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Exact Solutions of the Generalized- Zakharov (GZ) Equation by the Infinite Series Method

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Abstract

The infinite series method is an efficient method for obtaining exact solutions of some nonlinear partial differential equations. This method can be applied to nonintegrable equations as well as to integrable ones. In this paper, the direct algebraic method is used to construct new exact solutions of generalized- Zakharov equation.

Keywords: Infinite series method, Generalized- Zakharov equation

MSC (2000) No: 47F05, 35Q53, 35Q55, 35G25

1. Introduction

Investigation of the travelling wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics, fluid dynamics.

In order to better understand these nonlinear phenomena, many mathematicians and physical scientists search for more solutions. In the process several powerful methods have been proposed to obtain exact solutions of nonlinear evolution equations. These include the tanh-sech method

[Malfliet(1992), Khater et al. (2002) and Wazwaz (2006)], the extended tanh method [El-Wakil and Abdou (2007), Fan (2000) and Wazwaz (2005)], the hyperbolic function method [Xia et al. (2001)], the sine-cosine method [Wazwaz (2004), Yusufoglu and Bekir (2006)], the Jacobi elliptic function expansion method [Inc and Ergut (2005)], the F-expansion method [Sheng (2006)], and the direct algebraic method [Hereman et al. (1986)].

The technique we used in this paper is due to Hereman et al. [(1986)]. Here the solutions are developed as series in real exponential functions which physically corresponds to mixing of elementary solutions of the linear part due to nonlinearity. The method of Hereman et al. [(1986)] falls into the category of direct solution methods for nonlinear partial differential equations. This method is currently restricted to traveling wave solutions. In addition, depending on the number of nonlinear terms in the partial differential equation with arbitrary numerical coefficients, it is sometimes necessary to specialize to particular values of the velocity in order to find closed form solutions. On the other hand, the Hereman et al. series method does give a systematic means of developing recursion relations. Hereman et al. direct series method can be used to solve both dissipative and non dissipative equations [Hereman et al. (1986)]. They take solutions of the linear equation to be of the form

$$\exp[-k(c)(x-ct)],$$

where $k(c)$ is a function of the velocity c. The velocity though assumed constant is in general related to the wave amplitude. It is from the solutions of the linear part that the solution of the full nonlinear partial differential equation is synthesized. With wave number k, the dispersion relation $w = k(c)$ gives the angular frequency. Li et al. [(2008)] applied the Exp-function method to obtain exact solutions of generalized Zakharov equation. In this paper we apply the infinite series method for solving the generalized- Zakharov equation.

2. The Infinite Series Method

Consider the nonlinear partial differential equation:

$$F(u, u_t, u_x, u_{xx}, \ldots) = 0, \tag{1}$$

where $u = u(x,t)$ is the solution of the Eq. (1). We use transformations

$$u(x,t) = f(\xi), \qquad \xi = x + \lambda t, \tag{2}$$

where λ is constant. Based on this, we obtain

$$\frac{\partial}{\partial t}(.) = \lambda \frac{\partial}{\partial \xi}(.), \qquad \frac{\partial}{\partial x}(.) = \frac{\partial}{\partial \xi}(.), \qquad \frac{\partial^2}{\partial x^2}(.) = \frac{\partial^2}{\partial \xi^2}(.), \quad \ldots \tag{3}$$

We use (3) to change the nonlinear partial differential equation (1) to nonlinear ordinary differential equation

$$G(f(\xi), \frac{\partial f(\xi)}{\partial \xi}, \frac{\partial^2 f(\xi)}{\partial \xi^2}, \ldots) = 0. \tag{4}$$

Next, we apply the approach of Hereman et al. [(1986)]. We solve the linear terms and then suppose the solution in the form

$$f(\xi) = \sum_{n=1}^{\infty} a_n g^n(\xi), \tag{5}$$

where $g(\xi)$ is a solution of linear terms and the expansion coefficients $a_n (n = 1, 2, \ldots)$ are to be determined. To deal with the nonlinear terms, we need to apply the extension of Cauchy's product rule for multiple series.

Lemma 1. (Extension of Cauchy's product rule). If

$$F^{(i)} = \sum_{n_1=1}^{I} a_{n_1}^{(i)}, \qquad i = 1, \ldots, I, \tag{6}$$

represents I infinite convergent series then

$$\prod_{i=1}^{I} F^{(i)} = \sum_{n=1}^{\infty} \sum_{r=I-1}^{n-1} \ldots \sum_{m=2}^{k-1} \sum_{l=1}^{m-1} a_l^{(1)} a_{m-l}^{(2)} \ldots a_{n-r}^{(I)}. \tag{7}$$

Proof:

See [Hereman et al. (1986)].

Substituting (5) into (4) yields recursion relation which gives the values of the coefficients.

