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The Survivability of Symmetrical Hierarchical Networks with Radial Reserve

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Abstract

In this paper we shall consider the *Symmetrical Hierarchical Network* (SHN) and show that SHN possesses poor properties of survivability. There are several methods for raising the survivability of SHN. Here we consider the effectiveness of radial reserve to raise the survivability of SHN taking account of destruction of the main radial edges, and radial reserve.

Keywords: Network, Symmetrical, Hierarchical, Survivability, Reserve, Capacity, Radial edge

AMS 2000 Mathematical Subject Classifications: 90B18; 90B15

1. Introduction

An arbitrary Multicommodity Flow Network (MFN) is determined by two graphs, physical G and logical P on the same set V of nodes (cf. Phillips,D. and Garcia-Dias (1981)). Edges r_k of graph G mean physical lines of communications between the nodes from V, and they are ascribed with non-negative numbers c_k called capacity of edges $r_k, k = 1, \cdots, n$. Edges $p_i, i = 1, \cdots, m$ of graph P correspond to logical connection between certain pairs of nodes (source-sink pairs). This means that there are demands of flow transmission from one node to the other p_i through the edges of network graph G. Thus, each p_i is specified by source-sink pair $v_i^s, v_i^t \in V$ and positive demand d_i.

An MFN is called *Hierarchical Network (HN)* if its logical graph has the structure of a star (i.e., the source - sink pairs are given in the form $(v_0, v_i), i \in M \stackrel{def}{=} \{1, 2, \cdots, m\}$, with the common source

v_0). The physical graph of an HN usually repeats its logical structure and also is a star network: $G = \langle V, E \rangle$, where $V = \{v_0, v_1, ..., v_m\}$, $E = \{e_1, e_2, ..., e_m\}$ and $e_i = (v_0, v_i)$. A formal hierarchical network is considered by M.B. Ahmadi, Yu.E. Malashenko and N.M. Novikova (2001). An HN is called symmetrical if all demands are equal, i.e., $d_i = d, i = 1, ..., m.$

2. Survivability in network systems

Network survivability is critical component of the global telecommunication infrastructure (cf. T.H. Grubesic and A.T. Murray (2005)). Survivability is a key concept, but its definition can very depending on the context of applications. In many network design models, network survivability is defined as the ability of a network to maintain or restore an acceptable level of performance in the event of deterministic or random failures (cf. Shi, J. et al. (1995)).

Network planners have define the term survivability as the percentage of the total traffic surviving the failure of edges or nodes. Kolar and Wu (1988) have used this definition to evaluate the survivability of different network architectures. In particular, they have proposed several network architectures for fiber optic networks. Then, they compared the investment cost and edge survivability for each proposed architecture.

Here we define network survivability as the *guaranteed level of demand satisfaction* depending on capacity and demand vectors.

3. Evaluation of survivability of SHN

Let z_j be the amount flow between nodes v_0 and v_j. Denote by $Z(c)$ the set of all multiflows $z = (z_1, \cdots, z_m)$ in the network with capacity vector $c = (c_1, ..., c_m)$, i.e.,

$$Z(c) = \{z \mid 0 \le z_i \le c_i \quad \forall i \in M\}.$$

For every flow distribution, we define the value

$$\min_{i \in M} \frac{z_i}{d_i},$$

which is called the *demand satisfaction level* (d.s.l) given the distribution z (cf. Malashenko Yu.E. , Novikova N.M (1999)). An MFN performance efficiency measure is defined as *the maximum d.s.l.* attainable in the network

$$\theta_0 = \theta_0(c) = \max_{z \in Z(c)} \min_{i \in M} \frac{z_i}{d_i}. \quad (1)$$

For HN with the capacity $c = (c_1, c_2, ..., c_m)$, we have $\theta_0(c) = \min_i \frac{c_i}{d_i}$.

Let z^0 be the optimal solution of (1). In that case z^0 is called competitive distribution of flows. In the case $m=1$, (1) is a well known problem of the flow maximization. For $m>1$ it formalizes the concurrent flow problem. A multiflow achieves demand satisfaction if it ships an amount of each commodity equal to its demand from its source to its sink while obeying the capacity constraint. This corresponds to $\theta_0 \geq 1$. Otherwise, an arbitrary concurrent flow distribution z^0 [a maximizer in (1)] may not be the best for certain network users.

