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Cobb-Douglas Based Firm Production Model under Fuzzy Environment and its Solution using Geometric Programming

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Abstract

In this paper, we consider Cobb-Douglas production function based model in a firm under fuzzy environment, and its solution technique by making use of geometric programming. A firm may use many finite inputs such as labour, capital, coal, iron etc. to produce one single output. It is well known that the primary intention of using production function is to determine maximum output for any given combination of inputs. Also, the firm may gain competitive advantages if it can buy and sell in any quantities at exogenously given prices, independent of initial production decisions. On the other hand, in reality, constraints and/or objective functions in an optimization model may not be crisp quantities. These are usually imprecise in nature and are better represented by using fuzzy sets. Again, geometric programming has many advantages over other optimization techniques. In this paper, Cobb-Douglas production function based models are solved by applying geometric programming technique under fuzzy environment. Illustrative numerical examples further demonstrates the feasibility and efficiency of proposed model under fuzzy environment. Conclusions are drawn at last.

Keywords: Cobb-Douglas production function; firm production model; geometric programming technique; fuzzy decision making; fuzzy geometric programming; fuzzy mathematical programming

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1. Introduction

The Cobb-Douglas production function is widely used to represent the relationship of an output to inputs. Knut Wicksell (1906) had initially proposed the production function. It was tested against statistical evidence by Cobb et al. (1928). Cobb et al. (1928) published a study in which they modelled the growth of American economy during the period 1899-1922. They considered a simplified view of the economy in which production output is determined by the amount of labour involved and the amount of capital invested. While there are many other factors affecting the production output, their model was remarkably accurate. The production function used in that model was as follows:

$$P(L,K) = aL^{\alpha}K^{\beta} \ .$$

Here,

P: Total monetary value of all production (goods produced in a year)

L: Total labour input (number of person–hours worked in a year)

K: Total capital input (the monetary worth of all machinery, equipment and buildings)

a: Total factor productivity,

α, β: The output elasticity of labour and capital respectively.

Available technology may determine these values and they are usually constants. It may be noted that output elasticity measures the response of output to change in level of either labour or capital used in production, e.g., for α =0.25, single 1% increase in labour may lead to approximately 0.25% increase in output. When $\alpha+\beta=1$, the production function has constant *returns to scale*. Hence, an increase of 10% in both *L* and *K* increases *P* by 10%. Here *returns to scale* is a technical property of production, which examines changes in output subsequent to proportional change in all inputs, where all inputs increase by a constant factor. Again for $\alpha+\beta<1$, *returns to scale* are decreasing; and for $\alpha+\beta>1$, *returns to scale* are increasing. In the case of perfect competition, α and β are labours' and capitals' share of output. Analogous to Shivanian et al. (2013), inference is viewed as a process of propagation of elastic constraints.

One important modification or change in classical set theory that guided a paradigm shift in mathematics is the concept of fuzzy set theory. It was introduced by Lotfi Asker Zadeh in 1965. According to Bellman et al. (1970), a fuzzy set is a better representation of real life situations than classical crisp set. In production planning, Cobb-Douglas production function may also be considered under fuzzy environment. As Cao (2010) mentioned, it is well known that geometric programming technique provides us with a systematic approach for solving a class of non-linear optimization problems by finding the optimal value of the objective function and then the optimal values of decision variables are obtained. Consequently, as Guney et al. (2010) suggested, geometric programming technique can be applied in Cobb-Douglas based firm production model under fuzzy environment.

This paper is arranged as follows. In Section 2, Cobb-Douglas based firm production model is discussed in detail by applying different approaches under fuzzy environment. Next, in Section 3, a numerical example using these fuzzy optimization techniques is solved. We also compare the results in Section 3. Finally, conclusions are drawn in Section 4.

2. Cobb-Douglas Based Firm Production Model in Fuzzy Environment

In this paper, we consider a firm that uses *n* inputs (e.g. labour, capital, coal, iron) to produce one single output *q*. Suppose *p* is the cost / unit of output. The firm production function may be expressed as $q = f(x_1, x_2, \ldots, x_n)$. It gives output as a function of inputs in the following form:

$$f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} a x_i^{\alpha_i} \; . \qquad (2.1)$$

Here, $\alpha_i \ (i = 1, 2, \ldots, n)$ denotes the output elasticity of input components $x_i \ (i = 1, 2, \ldots, n)$. Therefore, total revenue amount is of the form:

$$pq = \prod_{i=1}^{n} p a x_i^{\alpha_i} \; .$$

Again, if $r_i \ (i = 1, 2, \ldots, n)$ are the prices of the inputs $x_i \ (i = 1, 2, \ldots, n)$, total expenditure cost is given by the following expression:

$$C(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} r_i x_i \; .$$

In this paper, we plan to maximize total revenue under total limited expenditure cost. Consequently, as per Liu (2006), Cobb-Douglas based firm production model under crisp environment may be taken as follows:

$$\begin{aligned}
&\text{Maximize } R(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} p a x_i^{\alpha_i}, \\
&\text{subject to the constraints: } C(x_1, x_2, \ldots, x_n) \equiv \sum_{i=1}^{n} r_i x_i \le c, \; x_i > 0, i = 1, 2, \ldots, n \; .
\end{aligned} \qquad (2.2)$$

Using the method described by Duffin et al. (1967), geometric programming (GP) technique can be applied to solve model (2.2).

Next, we may consider the Cobb Douglas production model under fuzzy environment, where constraints

$$\tilde{C} = \left\{ \left(C(x_1, x_2, \ldots, x_n), \mu_{\tilde{C}}\left(C(x_1, x_2, \ldots, x_n) \right) \right) \right\}$$

are in fuzzy form as follows:

$$\begin{aligned}
&\text{Maximize } R(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} p a x_i^{\alpha_i}, \\
&\text{subject to the constraints: } C(x_1, x_2, \ldots, x_n) \equiv \sum_{i=1}^{n} r_i x_i \prec c, \\
&\text{with maximum allowable tolerances } c_0, \\
&x_i > 0, \; i = 1, 2, \ldots, n \; .
\end{aligned} \qquad (2.3)$$

Here, membership function of fuzzy constraint

$$\tilde{C}=\left\{\left(C\left(x_1,x_2,\ldots,x_n\right),\mu_{\tilde{C}}\left(C\left(x_1,x_2,\ldots,x_n\right)\right)\right)\right\}$$

is of the form:

$$\mu_{\tilde{C}}\left(C\left(x_1,x_2,\ldots,x_n\right)\right)=\begin{cases}0, & \text{if } C\left(x_1,x_2,\ldots,x_n\right)\geq c+c_0,\\ \dfrac{c+c_0-C\left(x_1,x_2,\ldots,x_n\right)}{c_0}, & \text{if } c\leq C\left(x_1,x_2,\ldots,x_n\right)\leq c+c_0,\\ 1, & \text{if } C\left(x_1,x_2,\ldots,x_n\right)\leq c\ .\end{cases}$$

Next, we may apply different fuzzy optimization techniques on model (2.3).