3. Generalized-Zakharov Equation

Let us consider the generalized-Zakharov equation [Li et al. (2008), Borhanifar et al. (2009) and Zhou et al. (2004)]:

$$iu_t + u_{xx} - 2a|u|^2 u + 2uv = 0, \tag{8}$$

$$v_{tt} - v_{xx} + (|u|^2)_{xx} = 0. \tag{9}$$

We introduce the transformations

$$u(x,t) = e^{i\theta}U(\xi), \qquad v(x,t) = V(\xi),$$

$$\theta = \alpha x + \beta t, \qquad \xi = x - 2\alpha t, \tag{10}$$

where α and β are real constants. Hence,

$$u_t = (i\beta U(\xi) - 2\alpha \frac{\partial U(\xi)}{\partial \xi})e^{i\theta}, \tag{11}$$

$$u_{xx} = (-\alpha^2 U(\xi) + 2i\alpha \frac{\partial U(\xi)}{\partial \xi} + \frac{\partial^2 U(\xi)}{\partial \xi^2})e^{i\theta}, \tag{12}$$

$$v_{tt} = 4\alpha^2 \frac{\partial^2 V(\xi)}{\partial \xi^2}, \qquad v_{xx} = \frac{\partial^2 V(\xi)}{\partial \xi^2}. \tag{13}$$

Substituting (10) into Equations (8)-(9), and using (11)-(13), we have the ordinary differential equations (ODEs) for $U(\xi)$ and $V(\xi)$

$$-(\beta + \alpha^2)U(\xi) + U''(\xi) - 2aU^3(\xi) + 2U(\xi)V(\xi) = 0, \tag{14}$$

$$(4\alpha^2 - 1)V''(\xi) + (U^2(\xi))'' = 0. \tag{15}$$

Integrating (15) twice with respect to ξ, then we have

$$(4\alpha^2 - 1)V(\xi) + U^2(\xi) = C,$$

where C is second integration constant and the first one is taken to zero. Rewrite this equation as follows

$$V(\xi) = \frac{C - U^2(\xi)}{4\alpha^2 - 1}. \tag{16}$$

Substituting the (16) into (14) yields

$$(\beta+\alpha^2-\frac{2C}{4\alpha^2-1})U(\xi)-U''(\xi)+2(a+\frac{1}{4\alpha^2-1})U^3(\xi)=0. \tag{17}$$

The linear equation from (17) has the solution in the form

$$g(\xi)=\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi).$$

Thus, we look for the solution of (17) in the form

$$U(\xi)=\sum_{n=1}^{\infty}a_n\exp(n\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi). \tag{18}$$

Substituting (18) into (17) and by using Lemma 1, we obtain the recursion relation follows a_1 is arbitrary, $a_2=0$, and

$$\frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{4\alpha^2-1}(n^2-1)a_n+2(a+\frac{1}{4\alpha^2-1})\sum_{m=2}^{n-1}\sum_{l=1}^{m-1}a_l a_{m-l}a_{n-m}, \quad n\geq 3. \tag{19}$$

Then by (19), we have

$$\begin{aligned} a_{2d}&=0,\\ a_{2d+1}&=(-1)^d(\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)})^d\frac{a_1^{2d+1}}{2^{3d}}, \quad d=1,2,3,.... \end{aligned} \tag{20}$$

Substituting (20) into (18) gives

$$\begin{aligned} &U(\xi)\\ &=\sum_{d=0}^{\infty}(-1)^d(\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)})^d\frac{a_1^{2d+1}}{2^{3d}}\exp((2d+1)\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi)\\ &=\frac{a_1\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi)}{1+\frac{a_1^2}{8}\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)}\exp(2\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi)}. \end{aligned}$$

By using (16), we get

$$V(\xi)=\frac{C}{4\alpha^2-1}$$
$$-\frac{1}{4\alpha^2-1}\left(\frac{a_1\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi)}{1+\frac{a_1^2}{8}\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)}\exp(2\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}\xi)}\right)^2.$$

In $(x,t)-$ variables, we have the exact soliton solution of the generalized -Zakharov equation in the following form

$$u(x,t)=e^{i(\alpha x+\beta t)}\times$$
$$\frac{a_1\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))}{1+\frac{a_1^2}{8}\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)}\exp(2\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))},\tag{21}$$

$$v(x,t)=\frac{C}{4\alpha^2-1}-\frac{1}{4\alpha^2-1}\times$$
$$\left(\frac{a_1\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))}{1+\frac{a_1^2}{8}\frac{2+2a(4\alpha^2-1)}{2C-(4\alpha^2-1)(\beta+\alpha^2)}\exp(2\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))}\right)^2.\tag{22}$$

In (21) and (22) if we choose $a_1=\pm 2\sqrt{\frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{1+a(4\alpha^2-1)}}$, then

$$u(x,t)$$
$$=\pm e^{i(\alpha x+\beta t)}\sqrt{\frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{1+a(4\alpha^2-1)}}\frac{2\exp(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))}{1+\exp(2\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t))}$$
$$=\pm e^{i(\alpha x+\beta t)}\sqrt{\frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{1+a(4\alpha^2-1)}}\sec h(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t)).$$

Since

$$\sec h\phi = \frac{1}{\cosh\phi} = \frac{2}{e^{\phi} + e^{-\phi}} = \frac{2e^{\phi}}{1+e^{2\phi}}.$$

Thus, the exact solution of the generalized- Zakharov equation can be expressed as:

$$u(x,t) = \pm e^{i(\alpha x+\beta t)} \sqrt{\frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{1+a(4\alpha^2-1)}} \sec h(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t)),$$

$$v(x,t) = \frac{C}{4\alpha^2-1} - \frac{2C-(\beta+\alpha^2)(4\alpha^2-1)}{(1+a(4\alpha^2-1))(4\alpha^2-1)} \sec h^2(\sqrt{\frac{(\beta+\alpha^2)(4\alpha^2-1)-2C}{4\alpha^2-1}}(x-2\alpha t)).$$

4. Conclusion

The infinite series method has been successfully applied in solving the generalized-Zakharov equation. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas as well.

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