor, adaptive approaches result in better efficiency by taking advantage of the favorable channel conditions. In [5], [7], adaptive modulation techniques in MIMO-OFDM systems is presented with the objective to minimize the transmission power or improve spectral efficiency. The performance gain by combining adaptive modulation and power control is studied in [8], where significant throughput advantage has been demonstrated in the multi-user cellular system. Energy efficient modulation optimization problem in single carrier system is investigated in [10] for uncoded and coded MQAM and MFSK. In [6], a variable-rate and variable-power MQAM modulation scheme for high speed data transmission is presented over fading channels.

In this paper, adaptive modulation is employed to optimally distribute bits to the transmitted symbols, hence determining the constellation size on each of the available subcarriers. Adapting the constellation size will directly influence the power consumption of each node, and in turn will affect the lifetime of the network. The optimality is defined as maximizing energy efficiency subject to a pre-specified bit error rate (BER) and delay constraint. Specifically, given

*Corresponding author. Phone: 936-261-9915; Fax: 936-261-9930; E-mail: lijunqian@ieee.org

required BER and delay constraints, and assuming QAM is adopted as the modulation scheme, the optimal modulation order of QAM on each selected subcarrier that minimize the energy for delivery of L bits per slot (thus maximize the network lifetime) is derived analytically and is verified by simulations.

The remainder of this paper is organized as follows. In section II, the system model and the problem formulation are given. A fully distributed adaptive modulation algorithm for each individual user is proposed in section III. In Section IV, a distributed power control algorithm is suggested to manage potential co-channel interference caused by concurrent *new* users. Section V contains the simulation results and discussions. Section VI gives concluding remarks.

II. System Model

We consider an energy constrained CR OFDMA network of N communicating user-pairs. Both transmitter i and receiver j is indexed by $\mathcal{N} := \{1, 2, ..., N\}$. If $j = i$, receiver j is said to be the intended receiver of transmitter i. The transmission system is assumed to be a time-slotted OFDMA system with fixed time slot duration T_S. Slot synchronization is assumed to be achieved through a beaconing mechanism. Before each time slot, a guard interval is inserted to achieve synchronization, perform spectrum detection as well as resource allocation (based on the proposed scheme). Inter-carrier interference (ICI) caused by frequency offset from the side lobes is not considered in this work (which can be mitigated by windowing the OFDM signal in the time domain or adaptively deactivating adjacent subcarriers [11]).

A frequency selective Rayleigh fading channel is assumed at the physical layer, and the entire spectrum is appropriately divided into K subcarriers to guarantee each subcarrier experiencing flat Rayleigh fading. We label the subcarrier set available to the transmitter receiver pair i after spectrum detection by $\mathcal{L}_i \subset \{1, 2, ..., K\}$ which is a common subcarrier set available to both transmitter and intended receiver. Let $\mathbf{G} := \left\{G_{i,j}^k, i, j \in \mathcal{N}, k \in \mathcal{L}_i\right\}$ denote the subcarrier fading coefficient matrix, where $G_{i,j}^k$ stands for the sub-channel coefficient gain from transmitter i to receiver j over subcarrier k. $G_{i,j}^k = |H_{i,j}^k(f)|^2$, where $|H_{i,j}^k(f)|$ is the transfer function. It is assumed that $\mathbf{G}$ adheres to a block fading channel model which remains invariant over blocks (coherence time slots) of size T_S and uncorrelated across successive blocks. The noise is assumed to be additive white Gaussian noise (AWGN) and to be independent of the symbols, with variance σ^2 for all receivers over the entire available subcarrier set.

We define p_i^k to be the power allocated over subcarrier k for transmitter i. The signal to interference plus noise ratio (*SINR*) of receiver i over subcarrier k, γ_i^k, can be expressed as

$$\begin{aligned}\gamma_i^k(p_i^k) &= \alpha_i^k(p_j^k) \cdot p_i^k \\ \alpha_i^k(p_j^k) &= \frac{G_{i,i}^k}{\sum_{j \neq i, j \in \mathcal{N}} G_{j,i}^k \cdot p_j^k + \sigma^2}\end{aligned} \tag{1}$$

where α_i^k is the channel state information (CSI). α_i^k can be measured at the receiver side and is assumed to be known by the corresponding transmitter through a reciprocal common control channel. We assume the coherent reception with perfect carrier-phase estimation and perfect fading-value estimation at the receiver and the transmitter, thus the delays involved in obtaining the channel estimate at receiver and of providing the information to the corresponding transmitter can be neglected.