Method 2.1. Verdegay's approach (1982)

According to Verdegay's approach (1982) on fuzzy optimization technique, model (2.3) reduces to following parametric optimization model:

$$\text{Maximize } R(x_1,x_2,\ldots,x_n)=\prod_{i=1}^{n} pax_i^{\alpha_i},$$
$$\text{subject to the constraints: } \sum_{i=1}^{n} r_i x_i \leq c+(1-\beta)c_0,$$
$$\beta\in[0,1],\ x_i>0,\ i=1,2,\ldots,n\ .$$

The primal geometric programming problem (PGPP) of the above model is as follows:

$$\begin{aligned}&\text{Minimize } \prod_{i=1}^{n}\frac{1}{pa}x_i^{-\alpha_i},\\ &\text{subject to the constraints: } \frac{1}{c+(1-\beta)c_0}\sum_{i=1}^{n} r_i x_i \leq 1,\\ &x_i>0,\ i=1,2,\ldots,n\ .\end{aligned}\tag{2.4}$$

Model (2.4) is a posynomial geometric programming problem whose degree of difficulty (DD) is zero. Its dual geometric programming problem (DGPP) is as follows:

$$\text{Maximize } d\left(\delta_{01},\delta_{11},\delta_{12},\ldots,\delta_{1n}\right)=\left(\frac{1}{pa\delta_{01}}\right)^{\delta_{01}}$$
$$\times\prod_{i=1}^{n}\left(\frac{r_i}{(c+(1-\beta)c_0)\delta_{1i}}\right)^{\delta_{1i}}\left(\sum_{i=1}^{n}\delta_{1i}\right)^{\sum_{i=1}^{n}\delta_{1i}},$$
$$\text{subject to the constraints: } \delta_{01}=1, -\alpha_i\delta_{01}+\delta_{1i}=0,\ \delta_{1i}>0,\ \forall\ i=1,2,\ldots,n\ .$$

The optimal solution of this problem is obtained as $\delta_{01}^{*}=1,\ \delta_{1i}^{*}=\alpha_i$ for $i=1,2,\ldots,n$. It may be noted that although software can be used to find optimal solutions, we have used only pen and paper to find optimal solutions. Again, from the primal dual relations, we have:

$$\prod_{i=1}^{n} \frac{1}{pa} x_i^{-\alpha_i} = \delta_{01}^* \; d^*\left(\delta_{01}^*, \delta_{11}^*, \delta_{12}^*, \ldots, \delta_{1n}^*\right)$$

and

$$\frac{r_i x_i}{c+(1-\beta)c_0} = \frac{\delta_{1i}^*}{\sum_{i=1}^{n} \delta_{1i}^*}, \; \forall \; i = 1, 2, \ldots, n .$$

Hence, optimal inputs are

$$x_i^*(\beta) = \frac{(c+(1-\beta)c_0)\alpha_i}{r_i \sum_{i=1}^{n} \alpha_i}, \; \forall i = 1, 2, \ldots, n .$$

The optimal revenue is as follows:

$$R^*\left(x_1^*, x_2^*, \ldots, x_n^*; \beta\right) = pa\left(\frac{c+(1-\beta)c_0}{\sum_{i=1}^{n} \alpha_i}\right)^{\sum_{i=1}^{n} \alpha_i} \prod_{i=1}^{n} \left(\frac{\alpha_i}{r_i}\right)^{\alpha_i} .$$

Method 2.2. Werner's approach (1987)

First, model (2.3) is solved without tolerance by GP technique. Then, it is solved with tolerance by GP technique. Suppose that revenue without tolerance and with tolerance is R_0 and R_1, respectively. Finally, fuzzy non-linear programming problem is obtained as follows:

Maximize $R\left(x_1, x_2, \ldots, x_n\right) = \prod_{i=1}^{n} pax_i^{\alpha_i} \in [R_0, R_1]$,
subject to the constraints:
$C\left(x_1, x_2, \ldots, x_n\right) \equiv \sum_{i=1}^{n} r_i x_i \leq c,$ with maximum allowable tolerances c_0,
$x_i > 0, \; i = 1, 2, \ldots, n$.

Therefore, our task is to find:

$x_i, \; i = 1, 2, \ldots, n,$

subject to the constraints:

$R\left(x_1, x_2, \ldots, x_n\right) \equiv \prod_{i=1}^{n} pax_i^{\alpha_i} \succ R_1$, with maximum allowable tolerance $(R_1 - R_0)$,

$C\left(x_1, x_2, \ldots, x_n\right) \equiv \sum_{i=1}^{n} r_i x_i \prec c,$ with maximum allowable tolerance c_0,

$x_i > 0, \; i = 1, 2, \ldots, n$.

The fuzzy goal objective function is given by

$$\tilde{R} = \{R\left(x_1, x_2, \ldots, x_n\right), \mu_{\tilde{R}}(R\left(x_1, x_2, \ldots, x_n\right))\} ;$$

its linear membership function is as follows:

$$\mu_{\tilde{R}}(R(x_1,x_2,\dots,x_n)) = \begin{cases} 0, & \text{if } R(x_1,x_2,\dots,x_n) \le R_0, \\ \dfrac{R(x_1,x_2,\dots,x_n)-R_0}{R_1-R_0}, & \text{if } R_0 \le R(x_1,x_2,\dots,x_n) \le R_1, \\ 1, & \text{if } R(x_1,x_2,\dots,x_n) \ge R_1. \end{cases}$$

The constraint is also fuzzy and is given by

$$\tilde{C} = \left\{\left(C(x_1,x_2,\dots,x_n), \mu_{\tilde{C}}\left(C(x_1,x_2,\dots,x_n)\right)\right)\right\}.$$

