



## Kaluza-Klein Type Cosmological Model of the Universe with Inhomogeneous Equation of State

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### Abstract

In this paper we study Kaluza-Klein type cosmological model of the universe filled with an ideal fluid obeying an inhomogeneous equation of state depending on time. It is shown that there appears a quasi-periodic universe, which repeats the cycles of phantom type space acceleration.

**Keywords:** Cosmology; Kaluza-Klein theory; time dependent EOS

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### 1. Introduction

Kaluza-Klein (KK) theories have shown how gravity and electromagnetism can be unified from Einstein's field equations generalized to five dimensions. The idea has been later extended to include other types of interaction in  $4 + d$  dimensional models where the isometrics of the spontaneously compactified  $d$  space-like dimensions account for the gauge symmetries of the effective four dimensional  $4D$  theory [Wetterich (1982); Salam and Strathdee (1982)]. It is generally known that in KK theories with spontaneous dimensional reduction, a very large cosmological constant in four dimensions almost always arises, whereas from observational data we note that its value at present is too small  $\Lambda \sim 10^{-56} \text{cm}^{-2}$ . However, for non-compact internal space one can envision dynamical mechanisms leading to a vanishing  $4D$  cosmological constant [Wetterich (1985)]. It is interesting to point out that geometrically such solutions do not correspond to the topology of direct product of  $4D$  and extra dimensional spaces.

There is a lot of interest in the study of the nature of dark energy (for a review, see [Copeland et al. (2006); Padmanabhan (2003)]), responsible for the acceleration of the cosmic expansion,

which was initiated not far in the past and which is expected to continue to later times. Among the different possible models that have been considered in the literature, one that has a good probability to react (or at least to conveniently parameterize) what may be going on there, is a model in which dark energy is described by some rather complicated ideal fluid with an unusual equation of state (EOS). Very general dark fluid models can be described by means of an inhomogeneous equation of state [Nojiri and Odintsov (2005, 2006)]. Some particular examples of such kinds of equations have been considered in Brevik et al. (2004), Nojiri and Odintsov (2006), Brevik and Gorbunova (2005), Brevik et al. (2007), Ren and Meng (2006), Hu and Meng (2006), Cardone (2006), Elizalde et al. (2005), and Capozziello et al. (2006). Also, some cases of observational consequences of the corresponding generalized dark fluids Capozziello et al. (2006) have been considered. Moreover, a dark energy fluid obeying a time-dependent equation of state of Nojiri and Odintsov (2006), Brevik et al. (2007), Ray et al. (2007), Ray and Mukhopadhyay (2007), Ray et al. (2009), Ghosh et al. (2013), and Mukhopadhyay et al. (2015) may also be successfully used with the purpose of mimicing the classical string landscape picture, which is very interesting in order to establish a connection with a different fundamental approach. It is known as well that a dark fluid satisfying a time dependent equation of state can rather naturally lead to a phantom era [McInnes (2002)], where the phantom field is able to mimic some features of the underlying quantum field theory, Nojiri and Odintsov (2003).

In this study we consider Kaluza-Klein type cosmological model in a class of inhomogeneous universe. We have found the transition of the universe between a phantom and non-phantom phases. In a non-phantom phase, the energy density  $\rho$  decreases while in a phantom phase, it grows up leading to singularities with inhomogeneous time dependent equation of state.

## 2. Model and Field Equations

We consider flat Kaluza-Klein type cosmological model of the form

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] + d\psi^2, \quad (1)$$

where  $R(t)$  is the scale factor.

The Einstein field equations are given as

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -8\pi GT_{ij}, \quad (2)$$

the energy momentum tensor for matter source is given by

$$T_{ij} = (p + \rho)\mu_i\mu_j - pg_{ij},$$

where  $\rho$  and  $p$  are the energy density and pressure.