For MQAM, the number of bits per symbol is denoted as $b = \log_2 M$ which is defined over positive integer values. A bound on the probability of bit error rate for MQAM is given by

$$P_b \leq \frac{4}{b}\left(1 - \frac{1}{\sqrt{2^b}}\right) Q\left(\sqrt{\frac{3}{2^b - 1} \cdot \frac{\varepsilon_{av}}{N_0}}\right), \tag{2}$$

where P_b is the bit error probability (BER), ε_{av} is the average energy per symbol, and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(\frac{-u^2}{2}) du$. Since $Q(x) \leq \frac{1}{2}\exp(\frac{-x^2}{2})$, by approximating the bound as equality we obtain

$$P_b = \frac{2}{b}\left(1 - \frac{1}{\sqrt{2^b}}\right) \exp\left(\frac{-3}{2\left(2^b - 1\right)} \cdot \gamma\right). \tag{3}$$

Thus, the transmission power to guarantee a required BER $\mathcal{P}_{i,b}$ on each selected subcarriers can be derived as

$$p_i^k = \frac{2\left(2^{b_i^k} - 1\right)}{3 \cdot \alpha_i^k} \ln\left(\frac{2\left(1 - 1/\sqrt{2^{b_i^k}}\right)}{b_i^k \cdot \mathcal{P}_{i,b}}\right). \tag{4}$$

In each time slot, we assume L bits need to be transmitted by transmitter i to achieve QoS requirements. On each subcarrier, the allocated bits is denoted as L_i^k, thus $\sum_{k \in \mathcal{L}_i} L_i^k = L$. The number of MQAM symbols required to send L_i^k bits on subcarrier k is denoted as $S_i^k = L_i^k / b_i^k$. If the symbol period is T_{ls}, then $S_i^k = T_{i,on}/T_{i,ls}$, $T_{i,on}$ is the transmission time per slot and it is identical for all subcarriers. Furthermore,

$$L_i^k / b_i^k = T_{i,on}/T_{i,ls} \tag{5}$$

If square pulses are used, $T_{i,ls}$ can be approximated by $\frac{1}{B_i^k}$, where B_i^k is the bandwidth of subcarrier k for transmitter i. From (5), we obtain

$$T_{i,on} = L_i^k / (B_i^k \cdot b_i^k) \tag{6}$$

In this paper, without loss of generality, we assume the subcarriers are equally divided, and $B_i^k = 1$, $\forall k$.

Remark: From (5) and (6), the essential idea behind the adaptive modulation algorithm with equally divided bandwidth in OFDMA based CR networks is to allocate the same amount of symbols on each selected subcarrier while with different constellation size according to the varying subcarrier conditions.

Hence, the transmission energy consumption for transmitter i over subcarrier k can be expressed as

$$\begin{aligned} e_i^k &= p_i^k \cdot T_{i,on} \\ &= T_{i,on} \cdot \left(\frac{2\left(2^{b_i^k}-1\right)}{3\alpha_i^k} \ln\left(\frac{2\left(1-1/\sqrt{2^{b_i^k}}\right)}{b_i^k \cdot \mathcal{P}_{i,b}} \right) \right) \\ &= \left(\frac{2\left(2^{b_i^k}-1\right)}{3\alpha_i^k} \ln\left(\frac{2\left(1-1/\sqrt{2^{b_i^k}}\right)}{b_i^k \cdot \mathcal{P}_{i,b}} \right) \right) \cdot \frac{L_i^k}{b_i^k} \end{aligned} \quad (7)$$

In an energy constrained network (e.g. a wireless sensor network), reception power is not negligible since it is generally comparable to the transmission power [13]. In this work, we denote the receiving power as p_i^r which is treated as a constant value for all receivers [12]. Thus, for transmitter and receiver pair i, the total energy consumption for the delivery of L bits is given by

$$\begin{aligned} \boldsymbol{e}_i &= \sum_{k \in \mathcal{L}_i} e_i^k + p_i^r \cdot T_{i,on} \\ &= \left(\sum_{k \in \mathcal{L}_i} \frac{2\left(2^{b_i^k}-1\right)}{3\alpha_i^k} \cdot \eta(b_i^k) + p_i^r \right) \cdot T_{i,on} \\ \eta(b_i^k) &= \ln\left(\frac{2\left(1-1/\sqrt{2^{b_i^k}}\right)}{b_i^k \cdot \mathcal{P}_{i,b}} \right) \end{aligned} \quad (8)$$

Based on (6) and $B_i^k = 1,\ \forall k$, $T_{i,on}$ can be further expressed as

$$T_{i,on} = \frac{\sum_{k \in \mathcal{L}_i} L_i^k}{\sum_{k \in \mathcal{L}_i} (B_i^k \cdot b_i^k)} = \frac{L}{\sum_{k \in \mathcal{L}_i} b_i^k} \quad (9)$$

Taking (9) into (8), the energy consumption for the delivery of L bits can be re-organized as

$$\boldsymbol{e}_i = \frac{L}{\sum_{k \in \mathcal{L}_i} b_i^k} \cdot \left(\sum_{k \in \mathcal{L}_i} \frac{2\left(2^{b_i^k}-1\right)}{3\alpha_i^k} \eta(b_i^k) + p_i^r \right) \quad (10)$$