Let $\gamma \in (0,1)$ be a parameter which characterizes the strength of a failure: it indicates what part of the total capacity of edges may be lost.

Let $c=(c_1,...,c_m)$ be the initial capacity vector of HN. Denote by $Y_\gamma(c)$, the set of possible values of edge capacities after the fault

$$Y_\gamma(c)=\{y\in R_+^m \mid \sum_{i=1}^m y_i = (1-\gamma)\sum_{i=1}^m c_i,\, y_i \leq c_i,\, i=1,...,m\}.$$

The function $\theta_\gamma^g(c)$ denotes the *guaranteed level of demand satisfaction* depending on capacity vector c and demand d and it is defined as follows

$$\theta_\gamma^g(c)=\min_{y\in Y_\gamma(c)} \theta_0(y)=\min_{y\in Y_\gamma(c)} \max_{z\in Z(y)} \min_{i\in M} \frac{z_i}{d_i}$$

The survivability of HN is defined by $\theta_\gamma^g(c)$ (cf. Ahmadi (2007)).

Lemma 1: Let $c=(c_1,...,c_m)$ be the capacity vectors of SHN. Then

$$\theta_\gamma^g(c)=\frac{[\min_i c_i - \gamma\sum_{i=1}^m c_i]^+}{d}$$

Proof: Let $\min_i c_i = c_k$. First we show that $\theta_\gamma^g(c)=0$, for $\gamma=\frac{c_k}{\sum_{i\in M} c_i}$.

Let $\bar{y}=(c_1,\ldots,c_{k-1},0,c_{k+1},c_{k+1},...,c_m)$. We have

$$\sum_{i=1}^m \bar{y}_i = \sum_{\substack{i\in M\\ i\neq k}} c_i = (1-\frac{c_k}{\sum_{i\in M} c_i})\sum_{i\in M} c_i = (1-\gamma)\sum_{i\in M} c_i\ .$$

Thus, $\bar{y}\in Y_\gamma(c)$. In this case there is no path from v_0 to v_k. Hence, $z_k=0$, and $\theta_\gamma^g(c)=0$. It is clear that $\theta_\gamma^g(c)=0$ for $\gamma > \frac{c_k}{\sum_{i\in M} c_i}$.

Now suppose that $c_k - \gamma \sum_{i \in M} c_i > 0$. Let $\bar{y} = (c_1, \ldots, c_{k-1}, c_k - \gamma \sum_{i \in M} c_i, c_{k+1}, c_{k+1}, \ldots, c_m)$. We have

$$\sum_{i=1}^{m} \bar{y}_i = \sum_{i \in M} c_i - \gamma \sum_{i \in M} c_i = (1-\gamma) \sum_{i \in M} c_i \text{ . Thus } \bar{y} \in Y_\gamma(c).$$

It is clear that $\theta_0(\bar{y}) = \max_{z \in Z(\bar{y})} \min_{i \in M} \frac{z_i}{d} = \frac{c_k - \gamma \sum_{i \in M} c_i}{d}$, and consequently $\theta_\gamma^g(c) \le \frac{c_k - \gamma \sum_{i \in M} c_i}{d}$.

Now, suppose that $\exists \hat{y} \in Y_\gamma(c) : \theta_0(\hat{y}) < \frac{c_k - \gamma \sum_{i \in M} c_i}{d}$.Thus $\exists l \in M : \hat{y}_l < c_k - \gamma \sum_{i \in M} c_i$, and then

$$\sum_{i=1}^{m} \hat{y}_i < \sum_{i \in M} c_i - \gamma \sum_{i \in M} c_i = (1-\gamma) \sum_{i \in M} c_i \Rightarrow \hat{y} \notin Y_\gamma(c).$$

Hence, we have $\theta_\gamma^g(c) = \theta_0(\bar{y}) = \frac{[\min_i c_i - \gamma \sum_{i=1}^{m} c_i]^+}{d}$.■

Denote by $\bar{\gamma}(c)$ the smallest value γ that is enough for loss of connectivity of HN with initial capacity vector c, i.e.,

$$\begin{aligned} \bar{\gamma}(c) \overset{def}{=} \bar{\gamma} \quad &= \min\{\gamma \in (0,1) \mid \theta_\gamma^g(c) = 0\} \\ &= \min\{\gamma \in (0,1) \mid \exists y \in Y_\gamma(c) : \theta_0(y) = 0\}. \end{aligned} \tag{2}$$

Corollary 1: For SHN with the capacity vector $c = (\overbrace{d, \ldots, d}^{m-times})$, we have $\bar{\gamma}(c) = \frac{1}{m}$.