Here, our task is to find x_i so as to maximize the minimum of

$$\mu_{\tilde{R}}\left(R(x_1,x_2,\dots,x_n)\right) \text{ and } \mu_{\tilde{C}}\left(C(x_1,x_2,\dots,x_n)\right)$$
$$\text{and } x_i > 0,\ i = 1,2,\dots,n .$$

Method 2.3. Zimmermann's approach (1976)

Next, model (2.3) is solved by using max-min operator developed by Zimmermann (1976). Suppose

$$\beta = \left(\text{minimum } \left\{\mu_{\tilde{R}}(R(x_1,x_2,\dots,x_n)), \mu_{\tilde{C}}\left(C(x_1,x_2,\dots,x_n)\right)\right\}\right).$$

Then, the single objective optimization problem is as follows:

$$\begin{aligned} &\text{Maximize } \beta, \\ &\text{subject to the constraints: } \frac{c+c_0-C(x_1,x_2,\dots,x_n)}{c_0} \ge \beta, \frac{R(x_1,x_2,\dots,x_n)-R_0}{R_1-R_0} \ge \beta, \\ &x_i > 0, \forall i = 1,2,\dots,n . \end{aligned}$$

Then, taking the inverse of the objective function, we obtain the posynomial geometric programming problem, whose DD is 2. We solve it by using GP technique. The dual of the problem is obtained as follows:

$$\begin{aligned} \text{Maximize } d\left(\delta_{01},\delta_{11},\delta_{12},\dots,\delta_{1n+1},\delta_{21},\delta_{22}\right) &= \left(\frac{1}{\delta_{01}}\right)^{\delta_{01}} \prod_{i=1}^{n}\left(\frac{r_i}{(c+c_0)\delta_{1i}}\right)^{\delta_{1i}} \\ &\times\left(\frac{c_0}{(c+c_0)\delta_{1n+1}}\right)^{\delta_{1n+1}} \left(\frac{R_0}{pa\delta_{21}}\right)^{\delta_{21}} \\ &\times\left(\frac{R_1-R_0}{pa\delta_{22}}\right)^{\delta_{22}} \left(\sum_{i=1}^{n+1}\delta_{1i}\right)^{\sum_{i=1}^{n+1}\delta_{1i}} \left(\sum_{i=1}^{2}\delta_{2i}\right)^{\sum_{i=1}^{2}\delta_{2i}}, \end{aligned}$$

$$\text{subject to the constraints : } \delta_{01}=1,\ -\delta_{01}+\delta_{1n+1}+\delta_{22} = 0,\ \delta_{1i}-\alpha_i\delta_{21}-\alpha_i\delta_{22}=0,\ \forall\ i=1,2,\ldots,n\ . \tag{2.5}$$

Again, by using pen and paper, the optimal solution of model (2.5) is obtained as follows:

$$\delta_{01}^{*}=1,\ \delta_{1n+1}=(1-\delta_{22}),\ \delta_{1i}=\alpha_i(\delta_{21}+\delta_{22}),\ \forall\ i=1,2,\ldots,n..$$

Now substituting δ_{01}^{*}, δ_{1i} $(i=1,2,\ldots,n)$, δ_{1n+1} in model (2.5), the dual function is obtained as follows:

$$\begin{aligned}\text{Maximize } d(\delta_{21},\delta_{22}) = &\prod_{i=1}^{n}\left(\frac{r_i}{(c+c_0)(\delta_{21}+\delta_{22})\alpha_i}\right)^{(\delta_{21}+\delta_{22})\alpha_i} \\ &\times\left(\frac{c_0}{(c+c_0)(1-\delta_{22})}\right)^{(1-\delta_{22})}\left(\frac{R_0}{pa\delta_{21}}\right)^{\delta_{21}} \\ &\times\left(\frac{R_1-R_0}{pa\delta_{22}}\right)^{\delta_{22}}(\delta_{21}+\delta_{22})^{(\delta_{21}+\delta_{22})} \\ &\times\left[\left(\sum_{i=1}^{n}\alpha_i\right)\left(\delta_{21}+\delta_{22}\right)+\left(1-\delta_{22}\right)\right]^{\left[\left(\sum_{i=1}^{n}\alpha_i\right)\left(\delta_{21}+\delta_{22}\right)+\left(1-\delta_{22}\right)\right]}.\end{aligned} \tag{2.6}$$

To find the optimal values of δ_{21},δ_{22}, we have to maximize the dual objective function $d(\delta_{21},\delta_{22})$. Taking logarithms on both sides of model (2.6) and differentiating partially with respect to δ_{21}, δ_{22} one by one, and next equating those to zero, we obtain:

$$\frac{\partial}{\partial\delta_{21}}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=0 \text{ and } \frac{\partial}{\partial\delta_{22}}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=0,$$

i.e.,

$$\begin{aligned}&\sum_{i=1}^{n}\alpha_i\log\left(\frac{r_i}{(c+c_0)(\delta_{21}+\delta_{22})\alpha_i}\right)+\log\left(\frac{R_0}{pa\delta_{21}}\right) \\ &\quad+\left(\sum_{i=1}^{n}\alpha_i\right)\log\left[\left(\sum_{i=1}^{n}\alpha_i\right)(\delta_{21}+\delta_{22})+(1-\delta_{22})\right] \\ &\quad+\log(\delta_{21}+\delta_{22})=0\end{aligned}$$

and

$$\sum_{i=1}^{n}\alpha_i \log\left(\frac{r_i}{(c+c_0)(\delta_{21}+\delta_{22})\alpha_i}\right)-\log\left(\frac{c_0}{(c+c_0)(1-\delta_{22})}\right)$$
$$+\log\left(\frac{R_1-R_0}{pa\delta_{22}}\right)+\left(\sum_{i=1}^{n}\alpha_i-1\right)\log\left(\left(\sum_{i=1}^{n}\alpha_i\right)\left(\delta_{21}+\delta_{22}\right)+\left(1-\delta_{22}\right)\right)$$
$$+\log\left(\delta_{21}+\delta_{22}\right)=0.$$