From the expression of $\boldsymbol{e}_i$, it is observed that the transmission energy consumption is increasing with respect to b_i^k, on the contrary, the reception energy is decreasing w.r.t. b_i^k. Therefore, energy efficient bit loading algorithm needs to locate an optimal trade-off between transmission and reception energy. In each time slot, transmission is required to be bounded by the delay constraints, i.e., $T_{i,on} \le T_S$, which leads to

$$\sum_{k \in \mathcal{L}_i} b_i^k \ge \left\lceil \frac{L}{T_S} \right\rceil \quad (11)$$

Define $\lceil L/T_S \rceil := b_{min}$. Given the above system assumptions, we end up with the following constrained optimization problem for transmitter receiver pair i:

$$\begin{aligned} &\min_{b_i^k \in \boldsymbol{Z}^+} \ \boldsymbol{e}_i \\ &s.t. \sum_{k \in \mathcal{L}_i} b_i^k \ge b_{min}, \\ &P_b \le \mathcal{P}_{i,b}, \forall i \in \mathcal{N} \end{aligned} \quad (12)$$

III. Energy Efficient Adaptive Modulation Scheme

Since the design variable b_i^k is defined over positive integers, (12) is a constrained integer fractional programming problem [9]. Due to the non-convex nature of the problem, high computational complexity is expected by exhaustive search of the optimal solution, especially with the increase of the number of available subcarriers. In this paper, we propose an efficient one-dimensional sub-gradient search algorithm by relaxing b_i^k to positive real numbers $\boldsymbol{R}^+$ which will provide the (best) performance bound of energy efficiency.

For uncoded MQAM, if we approximate $\eta(b_i^k)$ by

$$\ln\left(\frac{2\left(1-1/\sqrt{2^{b_i^k}}\right)}{b_i^k \cdot P_b} \right) \approx \ln\left(\frac{3}{2 \cdot b_i^k \cdot P_b} \right) \quad (13)$$

We can simplify the representation of total energy consumption (10) as

$$\boldsymbol{e}_i = \frac{L \cdot \left(\sum_{k \in \mathcal{L}_i} \frac{2\left(2^{b_i^k}-1\right)}{3\alpha_i^k} \ln\left(\frac{3}{2 \cdot b_i^k \cdot \mathcal{P}_{i,b}} \right) + p_i^r \right)}{\sum_{k \in \mathcal{L}_i} b_i^k} \quad (14)$$

The relative looseness caused by the bound (13) is less than 7% (see Fig.1) when b is within the range $[2, 10]$ (which is a reasonable region for practical MQAM in wireless networks). After we find the optimal solution $\hat{b}_i^{k*}$ for the relaxed optimization problem, the optimal integer result b_i^{k*} can be located by evaluating the energy efficiency at the two neighboring integer points. Note that generally for integer programming problems, the optimal solution may not be one of the neighboring integer points. However, in the studied cases, the simulations demonstrate the effectiveness of the proposed relaxation algorithm for finding the true optimal solution, and the obtained optimal solution in real domain can be treated as the performance bound.

A. Unconstrained Energy Efficient Allocation Algorithm

In this paper, we decouple the constrained fractional programming problem into an unconstrained one and a branch-and-bound algorithm is applied thereafter to gain insights of the optimal adaptive modulation order. Based on the

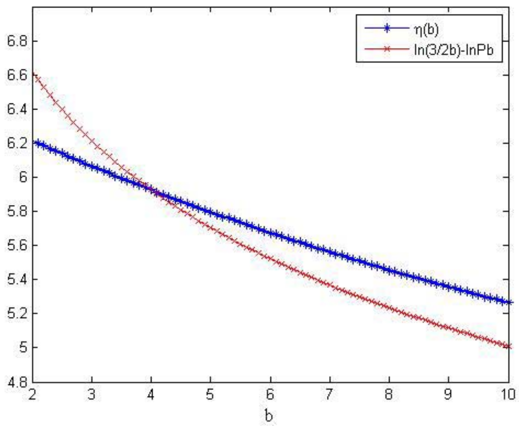

Fig. 1. Looseness of bound approximation (equation (13))

Dinkelback-type algorithm [9], we define

$$\lambda_i(\hat{\boldsymbol{b}}_i){:=}\frac{f_i(\hat{\boldsymbol{b}}_i)}{g_i(\hat{\boldsymbol{b}}_i)}, \quad \overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*) := \min_{b_i^k \in \boldsymbol{R}^+} \boldsymbol{e}_i$$
$$f_i(\hat{\boldsymbol{b}}_i){:=}L \cdot \left(\sum_{k \in \mathcal{L}_i} \frac{2\left(2^{\hat{b}_i^k}-1\right)}{3\alpha_i^k} \ln\left(\frac{3}{2 \cdot \hat{b}_i^k \cdot \mathcal{P}_{i,b}}\right) + p_i^r \right)$$
$$g_i(\hat{\boldsymbol{b}}_i){:=}\sum_{k \in \mathcal{L}_i} \hat{b}_i^k \tag{15}$$