Proof: According to Lemma 1, we have

$$\bar{\gamma}(c) = \min\left\{\gamma \mid [\min_i c_i - \gamma \sum_{i=1}^{m} c_i]^+ = 0\right\}$$

$$= \min\{\gamma \mid [d - \gamma md]^+ = 0\}$$

$$= \min\{\gamma \mid d \le \gamma md\}$$

$$= \min\left\{\gamma \mid \frac{1}{m} \le \gamma\right\} = \frac{1}{m}.$$■

Corollary 1 shows that SHN possesses poor properties of survivability. Extending the capacity of the radial edge or creating an additional radial reserve, makes it possible to raise the survivability of SHN.

Denote by $\overline{G}$ the SHN with additional radial reserve, i.e., $\overline{G} = < V, \overline{E} >, \overline{E} = E \cup E^0$, $E^0 = \{e_1^0 e_2^0, ..., e_m^0\}$. E^0 representing m new radial edges each with a capacity of t. Here t is a nonnegative number representing the capacity of radial reserve.

From now on, we do not distinguish between G and $\overline{G}$, assuming formally that for graph G, the capacity of radial reserve is equal to zero.

4. The Survivability of SHN with Radial Reserve

Let $c = (c_1, ..., c_m)$, $c_T = (\overbrace{t, ..., t}^{m-times})$ denote the initial capacity vector of radial edges and the radial reserve respectively, and also $\overline{c} = (c, c_T)$. From now on, the capacity vectors of radial edges and radial reserve have m dimensions.

To analyze survivability of SHN with radial reserve, we consider two cases:

- ***a.*** Failures happen only in radial edges and the radial reserve is failure-free.
- ***b.*** Failures happen both in radial edges and in radial reserve.

Denote by $Y_\gamma(\overline{c})$, the set of possible values of edge capacities after the fault. In the case ***a***

$$Y_\gamma(\overline{c}) = \{y \in R_+^{2m} \mid \sum_{i=1}^{m} y_i = (1-\gamma)\sum_{i=1}^{m} c_i, y_i \le c_i, y_{m+i} = t, i = 1, ..., m\},$$

and in the case ***b***

$$Y_\gamma(\overline{c}) = \{y \in R_+^{2m} \mid \sum_{i=1}^{2m} y_i = (1-\gamma)\sum_{i=1}^{m} c_i + mt, y_i \le c_i, y_{m+i} \le t, i = 1, ..., m\}.$$

Example 1. Let $c = (10,14,20,16)$ and $c_T = (12,12,12,12)$ be the capacity vectors of radial edges and radial reserve respectively, and $\gamma = 0.2$. Then in the case ***a***

$$Y(c, c_T) = \left\{ y = (y_1, y_2, y_3, y_4, 12,12,12,12) \mid \sum_{i=1}^{4} y_i = 0.8(60) = 48, 0 \le y_i \le c_i, i = 1,2,3,4 \right\}.$$

For instance $(0,12,20,16,12,12,12,12) \in Y(c, c_T)$. In the case ***b***

$$Y(c,c_T)=\left\{y=(y_1,y_2,...,y_8)\,|\sum_{i=1}^{8}y_i=0.8(60)+4(12)=96, 0\le y_i\le c_i, i=1,...,4, 0\le y_i\le 12, i=5,...,8\right\}$$

For example, the following vectors belong to $Y(c,c_T)$.

$$(0,14,20,16,10,12,12,12),\quad (10,2,20,16,12,12,12,12),\quad (10,14,20,16,12,0,12,12).$$

First, we consider the case ***a*.**

Lemma 2: Let $\overline{c}=(c,c_T)$ be the capacity vectors of SHN with radial reserve. Then

$$\theta_\gamma^g(\overline{c})=\frac{[\min_i c_i-\gamma\sum_{i=1}^{m}c_i]^+ +t}{d}.$$

Proof: Let $\min_i c_i=c_k$, $\overline{y}=(\overline{c}_R,c_T), \overline{c}_R=(c_1,...,c_{k-1},c_k-\gamma\sum_{i\in M}c_i,c_{k+1},...,c_m)$ and $c_k-\gamma\sum_{i\in M}c_i>0$. We have

$$\sum_{i=1}^{m}\overline{y}_i=\sum_{i\in M}c_i-\gamma\sum_{i\in M}c_i=(1-\gamma)\sum_{i\in M}c_i\ .$$

Thus, $\overline{y}\in Y_\gamma(\overline{c})$.