Again, we get:

$$\frac{\partial^2}{\partial\delta_{21}^2}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=-\frac{\delta_{22}}{\delta_{21}\left(\delta_{21}+\delta_{22}\right)}-\frac{(1-\delta_{22})\left(\sum_{i=1}^{n}\alpha_i\right)}{(\delta_{21}+\delta_{22})\left(\left(\sum_{i=1}^{n}\alpha_i\right)(\delta_{21}+\delta_{22})+(1-\delta_{22})\right)}<0,$$

$$\frac{\partial^2}{\partial\delta_{22}^2}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=-\frac{\delta_{21}+\delta_{22}\left(\sum_{i=1}^{n}\alpha_i\right)}{\delta_{22}\left(\delta_{21}+\delta_{22}\right)}-\frac{(1-\delta_{22})\left(\sum_{i=1}^{n}\alpha_i\right)\left(2-\left(\sum_{i=1}^{n}\alpha_i\right)\right)+\left(\sum_{i=1}^{n}\alpha_i\right)(\delta_{21}+\delta_{22})}{(1-\delta_{22})\left(\left(\sum_{i=1}^{n}\alpha_i\right)(\delta_{21}+\delta_{22})+(1-\delta_{22})\right)}<0,$$

$$\frac{\partial^2}{\partial\delta_{21}\partial\delta_{22}}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=\frac{\partial^2}{\partial\delta_{22}\partial\delta_{21}}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]=\frac{(1-\delta_{22})\left(1-\sum_{i=1}^{n}\alpha_i\right)}{(\delta_{21}+\delta_{22})\left(\left(\sum_{i=1}^{n}\alpha_i\right)(\delta_{21}+\delta_{22})+(1-\delta_{22})\right)}\ .$$

Here, we may observe that

$$\frac{\partial^2}{\partial\delta_{21}^2}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right].\frac{\partial^2}{\partial\delta_{22}^2}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]-\left(\frac{\partial^2}{\partial\delta_{21}\partial\delta_{22}}\left[\log\left\{d\left(\delta_{21},\delta_{22}\right)\right\}\right]\right)^2>0.$$

Method 2.4. Sakawa's method (1993)

Next, model (2.3) is solved by using Sakawa's (1993) method. Assuming that

$$\mu_{\tilde{R}}(R(x_1,x_2,\ldots,x_n))=\text{minimum }\left\{\mu_{\tilde{R}}(R(x_1,x_2,\ldots,x_n)),\mu_{\tilde{C}}(C(x_1,x_2,\ldots,x_n))\right\},$$

model (2.3) becomes:

Maximize $\mu_{\tilde{R}}(R(x_1,x_2,\ldots,x_n)),$
subject to the constraints: $\mu_{\tilde{C}}(C(x_1,x_2,\ldots,x_n))\ge\mu_{\tilde{R}}(R(x_1,x_2,\ldots,x_n)),$

$x_i>0,$ for $i=1,2,\ldots,n,$ $\mu_{\tilde{R}}(R(x_1,x_2,\ldots,x_n)),\mu_{\tilde{C}}(C(x_1,x_2,\ldots,x_n))\in[0,1].$

i.e.,

$$\text{Maximize } \mu_{\tilde{R}}\left(R(x_1,x_2,\ldots,x_n)\right)=\frac{\prod_{i=1}^{n} pax_i^{\alpha_i}-R_0}{R_1-R_0},$$
$$\text{subject to the constraints:} \frac{R_1-R_0}{R_1c_0+(R_1-R_0)c}\sum_{i=1}^{n} r_i x_i \tag{2.7}$$
$$+\frac{pac_0}{R_1c_0+(R_1-R_0)c}\prod_{i=1}^{n} x_i^{\alpha_i}\le 1,\ x_i>0,\ \text{for } i=1,2,\ldots,n\ .$$

Here,

$$\mu_{\tilde{R}}\left(R(x_1,x_2,\ldots,x_n)\right)=\mu_{\tilde{R}'}\left(R'(x_1,x_2,\ldots,x_n)\right)-\frac{R_0}{R_1-R_0}.$$

To solve model (2.7) by geometric programming technique, we rewrite the problem as follows:

$$\begin{aligned}&\text{Minimize } \theta\prod_{i=1}^{n} x_i^{-\alpha_i},\\ &\text{subject to the constraints: } \theta_1\sum_{i=1}^{n} r_i x_i+\theta_2\prod_{i=1}^{n} x_i^{\alpha_i}\le 1,\ x_i>0,\ i=1,2,\ldots,n\ .\end{aligned} \tag{2.8}$$

where,

$$\theta=\frac{R_1-R_0}{pa},\ \theta_1=\frac{R_1-R_0}{R_1c_0+\left(R_1-R_0\right)c},\ \theta_2=\frac{pac_0}{R_1c_0+\left(R_1-R_0\right)c}.$$

We find that model (2.8) is a posynomial PGPP with DD being 1. Its DGPP form is as follows:

$$\begin{aligned}&\text{Maximize } d\left(\delta_{01},\delta_{11},\delta_{12},\ldots,\delta_{1n+1}\right)=\left(\frac{\theta}{\delta_{01}}\right)^{\delta_{01}}\prod_{i=1}^{n}\left(\frac{r_i\theta_i}{\delta_{1i}}\right)^{\delta_{1i}}\left(\frac{\theta_2}{\delta_{1n+1}}\right)^{\delta_{1n+1}}\left(\sum_{i=1}^{n+1}\delta_{1i}\right)^{\sum_{i=1}^{n+1}\delta_{1i}},\\ &\text{subject to the constraints: } \delta_{01}=1,\ -\alpha_i\delta_{01}+\delta_{1i}+\alpha_i\delta_{1n+1}=0,\ \forall i=1,2,\ldots,n\ .\end{aligned} \tag{2.9}$$

The optimal solution to model (2.9) is obtained as follows:

$$\delta_{01}^{*}=1,\ \delta_{1i}=\alpha_i\left(1-\delta_{1n+1}\right),\forall\ i=1,2,\ldots,n\,.$$

Now substituting $\delta_{01}^{*},\ \delta_{1i},\ \text{for } i=1,2,\ldots,n$ in model (2.9), the dual function is obtained as follows:

$$\begin{aligned}d\left(\delta_{1n+1}\right)=\theta\prod_{i=1}^{n}&\left(\frac{r_i\theta_i}{\alpha_i\left(1-\delta_{1n+1}\right)}\right)^{\alpha_i\left(1-\delta_{1n+1}\right)}\left(\frac{\theta_2}{\delta_{1n+1}}\right)^{\delta_{1n+1}}\\ &\times\left(\left(\sum_{i=1}^{n}\alpha_i\right)\left(1-\delta_{1n+1}\right)+\delta_{1n+1}\right)^{\left(\left(\sum_{i=1}^{n}\alpha_i\right)\left(1-\delta_{1n+1}\right)+\delta_{1n+1}\right)}.\end{aligned} \tag{2.10}$$