where ˆ is used to represent the variables in the unconstrained optimization domain, $\hat{\boldsymbol{b}}_i^* = [\hat{b}_i^{1*}, \hat{b}_i^{2*}, \ldots, \hat{b}_i^{k*}]$ denotes the unconstrained optimal modulation orders over available subcarrier set for transmitter receiver pair i. In the practical region of MQAM constellation size, it can be seen that $f_i(\hat{\boldsymbol{b}}_i), g_i(\hat{\boldsymbol{b}}_i)$ are positive, continuous, and convex (the proof is given in **Appendix A**). Given the channel matrix $\mathbf{G}$ and noise variance σ^2, $\hat{\boldsymbol{b}}_i^*$ is defined as the *optimal point* by satisfying

$$\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*) \leq \lambda_i(\hat{\boldsymbol{b}}_i), \quad \forall \hat{\boldsymbol{b}}_i \subset \boldsymbol{R}^{J+} \tag{16}$$

where J is the cardinality of the available subcarrier set. To analyze the properties of the fractional programming optimization, we introduce the function $\rho_i(\lambda_i, \hat{\boldsymbol{b}}_i) : \boldsymbol{R}^+ \times \boldsymbol{R}^{J+} \rightarrow \boldsymbol{R}^+$ which is given by

$$\rho_i(\lambda_i, \hat{\boldsymbol{b}}_i) := f_i(\hat{\boldsymbol{b}}_i) - \lambda_i(\hat{\boldsymbol{b}}_i) \cdot g_i(\hat{\boldsymbol{b}}_i) \tag{17}$$

Therefore, the optimal solution of the unconstrained energy efficiency optimization problem can by obtained by differentiating $\rho_i(\lambda_i, \hat{\boldsymbol{b}}_i)$ with respect to $\hat{b}_i^k$

$$\frac{\partial \rho_i(\lambda_i, \hat{\boldsymbol{b}}_i)}{\partial \hat{b}_i^k} = \frac{\partial f_i(\hat{\boldsymbol{b}}_i)}{\partial \hat{b}_i^k} - \lambda_i(\hat{\boldsymbol{b}}_i) \tag{18}$$

By setting $\partial \rho_i(\lambda_i, \hat{\boldsymbol{b}}_i)/\partial \hat{b}_i^k = 0$, we obtain

$$\frac{2}{3\alpha_i^k}\left(2^{\hat{b}_i^k} \ln 2 \ln\left(\frac{1.5}{\mathcal{P}_{i,b} \cdot \hat{b}_i^k}\right) - \frac{\left(2^{\hat{b}_i^k}-1\right)}{\hat{b}_i^k}\right) = \lambda_i(\hat{\boldsymbol{b}}_i) \tag{19}$$

Because computing the entire optimal constellation size $\hat{\boldsymbol{b}}_i^*$ is too expensive, a more efficient approach is proposed here to iteratively search for $\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*)$. From (19), it can be observed that original J dimensional fractional programming problem can be reduced to one dimensional optimization problem if $\lambda_i(\hat{\boldsymbol{b}}_i)$ is considered to be a constant value for each subcarrier k. Thus, we propose a one dimensional efficient sub-gradient search algorithm to locate the optimal $\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*)$ which will determine the optimal modulation order vector $\hat{\boldsymbol{b}}_i^*$ in reverse. Specifically, we propose the following sub-gradient projection iteration (indexed by n) for locating $\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*)$

$$\lambda_i(n+1) = \lambda_i(n) + \beta(n) \cdot \left(\frac{f_i(\hat{\boldsymbol{b}}_i)(n)}{g_i(\hat{\boldsymbol{b}}_i)(n)} - \lambda_i(n)\right) \tag{20}$$

where $\beta(n)$ is a positive step size. Each λ_i encountered during the search process must be evaluated to determine the constellation size associated with it which can be used to update $f_i(\hat{\boldsymbol{b}}_i)(n)$ and $g_i(\hat{\boldsymbol{b}}_i)(n)$, respectively. From (20), the tentative optimal modulation order $\tilde{b}_i^k(n)$ for the n^{th} iteration can be derived over each subcarrier $k \in \mathcal{L}_i$. And the temporary energy consumption which we denote as $\tilde{\lambda}_i(n)$ is updated with the corresponding $\tilde{b}_i^k$. The ϵ-optimal constellation size b_i^{k*} has been reached if the following condition holds for the subsequent $\{\lambda_i(n)\}$.

$$\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*) = \tilde{\lambda}(n) = \frac{f_i(\hat{\boldsymbol{b}}_i)(n)}{g_i(\hat{\boldsymbol{b}}_i)(n)} - \lambda_i(n) \leq \epsilon \tag{21}$$

where ϵ is an arbitrary small positive number. Simulations demonstrate that the presented sub-gradient search algorithm provides fast convergence and easy verification when $\overline{\lambda}_i$ has been reached. The entire algorithm is summarized as follows.