It is clear that

$$\theta_0(\overline{y})=\max_{z\in Z(\overline{y})}\min_{i\in M}\frac{z_i}{d}=\frac{(c_k-\gamma\sum_{i=1}^{m}c_i)+t}{d},$$

and consequently

$$\theta_\gamma^g(\overline{c})\le\frac{(c_k-\gamma\sum_{i=1}^{m}c_i)+t}{d}.$$

Now, suppose that

$$\exists\hat{y}\in Y_\gamma(\overline{c}):\theta_0(\hat{y})<\frac{(c_k-\gamma\sum_{i=1}^{m}c_i)+t}{d}.$$

Thus,

$$\exists l\in M:\hat{y}_l<c_k-\gamma\sum_{i\in M}c_i,$$

and, then

$$\sum_{i=1}^{m} \hat{y}_i < \sum_{i \in M} c_i - \gamma \sum_{i \in M} c_i = (1-\gamma)\sum_{i \in M} c_i \Rightarrow \hat{y} \notin Y_\gamma(\bar{c}).$$

Hence, in the case ***a*** , for all $\gamma < \frac{\min_i c_i}{\sum_{i \in M} c_i}$, we have $\theta_\gamma^g(\bar{c}) = \theta_0(\bar{y}) = \frac{(c_k - \gamma \sum_{i=1}^{m} c_i) + t}{d}$.

Now, to prove the Lemma 2 it remains to show that $\theta_\gamma^g(\bar{c}) = \frac{t}{d}$ for $\gamma \geq \frac{\min_i c_i}{\sum_{i \in M} c_i}$.

For $\gamma = \frac{\min_i c_i}{\sum_{i \in M} c_i}$, we introduce

$$\tilde{y} = (\tilde{c}_R, c_T), \tilde{c}_R = (\tilde{c}_1, \ldots, \tilde{c}_{k-1}, 0, \tilde{c}_{k+1}, \ldots, \tilde{c}) \text{ and } c_T = (t, \ldots, t),$$

where

$$\sum_{i \in M} \tilde{c}_i = (1-\gamma)\sum_{i \in M} c_i .$$

We have

$$\sum_{i=1}^{m} \tilde{y}_i = (1-\gamma)\sum_{i \in M} c_i \Rightarrow \tilde{y} \in Y_\gamma(\bar{c}).$$

It is obvious that $\theta_0(\tilde{y}) = \frac{t}{d}$. Thus $\theta_\gamma^g(\bar{c}) \leq \frac{t}{d}$ for $\gamma \geq \frac{\min_i c_i}{\sum_{i \in M} c_i}$.

On the other hand, in the case ***a***, we may choose $z_i \geq t$ for all $i \in M$. Then we have

$$\theta_0(y) \geq \frac{t}{d} \qquad \forall \gamma \in (0,1), \forall y \in Y_\gamma(\bar{c}).$$

Therefore, $\theta_\gamma^g(\bar{c}) = \frac{t}{d}$ for $\gamma \geq \frac{\min_i c_i}{\sum_{i \in M} c_i}$. ■

Now assume that the original SHN structure is optimal: the radial edge capacity vector is equal to demand vector, i.e., all the demanded flows are transmitted by network and there is no excess capacity. In this case $c = (d, \ldots, d)$.