To find optimal solution δ_{1n+1}, we have to maximize the dual function $d(\delta_{1n+1})$. Taking logarithms on both sides of equation (2.10) and differentiating with respect to δ_{1n+1} and then equating to zero, we get:

$$\frac{d}{d\delta_{1n+1}}\left[\log\left\{d\left(\delta_{1n+1}\right)\right\}\right]=0,$$

that is,

$$\log\left(\frac{\theta_2}{\delta_{1n+1}}\right)+\left(1-\sum_{i=1}^{n}\alpha_i\right)\log\left\{\left(\sum_{i=1}^{n}\alpha_i\right)(1-\delta_{1n+1})+\delta_{1n+1}\right\}$$
$$-\sum_{i=1}^{n}\alpha_i\log\left(\frac{r_i\theta_1}{\alpha_i\left(1-\delta_{1n+1}\right)}\right)=0.$$

Again, since $0<\delta_{1n+1}<1$ as $\delta_{1i}>0,\ \forall i,$

$$\frac{d^2}{d\delta_{1n+1}^2}\left[\log\left\{d\left(\delta_{1n+1}\right)\right\}\right]=-\frac{1}{\delta_{1n+1}}-\frac{\sum_{i=1}^{n}\alpha_i}{1-\delta_{1n+1}}+\frac{\left(1-\sum_{i=1}^{n}\alpha_i\right)^2}{\left(\sum_{i=1}^{n}\alpha_i\right)(1-\delta_{1n+1})+\delta_{1n+1}}$$
$$=-\left[\frac{\sum_{i=1}^{n}\alpha_i}{1-\delta_{1n+1}}+\left(\frac{\left(\sum_{i=1}^{n}\alpha_i\right)\left\{1+\delta_{1n+1}\left(1-\sum_{i=1}^{n}\alpha_i\right)\right\}}{\delta_{1n+1}\left\{\left(\sum_{i=1}^{n}\alpha_i\right)(1-\delta_{1n+1})+\delta_{1n+1}\right\}}\right)\right]<0.$$

Although Sakawa's (1993) approach and Zimmermann's (1976) approach are similar, one main disadvantage of Zimmermann's (1976) method over Sakawa's (1993) method is the increase in degree of difficulty of the model in Zimmermann's (1976) method. It makes the model difficult to solve in GP technique under fuzzy environment. On the other hand, the advantage of Zimmermann's (1976) method over Sakawa's (1993) method is that only one problem needs to be solved in Zimmermann's (1976) method but two problems need to be solved in Sakawa's (1993) method. In this paper, intentionally, we have solved only one problem. The other problem of Sakawa's (1993) method can be solved similarly.

Method 2.5. Max-additive operator (1987)

Next, we solve model (2.3) using max-additive operator (1987) as follows:

maximize $\mu_{\tilde{R}}\left(R\left(x_1,x_2,\ldots,x_n\right)\right)+\mu_{\tilde{C}}\left(C\left(x_1,x_2,\ldots,x_n\right)\right),$
subject to the constraints: $\mu_{\tilde{R}}\left(R\left(x_1,x_2,\ldots,x_n\right)\right),\ \mu_{\tilde{C}}\left(C\left(x_1,x_2,\ldots,x_n\right)\right)\in[0,1],$
$x_i>0,$ for $i=1,2,\ldots,n.$

i.e.,

maximize $\dfrac{pa}{R_1-R_0}\prod_{i=1}^{n}x_i^{\alpha_i}-\dfrac{1}{c_0}\sum_{i=1}^{n}r_ix_i,$
subject to the constraints: $x_i>0, i=1,2,\ldots,n$.

Now if

$$\frac{pa}{R_1-R_0}\prod_{i=1}^{n}x_i^{\alpha_i}-\frac{1}{c_0}\sum_{i=1}^{n}r_ix_i\geq x_{n+1},$$

our problem becomes:

$$\begin{aligned}&\text{maximize} \quad x_{n+1},\\ &\text{subject to the constraints: } \frac{pa}{R_1-R_0}\prod_{i=1}^{n}x_i^{\alpha_i}-\frac{1}{c_0}\sum_{i=1}^{n}r_ix_i\geq x_{n+1},\\ &\qquad x_i>0, \quad \text{for } i=1,2,\ldots,n .\end{aligned} \tag{2.11}$$

Rewriting model (2.11) as PGPP form, we get:

$$\begin{aligned}&\text{minimize} \quad x_{n+1}^{-1},\\ &\text{subject to the constraints: } \frac{R_1-R_0}{c_0pa}\left(\prod_{i=1}^{n}x_i^{-\alpha_i}\right)\left(\sum_{i=1}^{n}r_ix_i\right)+\frac{R_1-R_0}{pa}x_{n+1}\prod_{i=1}^{n}x_i^{-\alpha_i}\leq 1,\\ &\qquad x_i>0, \quad \text{for } i=1,2,\ldots,n .\end{aligned} \tag{2.12}$$

Model (2.12) is a posynomial PGPP with DD being zero. Its DGPP is as follows:

$$\text{Maximize } d\left(\delta_{01},\delta_{11},\delta_{12},\ldots,\delta_{1n+1}\right)=\left(\frac{1}{\delta_{01}}\right)^{\delta_{01}}\prod_{i=1}^{n}\left(\frac{r_i\left(R_1-R_0\right)}{c_0pa\delta_{1i}}\right)^{\delta_{1i}}$$

$$\times\left(\frac{R_1-R}{pa\delta_{1n+1}}\right)^{\delta_{1n+1}}\left(\sum_{i=1}^{n+1}\delta_{1i}\right)^{\sum_{i=1}^{n+1}\delta_{1i}},$$

subject to the constraints: $\delta_{01}=1,$

$$\left(1-\sum_{j=1}^{n}\alpha_j\right)\delta_{1i}-\alpha_i\delta_{1n+1}=0,\ \forall i=1,2,\ldots,n,\ -\delta_{01}+\delta_{1n+1}=0 .$$

The optimal solutions to the problem are

$$\delta_{01}^*=1,\ \delta_{1n+1}^*=1,\ \delta_{1i}^*=\frac{\alpha_i}{1-\sum_{i=1}^{n}\alpha_i},\ i=1,2,\ldots,n.$$