Unconstrained Bit Loading Algorithm

1) *Initialization*
   - Initialize the unconstrained energy consumption λ_i=0, the temporary energy consumption $\tilde{\lambda}_i$=0, and the tentative constellation size vector $\tilde{\boldsymbol{b}}_i$=ϕ.
2) *Sub-Gradient search algorithm of* $\overline{\lambda}_i^*$
   - Select $\lambda_i(0) \in \mathbf{R}^+$ and $n := 0$
   - Take $\lambda_i(n)$ into (19) for tentative modulation order allocation $\tilde{\boldsymbol{b}}_i(n)$.
   - Deriving the temporary energy consumption $\tilde{\lambda}_i(n)$ by the corresponding $\tilde{\boldsymbol{b}}_i(n)$.
   - Determine if $\lambda_i(n)$ and $\tilde{\lambda}_i(n)$ satisfy (21) for convergence
   - If yes, stop and the unconstrained optimal constellation size over entire available subcarrier set is $\hat{\boldsymbol{b}}_i^* = \tilde{\boldsymbol{b}}_i(n)$, and the optimal energy consumption is $\overline{\lambda}_i(\hat{\boldsymbol{b}}_i^*) = \tilde{\lambda}_i(n)$.
   - If no, update $\lambda_i(n)$ based on (20), and go to the beginning of step 2).

Therefore, the optimal unconstrained constellation sizes set $\hat{\boldsymbol{b}}_i^*$ can be expressed as

$$\hat{\boldsymbol{b}}_i^* = \prod_{k \in \mathcal{L}_i} \max\{\hat{b}_i^{k*}, 0\}, \quad \forall k \in \mathcal{L}_i, i \in \mathcal{N} \tag{22}$$

B. Constrained Energy Efficient Allocation Algorithm

The last section provides an efficient sub-gradient search algorithm for the unconstrained optimal constellation size $\hat{\boldsymbol{b}_i^*} = [\hat{b}_i^{1*}, \hat{b}_i^{2*}, \ldots, \hat{b}_i^{k*}], \forall k \in \mathcal{L}_i$. Given such unconstrained optimal solution, the ultimate solution space can be partitioned into two sub-space based on the delay constraints.

Case 1). If $\sum_{k\in\mathcal{L}_i} \hat{b}_i^{k*} \geq b_{min}$

If the unconstrained solution $\hat{\boldsymbol{b}_i^*}$ satisfies the delay constraints, it is regarded as the constrained optimal solution by evaluating the energy efficiency at two neighboring integer points when both of them meet the delay constraints. Otherwise, the constellation size in real domain will be rounded up to the least greater integer[1] to ensure the non-violation of the delay constraints.

$$\boldsymbol{b}_i^* = \begin{cases} \prod_{k\in\mathcal{L}_i} \arg\min_{\hat{b}_i^{k*}} \{\boldsymbol{e}_i(\lfloor \hat{b}_i^{k*} \rfloor), \boldsymbol{e}_i(\lceil \hat{b}_i^{k*} \rceil)\}, & \sum_{k\in\mathcal{L}_i} \lfloor \hat{b}_i^{k*} \rfloor \geq b_{min} \\ \prod_{k\in\mathcal{L}_i} \lceil \hat{b}_i^{k*} \rceil, & \sum_{k\in\mathcal{L}_i} \lfloor \hat{b}_i^{k*} \rfloor < b_{min} \end{cases}$$

Case 2). If $\sum_{k\in\mathcal{L}_i} \hat{b}_i^{k*} \leq b_{min}$

When the unconstrained optimal solution $\hat{\boldsymbol{b}_i^*}$ cannot satisfy the delay constraints, additional bits need to be allocated on the subcarrier set. Constrained optimization techniques, such as the interior-point method [10], can be applied but with considerable computational complexity. In order to achieve tractable complexity and implementation simplicity, we present a constant bit allocation scheme based on the unconstrained optimal solution which will allocate the extra required bits equally over the entire selected subcarrier set.

Due to the optimality of the unconstrained solution, minimal increased bit allocation which is denoted as $\triangle b_i$ will result in the best energy efficiency since it has minimal deviation from the optimal point. Thus $\triangle b_i$ is given by

$$\triangle b_i = b_{min} - \sum_{k\in\mathcal{L}_i} \hat{b}_i^k$$
$$\triangle b_i^k = \frac{\triangle b_i}{\Gamma(\hat{\boldsymbol{b}}_i^*)}, \quad \forall k \in \mathcal{L}_i \tag{23}$$

where $\triangle b_i^k$ is the equally allocated bits on each subcarrier, and $\Gamma(X)$ is defined as the cardinality of the nonzero elements in vector X. Eventually, the constrained modulation order allocated on selected subcarrier can be expressed as

$$\hat{\boldsymbol{b}}_i^* = \prod_{k\in\mathcal{L}_i} b_i^{k*} = \prod_{k\in\mathcal{L}_i} \lceil \hat{b}_i^{k*} + \triangle b_i^k \rceil, \quad \forall k \in \mathcal{L}_i \tag{24}$$