Corollary 2: Let $\bar{c} = (c, c_T)$ be the capacity vectors of SHN with radial reserve. Then

$$\theta_\gamma^g(\bar{c}) = \begin{cases} 1 - \gamma m + \dfrac{t}{d} & if \quad \gamma < \dfrac{1}{m}, \\ \dfrac{t}{d} & if \quad \gamma \geq \dfrac{1}{m}. \end{cases}$$

Proof: According to Lemma 2, we have

$$\theta_\gamma^g(\bar{c}) = \frac{[\min_i c_i - \gamma \sum_{i=1}^{m} c_i]^+ + t}{d}$$

$$= \frac{([d - \gamma m d]^+ + t}{d}$$

$$= \begin{cases} \dfrac{d - \gamma m d + t}{d} & if \quad d - \gamma m d > 0 \\ \dfrac{t}{d} & if \quad d - \gamma m d \leq 0 \end{cases}$$

$$= \begin{cases} 1 - \gamma m + \dfrac{t}{d} & if \quad \gamma < \dfrac{1}{m}, \\ \dfrac{t}{d} & if \quad \gamma \geq \dfrac{1}{m}, \end{cases}$$

and the proof is complete now. ■

In particular, for SHN without radial reserve, i.e., $t = 0$, we have

$$\theta_\gamma^g(\bar{c}) = [1 - \gamma m]^+ = \begin{cases} 1 - \gamma m & if \quad \gamma < \dfrac{1}{m}, \\ 0 & if \quad \gamma \geq \dfrac{1}{m}. \end{cases}$$

Example 2. Let $c = (14,14,14,14)$ and $c_T = (10,10,10,10)$ be the capacity vectors of radial edges and radial reserve respectively, and $d = 14$, $\gamma = 0.2$. Then in the case ***a***, we have $\gamma = 0.2 < \dfrac{1}{4}$, and

$$\theta_{0.2}^g(c, c_T) = 1 - 0.2(4) + \frac{10}{14} = 0.9143 \qquad (4D)$$

$$\theta_{0.2}^g(c, 0) = 1 - 0.2(4) = 0.2.$$

For $\gamma = 0.25$, we obtain

$$\theta^g_{0.25}(c, c_T) = \frac{10}{14} = 0.7143 \qquad (4D)$$
$$\theta^g_{0.25}(c, 0) = 0.$$

In this example, it is clear that SHN without reserve is quite fragile.

Now we consider case ***b***.

Lemma 3: Let $\bar{c} = (c, c_T)$ be the capacity vectors of SHN with radial reserve. Then,

$$\theta^g_\gamma(\bar{c}) = \frac{[\min_i c_i - \gamma \sum_{i=1}^{m} c_i + t]^+}{d}.$$

Proof: Let $\min_i c_i = c_k$, $\bar{y} = (\bar{c}_R, \bar{c}_T), \bar{c}_R = (c_1, \ldots, c_{k-1}, \bar{c}_k, c_{k+1}, \ldots, c_m)$ and $\bar{c}_T = (t, \ldots, t_k, \ldots, t)$, where

$$\bar{c}_k + t_k = c_k + t - \gamma \sum_{i \in M} c_i \,, 0 \le \bar{c}_k \le c_k, 0 \le t_k \le t, \;\; \gamma < \frac{c_k + t}{\sum_{i \in M} c_i}.$$

We have

$$\sum_{i=1}^{2m} \bar{y}_i = \sum_{\substack{i \in M \\ i \ne k}} c_i + (m-1)t + c_k + t - \gamma \sum_{i \in M} c_i = (1-\gamma) \sum_{i \in M} c_i + mt.$$

Thus, $\bar{y} \in Y_\gamma(\bar{c})$.

It is clear that $\theta_0(\bar{y}) = \frac{\bar{c}_k + t_k}{d} = \frac{c_k + t - \gamma \sum_{i \in M} c_i}{d}$, and consequently $\theta^g_\gamma(c) \le \frac{c_k + t - \gamma \sum_{i \in M} c_i}{d}$.

Now, suppose that $\exists \hat{y} = (\hat{c}_R, \hat{c}_T) \in Y_\gamma(\bar{c}) : \theta_0(\hat{y}) < \frac{c_k + t - \gamma \sum_{i \in M} c_i}{d}$. Thus,

$$\exists l \in M : \hat{c}_{R_l} + \hat{c}_{T_l} < c_k + t - \gamma \sum_{i \in M} c_i \,,$$

and then,

$$\sum_{i=1}^{2m} \hat{y}_i < \sum_{i \in M} c_i + (m-1)t + c_k + t - \gamma \sum_{i \in M} c_i = (1-\gamma) \sum_{i \in M} c_i + mt \Rightarrow \hat{y} \notin Y_\gamma(\bar{c}).$$

Hence, for all $\gamma < \frac{c_k + t}{\sum_{i \in M} c_i}$, we have $\theta_\gamma^g(c) = \theta_0(\hat{y}) = \frac{c_k + t - \gamma \sum_{i \in M} c_i}{d}$.