Therefore,

$$d^*\left(\delta_{01}^*,\delta_{11}^*,\delta_{12}^*,\ldots,\delta_{1n+1}^*\right)=\frac{1}{1-\sum_{i=1}^{n}\alpha_i}\left(\frac{R_1-R_0}{pa}\right)^{\frac{1}{1-\sum_{i=1}^{n}\alpha_i}}\prod_{i=1}^{n}\left(\frac{r_i}{c_0\alpha_i}\right)^{\frac{\alpha_i}{1-\sum_{i=1}^{n}\alpha_i}} .$$

From primal dual relations, we get:

$$x_{n+1}^{-1} = \delta_{01}^{*} d^{*}\left(\delta_{01}^{*}, \delta_{11}^{*}, \delta_{12}^{*}, \ldots, \delta_{1n+1}^{*}\right)$$

and

$$\frac{R_1 - R_0}{c_0 pa}\left(\prod_{i=1}^{n} x_i^{-\alpha_i}\right) r_i x_i = \frac{\delta_{1i}^{*}}{\sum_{i=1}^{n+1} \delta_{1i}^{*}}, \ \forall i = 1, 2, \ldots, n.$$

Hence,

$$\frac{R_1 - R_0}{pa} x_{n+1} \prod_{i=1}^{n} x_i^{-\alpha_i} = \frac{\delta_{1n+1}^{*}}{\sum_{i=1}^{n+1} \delta_{1i}^{*}}.$$

Here, the optimal inputs are obtained as follows:

$$x_i^{*} = \left(\frac{\alpha_i}{r_i}\right) c_0^{\frac{1}{1-\sum_{i=1}^{n}\alpha_i}} \left(\frac{pa}{R_1 - R_0}\right)^{\frac{1}{1-\sum_{i=1}^{n}\alpha_i}} \prod_{i=1}^{n}\left(\frac{\alpha_i}{r_i}\right)^{\frac{\alpha_i}{1-\sum_{i=1}^{n}\alpha_i}}, \text{ for } i = 1, 2, \ldots, n.$$

and the optimal revenue is obtained as

$$R^{*}\left(x_1^{*}, x_2^{*}, \ldots, x_n^{*}\right) = pa \prod_{i=1}^{n}\left(\frac{\alpha_i}{r_i}\right)^{\alpha_i} \left\{ c_0^{\frac{1}{1-\sum_{i=1}^{n}\alpha_i}} \left(\frac{pa}{R_1 - R_0}\right)^{\frac{1}{1-\sum_{i=1}^{n}\alpha_i}} \prod_{i=1}^{n}\left(\frac{\alpha_i}{r_i}\right)^{\frac{\alpha_i}{1-\sum_{i=1}^{n}\alpha_i}} \right\}^{\sum_{i=1}^{n}\alpha_i}.$$

Method 2.6. Max-product operator (1978)

Next, we solve model (2.3) using max-product operator (1978). Applying max-product operator (1978), the model becomes:

maximize $\mu_{\tilde{R}}\left(R\left(x_1, x_2, \ldots, x_n\right)\right) . \mu_{\tilde{C}}\left(C\left(x_1, x_2, \ldots, x_n\right)\right)$,

subject to the constraints:

$$\mu_{\tilde{R}}\left(R\left(x_1, x_2, \ldots, x_n\right)\right),$$

$$\mu_{\tilde{C}}\left(C\left(x_1, x_2, \ldots, x_n\right)\right) \in [0,1], \ x_i > 0, \text{ for } i = 1, 2, \ldots, n.$$

i.e.,

$$\text{Maximize } \frac{\prod_{i=1}^{n} pax_i^{\alpha_i} - R_0}{R_1 - R_0} . \frac{c + c_0 - \sum_{i=1}^{n} r_i x_i}{c_0},$$

$$\text{subject to the constraints: } \frac{\prod_{i=1}^{n} pax_i^{\alpha_i} - R_0}{R_1 - R_0}, \ \frac{c + c_0 - \sum_{i=1}^{n} r_i x_i}{c_0} \in [0,1],$$

$$x_i > 0, \text{ for } i = 1, 2, \ldots, n.$$

Suppose that

$$\frac{\prod_{i=1}^{n} pax_i^{\alpha_i} - R_0}{R_1 - R_0} \geq x_{n+1}, \ \frac{c + c_0 - \sum_{i=1}^{n} r_i x_i}{c_0} \geq x_{n+2}.$$

Consequently, the above model becomes:

$$\text{Maximize } x_{n+1}.x_{n+2},$$

$$\text{subject to the constraints: } \frac{\prod_{i=1}^{n} pax_i^{\alpha_i} - R_0}{R_1 - R_0} \geq x_{n+1}, \ \frac{c + c_0 - \sum_{i=1}^{n} r_i x_i}{c_0} \geq x_{n+2}, \tag{2.13}$$

$$x_i > 0, \text{ for } i = 1, 2, \ldots, n+2.$$

Equation (2.13) can be written in PGPP form as follows:

$$\text{minimize } x_{n+1}^{-1} x_{n+2}^{-1},$$

subject to the constraints:

$$\frac{1}{c + c_0} \sum_{i=1}^{n} r_i x_i + \frac{c_0}{c + c_0} x_{n+2} \leq 1, \tag{2.14}$$

$$\frac{R_0}{pa} \prod_{i=1}^{n} x_i^{-\alpha_i} + \frac{R_1 - R_0}{pa} x_{n+1} \prod_{i=1}^{n} x_i^{-\alpha_i} \leq 1,$$

$$x_i > 0, \text{ for } i = 1, 2, \ldots, n.$$

Model (2.14) is a posynomial PGPP with DD being unity. Therefore, its DGPP is as follows:

$$\text{Maximize } d\left(\delta_{01},\delta_{11},\delta_{12},\dots,\delta_{1n+1},\delta_{21},\delta_{22}\right)=\left(\frac{1}{\delta_{01}}\right)^{\delta_{01}}\prod_{i=1}^{n}\left(\frac{r_i}{\left(c+c_0\right)\delta_{1i}}\right)^{\delta_{1i}}$$

$$\times\left(\frac{c_0}{\left(c+c_0\right)\delta_{1n+1}}\right)^{\delta_{1n+1}}\left(\frac{R_0}{pa\delta_{21}}\right)^{\delta_{21}}\left(\frac{R_1-R_0}{pa\delta_{22}}\right)^{\delta_{22}}$$

$$\times\left(\sum_{i=1}^{n+1}\delta_{1i}\right)^{\sum_{i=1}^{n+1}\delta_{1i}}\left(\sum_{i=1}^{2}\delta_{2i}\right)^{\sum_{i=1}^{2}\delta_{2i}},$$

subject to the constraints:

$$\delta_{01}=1,\ \delta_{1i}-\alpha_i\delta_{21}-\alpha_i\delta_{22}=0,\ \forall\ i=1,2,\dots,n,$$
$$-\delta_{01}+\delta_{22}=0,\ -\delta_{01}+\delta_{1n+1}=0. \tag{2.15}$$