IV. Distributed Power Control Algorithm

In section III, an adaptive modulation algorithm is proposed for each individual user pair. Whereas, in multi-user ad hoc networks, simultaneous spectrum access among multiple transmitter receiver pairs in the same time slot will cause co-channel interference and result in degradation of network performance. In order to guarantee the QoS requirements, such as the delay constraints and BER, a distributed power control algorithm is presented in this section to manage the potential co-channel interference. Each emerging *new* user initially obtain their constellation size individually with the algorithm proposed in section III, and then iterates the distributed power control algorithm to mitigate the interference from peers.

[1] A better solution may be obtained by searching all the combinations of $\boldsymbol{b}_i^*$ as long as $\sum_{k\in\mathcal{L}_i} b_i^{k*} \geq b_{min}$. However, the incurred complexity is high.

TABLE I
Convergence of the Proposed Sub-Gradient Search Algorithm

Iteration No.	$\tilde{\boldsymbol{b}}_i(n)$	$\tilde{\lambda}_i(n)$	$\tilde{\lambda}_i(n)$
1	[1.4, 1.5]	0.01	0.0345
2	[3.4, 3.5]	0.0345	0.0263
3	[2.9, 3.1]	0.0263	0.0255
4	[2.9, 3.1]	0.0255	0.0255

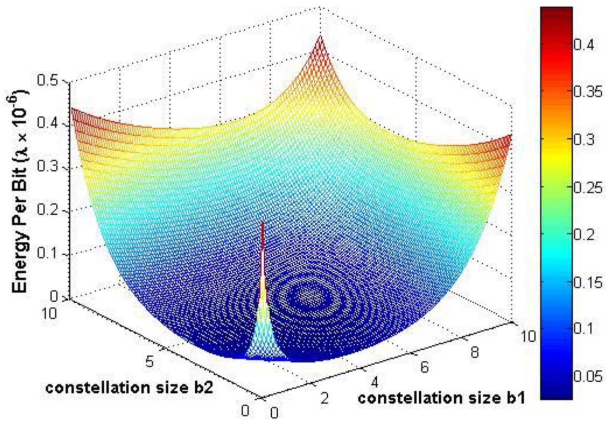

Fig. 2. Optimal solution for energy per bit in two subcarrier case

The power control algorithm is given by [14]

$$p_i^k(t+1) = \min\left\{\frac{\gamma_i^{k*}}{\gamma_i^k(t)} p_i^k(t), p_i^{max}\right\} \tag{25}$$

where γ_i^{k*} is the individual target *SINR* of transmitter i over each subcarrier k which can be determined by (3).

During the power control stage, if the target *SINR* γ_i^{k*} can not be maintained when transmitter i hits its power bound p_i^{max}, the network is considered to be unable to accommodate all the current *new* users. A multi-access control (MAC) scheme is then required to guarantee the fairness among the users.

V. Simulation Result

In this section, we consider a wireless sensor network with cognitive radio capability. The channel gains are assumed to be sampled from a Rayleigh distribution with mean equals to $0.4d^{-3}$, where d is the distance from the transmitter to the receiver. The entire spectrum is equally divided into subcarriers with bandwidth 100 KHz for each subcarrier. The duration of each time slot T_S is assumed to be $10ms$ in which L bits need to be transmitted. The thermal noise power is assumed to be the same over all subcarriers and equal to 10^{-8}W.

TABLE III
COMPARISON BETWEEN EXHAUSTIVE SEARCH RESULTS AND PROPOSED SUB-GRADIENT RESULTS

No. of Sub.	$\hat{\boldsymbol{b}}^*_{i,r}$	$\boldsymbol{b}^*_{i,z}$	$\boldsymbol{b}_{i,opt}$
1	[3.56]	[4]	[4]
2	[3.11, 2.9]	[3, 3]	[3, 3]
3	[2.75, 2.61, 2.52]	[3, 3, 2]	[3, 3, 2]
4	[2.5, 2.43, 2.32, 2.1]	[3, 2, 2, 2]	[3, 2, 2, 2]
5	[2.31, 2.27, 2.2, 2.1, 2]	[2, 2, 2, 2, 2]	[2, 2, 2, 2, 2]

We first investigate the fast convergence and computation efficiency of the proposed algorithm. Without loss of generality, we assume the available subcarrier set after spectrum sensing includes two subcarriers experiencing Ralyeigh fading. By adopting the step size $\beta(n) = 1$ of (20), the simulation (Table I) shows the convergence occurs in 4 steps. And through Fig.2, the minimal energy efficiency is achieved at the optimal constellation size vector in two-subcarrier case. Even though the subcarrier set is small in this simulation, comparable convergence speed is expected with the expansion of the available subcarier set as shown in (Table II) for a 10-subcarrier case.