For $\gamma = \frac{c_k + t}{\sum_{i \in M} c_i}$, we introduce

$$\tilde{y} = (\tilde{c}_R, \tilde{c}_T), \tilde{c}_R = (c_1, \ldots, c_{k-1}, 0, c_{k+1}, \ldots, c) \text{ and } \tilde{c}_T = (t_1, \ldots, t_{k-1}, 0, t_{k+1}, \ldots, t_m),$$

where $t_i = t \quad \forall i \neq k$. We have

$$\sum_{i=1}^{2m} \tilde{y}_i = \sum_{\substack{i \in M \\ i \neq k}} c_i + (m-1)t = \sum_{i \in M} c_i + mt - (c_k + t) = (1 - \frac{c_k + t}{\sum_{i \in M} c_i}) \sum_{i \in M} c_i + mt = (1 - \gamma) \sum_{i \in M} c_i + mt \Rightarrow \tilde{y} \in Y_\gamma(\bar{c})$$

In this case there is no path from v_0 to v_k. Thus, $z_k = 0,$ and $\theta_\gamma^g(c) = 0$.

It is obvious that $\theta_\gamma^g(\bar{c}) = 0$, for $\gamma \geq \frac{\min_i c_i + t}{\sum_{i \in M} c_i}$. ■

Corollary 3: For SHN with capacity vector $\bar{c} = (c_R, c_T)$, $c_R = (d, \ldots, d), c_T = (t, \ldots, t)$ we have

$$\gamma_b \overset{def}{=} \bar{\gamma}(\bar{c}) = \frac{1}{m} + \frac{t}{md}.$$

Proof: According to Lemma 3, we have

$$\gamma_b = \bar{\gamma}(\bar{c}) = \min\{\gamma \mid [d - \gamma md + t]^+ = 0\}$$

$$= \min\{\gamma \mid d + t \leq \gamma md\}$$

$$= \min\left\{\gamma \mid \frac{1}{m} + \frac{t}{md} \leq \gamma\right\}$$

$$= \frac{1}{m} + \frac{t}{md}. ■$$

Corollary 4: Let $\bar{c} = (c_R, c_T)$, $c_R = (d, \ldots, d)$ and $c_T = (t, \ldots, t)$ be the capacity vectors of SHN with radial reserve. Then,

$$\theta_\gamma^g(\bar{c}) = \begin{cases} 1-\gamma m+\dfrac{t}{d} & if \quad \gamma<\gamma_b, \\ 0 & if \quad \gamma \ge \gamma_b. \end{cases}$$

Proof: According to Lemma 3, we have

$$\theta_\gamma^g(\bar{c}) = \frac{[\min_i c_i - \gamma\sum_{i=1}^{m} c_i + t]^+}{d}$$

$$= \frac{[d-\gamma md+t]^+}{d}$$

$$= \begin{cases} \dfrac{d-\gamma md+t}{d} & if \quad d-\gamma md+t>0 \\ 0 & if \quad d-\gamma md+t\le 0 \end{cases}$$

$$= \begin{cases} 1-\gamma m+\dfrac{t}{d} & if \quad \gamma<\dfrac{1}{m}+\dfrac{t}{md}=\gamma_b, \\ 0 & if \quad \gamma\ge\dfrac{1}{m}+\dfrac{t}{md}=\gamma_b, \end{cases}$$

and the proof is complete now. ■

Example 3. In the case ***b,*** for SHN in the example 2, we have

$$\gamma_b = \frac{1}{4}+\frac{10}{4(14)} = 0.4286 \qquad (4D).$$

Since $\gamma = 0.2 < \gamma_b$, we obtain

$$\theta_{0.2}^g(c, c_t) = 1-0.2(4)+\frac{10}{14} = 0.9143 \qquad (4D)$$

5. Conclusion

Physical star structures have poor survivability characteristics. Reinforcing SHN by extending the capacity of the radial edge or creating an additional radial reserve is a method which makes it possible to raise the survivability of SHN and duplicate the messages in the event of loss of communication between the center and some other nodes.

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