Using pen and paper, optimal solutions of the model (2.15) are obtained as:

$$\delta_{01}^*=1,\ \delta_{22}^*=1,\ \delta_{1n+1}^*=1,\ \delta_{1i}=\alpha_i\left(\delta_{21}+1\right),\forall\ i=1,2,\dots,n\ .$$

Substituting $\delta_{01}^*,\ \delta_{22}^*,\ \delta_{1n+1}^*,\ \delta_{1i}$ for $i=1,2,\dots,n,$ in (2.15), the dual function is obtained as follows:

$$d\left(\delta_{21}\right)=\prod_{i=1}^{n}\left(\frac{r_i}{\left(c+c_0\right)\alpha_i(\delta_{21}+1)}\right)^{\alpha_i(\delta_{21}+1)}\left(\frac{c_0}{c+c_0}\right)\left(\frac{R_0}{pa\delta_{21}}\right)^{\delta_{21}}\left(\frac{R_1-R_0}{pa}\right)$$
$$\left\{1+(\delta_{21}+1)\sum_{i=1}^{n}\alpha_i\right\}^{\left\{1+(\delta_{21}+1)\sum_{i=1}^{n}\alpha_i\right\}}(\delta_{21}+1)^{(\delta_{21}+1)}. \tag{2.16}$$

Next, to find optimal value of δ_{21}, we maximize the dual function: $d(\delta_{21})$. Taking logarithms on both sides of model (2.16), and differentiating with respect to δ_{21} and next equating to zero, we find:

$$\frac{d}{d\delta_{21}}\left\{\ln(d(\delta_{21}))\right\}=0,$$

i.e.,

$$\sum_{i=1}^{n}\alpha_i\ln\frac{r_i}{a_i(c+c_0)(\delta_{21}+1)}+\ln\left(\frac{R_0}{pa\delta_{21}}\right)+\left(\sum_{i=1}^{n}\alpha_i\right)\ln\left\{1+(\delta_{21}+1)\sum_{i=1}^{n}\alpha_i\right\}+\ln\left(\delta_{21}+1\right)=0.$$

Hence, we have:

$$\frac{d^2}{d\delta_{21}^2}\left[\ln\left\{d\left(\delta_{21}\right)\right\}\right] = -\frac{\sum_{i=1}^{n}\alpha_i}{(\delta_{21}+1)} + \frac{1}{(\delta_{21}+1)} - \frac{1}{\delta_{21}} + \frac{\left(\sum_{i=1}^{n}\alpha_i\right)^2}{\left\{1+(\delta_{21}+1)\sum_{i=1}^{n}\alpha_i\right\}}$$

$$= -\frac{1}{\delta_{21}(\delta_{21}+1)} - \frac{\sum_{i=1}^{n}\alpha_i}{(\delta_{21}+1)\left(1+(\delta_{21}+1)\sum_{i=1}^{n}\alpha_i\right)} < 0\ .$$

We find that the second order derivative is always negative.

3. Numerical Examples

Now, we consider numerical examples on which we may apply these optimization techniques and solve Cobb-Douglas based firm production model under fuzzy environment. Analogous to Creese (2010), the input data are taken as given in Table 1. The output data obtained by using crisp optimization technique to solve Cobb-Douglas based firm production model are given in Table 2.

Table 1. Input data of Cobb-Douglas based firm production model

No. of Inputs	Output elasticity of the Input components			Prices of the input components			Selling price of a unit product	Total productivity	Available cost
n	α_1	α_2	α_3	r_1	r_2	r_3	p	a	c
3	0.1	0.3	0.2	20	24	30	20	40	8500

Table 2. Output data using crisp optimization technique

Dual Variables				Primal Variables			Revenue
δ_{01}^*	δ_{11}^*	δ_{12}^*	δ_{13}^*	x_1^*	x_2^*	x_3^*	R^*
1.0	0.1	0.3	0.2	70.8333	177.0833	94.4444	14374.82

Next, suppose that the input data under fuzzy environment is given in Table 3.

Table 3. Input data of Cobb-Douglas based firm production model in fuzzy environment

No. of Inputs	Output elasticity of the Input components			Prices of the input components (Rs.)			Selling Price of a unit product (Rs.)	Total productivity	Available cost (Rs.)	Available tolerance (Rs.)
n	α_1	α_2	α_3	r_1	r_2	r_3	p	a	c	c_0
3	0.1	0.3	0.2	20	24	30	20	40	8500	300

On solving the model under fuzzy environment by Verdegay's approach (1982), output data corresponding to different values of aspiration level β are obtained as given in Table 4.

Table 4. Output data of Cobb-Douglas based firm production model by Verdegay's approach (1982)

Aspiration Level	Dual Variables			Primal Variables			Cost (Rs.)	Revenue (Rs.)
β	δ_{01}^*	δ_{02}^*	δ_{03}^*	x_1^*	x_2^*	x_3^*	C^*	R^*
0.0	0.1	0.3	0.2	73.3333	183.3333	97.7778	8800	14677.11
0.1				73.0833	182.7083	97.4444	8770	14647.07
0.2				72.8333	182.0833	97.1111	8740	14616.99
0.3				72.5833	181.4583	96.7778	8710	14586.86
0.4				72.3333	180.8333	96.4444	8680	14556.70
0.5				72.0833	180.2083	96.1111	8650	14526.49
0.6				71.8333	179.5833	95.7778	8620	14496.24
0.7				71.5833	178.9583	95.4444	8590	14465.95
0.8				71.3333	178.3333	95.1111	8560	14435.61
0.9				71.0833	177.7083	94.7778	8530	14405.24
1.0				70.8333	177.0833	94.4444	8500	14374.82

On solving the same model with the same input data by max-min operator (Zimmermann 1976) under fuzzy environment, the output data is obtained as given in Table 5.