In Fig. 3 and Table III, we compared the performance between the optimal solution obtained by exhaustive search and the solution obtained by the proposed sub-gradient search algorithm. The system with the available subcarrier set ranging from 1 to 5 subcarriers is shown in this illustration. We assume $b_{min} = 4$. The dashed line marked with circle represents the optimal energy efficiency achieved when b is relaxed to real domain and it acts as the performance bound for the achievable energy efficiency. The solid line marked with square is the achieved energy efficiency when b is in the integer domain. It can be observed that with the increase of the number of subcarriers, the energy efficiency is improved from 3.84×10^{-7} to 1.88×10^{-7} at the expense of available bandwidth. In Table III, $\boldsymbol{b}_{i,opt}$ is the optimal solution obtained from exhaustive search that serves as a benchmark. It is observed that the optimal solution derived by relaxation ($\boldsymbol{b}^*_{i,z}$) is the same as that in exhaustive search ($\boldsymbol{b}_{i,opt}$), which demonstrates the effectiveness of the proposed algorithm. Furthermore, the performance gap between the performance bound obtained by $\hat{\boldsymbol{b}}^*_{i,r}$ and the achieved energy efficiency by using $\boldsymbol{b}^*_i$ is very small as shown in Fig. 3.

In Table IV, the constrained case (Section III-B) is demonstrated with the same system setting as the unconstrained case (Table III) except for $b_{min} = 10$. After obtaining the unconstrained optimal solution following the proposed sub-gradient search algorithm, the unfulfilled bits is equally loaded on the available subcarrier set which provides a suboptimal solution for the bit loading with tractable complexity. It can be observed that even the sub-optimal constant bit loading strategy offers close performance to the optimal solution achieved through exhaustive search. Another thing to be noted is that with the increase of available subcarriers, the constrained problem evolves into an unconstrained one with the increase of available bandwidth. In Table IV, the fist four cases are

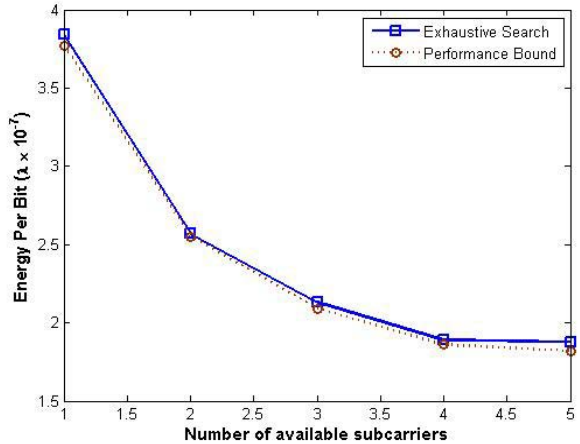

Fig. 3. Performance bound vs. achieved energy efficiency for $\boldsymbol{b}^*_i$

TABLE IV
COMPARISON BETWEEN CONSTRAINED SUB-GRADIENT RESULTS AND EXHAUSTIVE SEARCH RESULTS

No. of Sub.	$\boldsymbol{b}^*_{i,r}$	$\boldsymbol{b}^*_{i,z}$	$\boldsymbol{b}_{i,opt}$
1	[10]	[10]	[10]
2	[4.98, 5.02]	[5, 6]	[5, 5]
3	[3.3733, 3.3533, 3.2733]	[4, 4, 4]	[4, 3, 3]
4	[2.595, 2.575, 2.505, 2.325]	[3, 3, 3, 3]	[3, 3, 2, 2]
5	[2.27, 2.23, 2.22, 2.16, 1.99]	[2, 2, 2, 2, 2]	[2, 2, 2, 2, 2]

corresponding to the constrained problem *Case 2)*, and the last case is in *Case 1)*.

In multiple user scenario, in order to investigate the distributed power control for managing the co-channel interference, we consider two *emerging* new users who share two common subcarriers as shown in Fig.4. After each user conducting the adaptive modulation algorithm independently, distributed power control is performed to guarantee the BER ($\mathcal{P}_{i,b}$) requirment by increasing the transmission power. It can be observed that the system converges in 3-4 steps.

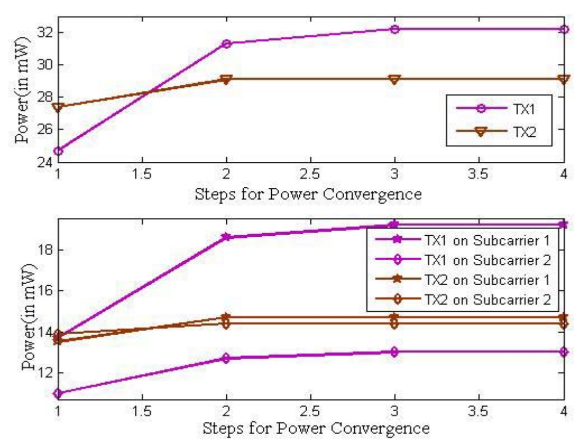

Fig. 4. Convergence of distributed power control

TABLE II
CONVERGENCE OF THE PROPOSED SUB-GRADIENT SEARCH ALGORITHM

Iteration No.	$\tilde{\boldsymbol{b}}_i(n)$	$\tilde{\lambda}_i(n)$	$\tilde{\lambda}_i(n)$
1	[2.25, 2.28 2.3, 2.32, 2.48, 2.481, 2.52, 2.53, 2.58, 2.69]	0.005	0.0125
2	[1.87, 1.91 1.92, 1.93, 2.08, 2.09, 2.14, 2.15, 2.18, 2.3]	0.0125	0.0098
3	[1.82, 1.86 1.87, 1.88, 2.04, 2.05, 2.11, 2.12, 2.14, 2.26]	0.0098	0.0096
4	[1.82, 1.86 1.87, 1.88, 2.04, 2.05, 2.11, 2.12, 2.14, 2.26]	0.0096	0.0096

VI. CONCLUSION

In this paper, a fully distributed energy efficient adaptive modulation algorithm is proposed for a wireless OFDMA cognitive radio ad hoc network. A Dinkelback-type algorithm is proposed to solve the unconstrained optimization problem such that energy consumption for data delivery (in terms of energy per bit) is minimized over the available subcarrier set. A sub-gradient search algorithm is proposed to locate the optimal constellation size for each individual user. Given the delay requirement, a sub-optimal constant bit allocation strategy is presented to resolve the constrained optimization problem. In addition, a distributed power control is performed to manage the co-channel interference among the new users when needed.

Although the proposed algorithm provides a sub-optimal solution to the constrained optimization problem, it has low computational complexity and implementation simplicity. Furthermore, for the unconstrained cases, the proposed sub-gradient algorithm turns a multi-dimensional optimization problem into one-dimensional search problem with fast convergence speed. When the network is unable to accommodate the traffic load of all *new* users in the current time slot, a scheduling algorithm with fairness consideration is desired, which will be one of our future efforts. In addition, large scale simulation experiments will be carried out as well.

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VII. APPENDIX A

The convexity of the function (over b).

From (15), it is apparent $g_i(\hat{\boldsymbol{b}}_i)$ can be regarded as convex function. For $f_i(\hat{\boldsymbol{b}}_i)$, which is expressed as

$$f_i(\hat{\boldsymbol{b}}_i) := L \cdot \left(\sum_{k \in \mathcal{L}_i} \frac{2\left(2^{\hat{b}_i^k} - 1\right)}{3\alpha_i^k} \ln\left(\frac{3}{2 \cdot \hat{b}_i^k \cdot \mathcal{P}_{i,b}}\right) + p_i^r \right)$$

For each subcarrier k, we define $f_i^k(b_i^k)$ as

$$f_i^k(b_i^k) := L \cdot \left(\frac{2\left(2^{\hat{b}_i^k} - 1\right)}{3\alpha_i^k} \ln\left(\frac{3}{2 \cdot \hat{b}_i^k \cdot \mathcal{P}_{i,b}}\right) + p_i^r \right)$$

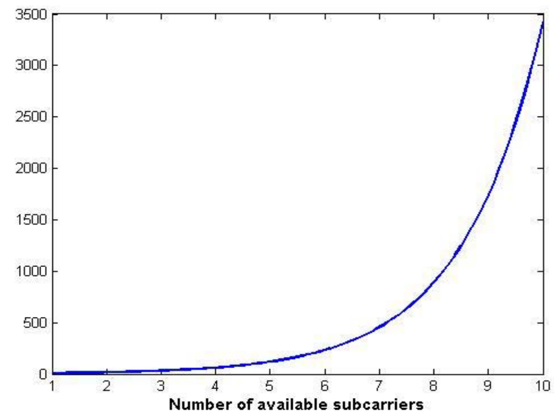

Fig. 5. convexity of function

The second derivative of $f_i^k(b_i^k)$ over practical region $[2, 10]$ is given as

$$\frac{3\alpha_i^k}{2 \cdot L} \cdot \frac{\partial^2 f_i^k(b_i^k)}{\left(\partial b_i^k\right)^2} = 2^{b_i^k} \cdot \ln 2 \cdot \ln\left(\frac{3}{2 \cdot \hat{b}_i^k \cdot \mathcal{P}_{i,b}}\right) - \frac{2^{b_i^k + 1} \cdot \ln 2}{b_i^k} + \frac{2^{b_i^k} - 1}{\left(b_i^{k2}\right)^2} \tag{26}$$

It is shown in Fig.5, that $\frac{\partial^2 f_i^k(b_i^k)}{\left(\partial b_i^k\right)^2} > 0$ for $[2, 10]$. Therefore, $f_i^k(b_i^k)$ is convex function which will result in the convexity of function $f_i(\hat{\boldsymbol{b}}_i)$.

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