Table 5. Output data using Zimmermann's approach (1976)

Dual Variables							Primal Variables			Optimal Revenue	Optimal Cost	Aspiration level
δ_{01}^*	δ_{11}^*	δ_{12}^*	δ_{13}^*	δ_{14}^*	δ_{21}^*	δ_{22}^*	x_1^*	x_2^*	x_3^*	R^*	C^*	$\mu_{\tilde{R}}{}^*(R(x_1, \dots, x_n))$ and $\mu_{\tilde{C}}{}^*(C(x_1, \dots, x_n))$
1.0	4.797	14.391	9.594	0.5	47.470	0.5	72.081	180.202	96.108	14526.19	8649.71	0.5 and 0.5

On solving the same model with the same input data by Sakawa's (1993) method under fuzzy environment, the output data is obtained as given in Table 6.

Table 6. Output data using Sakawa's (1993) method

Dual Variables					Primal Variables			Optimal Revenue	Optimal Cost	Aspiration level
δ_{01}^*	δ_{11}^*	δ_{12}^*	δ_{13}^*	δ_{14}^*	x_1^*	x_2^*	x_3^*	R^*	C^*	$\mu_{\tilde{R}}{}^*(R(x_1, \dots, x_n))$ and $\mu_{\tilde{C}}{}^*(C(x_1, \dots, x_n))$
1.00	0.050	0.150	0.100	0.499	72.254	180.634	96.338	14547.07	8670.44	0.57 and 0.43

On solving the same model with the same input data by max-additive (1987) operator under fuzzy environment, the output data is obtained as given in Table 7.

Table 7. Output data using max-additive (1987) operator

Dual variables					Primal variables			Optimal Revenue (Rs.)	Optimal Cost (Rs.)	Aspiration level
${\delta_{01}}^*$	${\delta_{11}}^*$	${\delta_{12}}^*$	${\delta_{13}}^*$	${\delta_{14}}^*$	${x_1}^*$	${x_2}^*$	${x_3}^*$	R^*	C^*	$\mu_{\tilde{R}}^*(R(x_1,\ldots,x_n))$ and $\mu_{\tilde{C}}^*(C(x_1,\ldots,x_n))$
1.00	0.25	0.75	0.50	1.00	72.080	180.201	96.107	14526.12	8649.63	0.5 and 0.5

On solving the same model with the same input data by max-product (1978) operator under fuzzy environment, the output data is obtained as given in Table 8.

Table 8. Output data using max-product (1978) operator

Dual Variables							Primal variables			Optimal Revenue (Rs.)	Optimal Cost (Rs.)	Aspiration Level
δ_{01}	${\delta_{11}}^*$	${\delta_{12}}^*$	${\delta_{13}}^*$	${\delta_{14}}^*$	${\delta_{21}}^*$	${\delta_{22}}^*$	${x_1}^*$	${x_2}^*$	${x_3}^*$	R^*	C^*	$\mu_{\tilde{R}}^*(R(x_1,\ldots,x_n))$ and $\mu_{\tilde{C}}^*(C(x_1,\ldots,x_n))$
1. 00	9.59	28.7	19.1	1.00	94.9	1.00	72.08	180.20	96.10	14526.22	8649.73	0.5 and 0.5

Finally, we may compare the results obtained by using different fuzzy optimization techniques to solve Cobb-Douglas based firm production model under fuzzy environment.

Table 9. Comparison of outcomes in different techniques

Method	Optimal Inputs			Optimal Revenue	Optimal Cost
	${x_1}^*$	${x_2}^*$	${x_3}^*$	R^*	C^*
Zimmermann's approach (1976)	72.081	180.202	96.108	14526.19	8649.71
Sakawa's method (1993)	72.254	180.634	96.338	14547.07	8670.44
max-additive operator (1987)	72.080	180.201	96.107	14526.12	8649.63
max-product operator (1978)	72.081	180.203	96.108	14526.22	8649.73

Hence, the optimal revenue in classical optimization technique is Rs. 14374.82 with optimal cost Rs. 8500. But if the same model is considered under fuzzy environment and solved by using max-min operator in Zimmermann's (1976) technique, the optimal revenue comes as Rs. 14526.19 with optimal cost being Rs. 8649.71. As maximizing revenue is a primary objective to decision makers, this outcome is more acceptable than the solution under crisp environment.

Again if max-min operator in Sakawa's (1993) technique is used to solve the same model under fuzzy environment, the optimal revenue is Rs. 14547.07, a far more acceptable solution than the solution under crisp environment.

If max-additive (1987) operator is used to solve the same model under fuzzy environment, the optimal revenue is Rs. 14526.12, another better optimal solution than crisp solution.

If max-product (1978) operator is used to solve the same model under fuzzy environment, the optimal revenue is Rs. 14526.22, again one better optimal solution than crisp solution.

4. Conclusion

In this paper, we have considered Cobb-Douglas production function based model in a firm under fuzzy environment and its solution technique by making use of geometric programming. Here, the objective is to maximize the revenue under limited total expenditure cost, and to minimize the total expenditure costs subject to target revenue. To match with reality, the model is considered under fuzzy environment and solved using different fuzzy optimization techniques.

In this paper, geometric programming is applied to solve the model obtained by fuzzy optimization techniques. The advantage of geometric programming over other optimization techniques is that it provides us with a systematic approach for solving a class of non-linear optimization problems by finding the optimal value of the objective function and then the optimal values of decision variables are obtained. Moreover, GP often reduces one complex optimization problem to set of simultaneous linear equations.

We know that a decision maker is the king and his decision is final. Accordingly, in this paper, we collect information from a decision maker; then based on such information, fuzzy optimization approach is chosen. Then GP is used to find optimal solution. The optimal solution is presented to the decision maker. If he/she is satisfied with the solution, stop. Otherwise, another fuzzy technique may be used. We stop when the decision maker is satisfied.

We have not used any software but only pen and paper to compute the optimal solutions by using geometric programming technique. Software available on the market can also be used to find the optimal solution.

We further plan to develop a few interesting results on Cobb-Douglas based firm production model in fuzzy environment. We also plan to use the proposed technique to generate optimal solutions in agro-industrial sector in near future.

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