Einstein field Equation (2) for the model Equation (1) are given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{\chi^2\rho}{6}, \quad (3)$$

$$\frac{\dot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 = \frac{\chi^2 p}{3}, \quad (4)$$

where the dot (.) denotes differentiation with respect to the proper time  $t$  and  $\chi = 8\pi G$  is the gravitational constant. From Equation (3) we get

$$\frac{6}{\chi^2} H^2 = \rho, \quad (5)$$

where  $H = \frac{\dot{R}}{R}$  the Hubble parameter. The energy conservation law gives

$$\dot{\rho} + 4H(P + \rho) = 0. \quad (6)$$

### 3. Inhomogeneous EOS for the Universe and its Solutions

Let us assume that the universe is filled with an ideal fluid (dark energy) obeying an inhomogeneous equation of state depending on time of the form

$$p = \omega(t)\rho + f(\rho) + \Lambda(t), \quad (7)$$

where  $\omega(t)$  and  $\Lambda(t)$  depend on the time  $t$  and  $f(\rho)$  is an arbitrary function, in the general case. Using Equation (5) and Equation (7), the conservation Equation (6) becomes

$$\dot{\rho} + \frac{4\chi}{\sqrt{6}} \left[ (1 + \omega(t))\rho^{\frac{3}{2}} + f(\rho)\rho^{\frac{1}{2}} + \Lambda(t)\rho^{\frac{1}{2}} \right] = 0. \quad (8)$$

We solve the above equation for the following three cases:

**3.1. Case (i):**  $\omega(t) = a_1(t) + b$ ,

**3.2. Case (ii):**  $\omega(t) = \left(\frac{t^n}{\tau^n} - 1\right)$ , and

**3.3. Case (iii):**  $\omega = \omega(t)$ , respectively, where  $a_1$ ,  $b$  and  $\tau$  are constants.

In all the above three cases we assume that  $f(\rho) = A\rho$ , where  $A$  is constant and in Case (i) and Case (ii) we assume that  $\Lambda(t) \propto H^2(t)$  and in Case (iii)  $\Lambda = \Lambda(t)$ .

For the case **3.1. Case (i):** For  $\omega(t) = a_1(t) + b$ , we solve Equation (8) for the following three different cases: case (a):  $f(\rho) = \Lambda(t) = 0$ , case (b):  $\Lambda(t) = 0$  and  $f(\rho) \neq 0$ , case (c):  $\Lambda(t)$  and  $f(\rho) \neq 0$ .

**3.1.1 Case (a):**  $f(\rho) = \Lambda(t) = 0$

In this case from Equation (8), the energy density takes the form

$$\rho(t) = \frac{6a_1^2}{\chi^2} \frac{1}{[(a_1 t + 1 + b)^2 - S_1^2]^2}. \quad (9)$$

Hubble's parameter becomes

$$H(t) = \frac{a_1}{(a_1 t + 1 + b)^2 - S_1^2}, \quad (10)$$

where  $S_1$  is the constant of integration. The time derivative of  $H(t)$  becomes

$$\dot{H}(t) = - \left[ \frac{2a_1^2(a_1 t + 1 + b)}{[(a_1 t + 1 + b)^2 - S_1^2]^2} \right], \quad (11)$$

and, hence, the scale factor takes the following form

$$R(t) = e \left( \frac{a_1 t + b + 1 - S_1}{a_1 t + b + 1 + S_1} \right)^{\frac{1}{2S_1}}, \quad (12)$$

where  $e$  is an integration constant. From Equation (11), it is observed that the time derivative of  $H(t)$  is zero when  $t = t_1 = \frac{-(b+1)}{a_1}$ . If  $a_1 > 0$ ,  $b > -1$  and  $t < t_1$ , then  $\dot{H} > 0$ ; that is, the universe is accelerating or the universe is in the phantom phase and if  $t > t_1$ , one gets  $\dot{H} < 0$  and correspondingly, a decreasing universe i.e. the universe is in a non-phantom phase. There is a transition from phantom epoch to a non-phantom one. At the moment when the universe passes from a phantom to a non-phantom era, Hubble's parameter equals

$$H = \frac{-a_1}{S_1^2}. \quad (13)$$

### 3.1.2. Case (b): $\Lambda(t) = 0$ and $f(\rho) \neq 0$

In this case from Equation (8), we get

$$\rho(t) = \frac{6a_1^2}{\chi^2} \frac{1}{[(a_1 t + 1 + b + A)^2 - S_2^2]^2}, \quad (14)$$

where  $S_2$  is the constant of integration. Hubble's parameter becomes

$$H(t) = \frac{a_1}{(a_1 t + 1 + b + A)^2 - S_2^2}. \quad (15)$$

The time derivative of  $H(t)$  becomes

$$\dot{H}(t) = - \left[ \frac{2a_1^2(a_1 t + 1 + b + A)}{[(a_1 t + 1 + b + A)^2 - S_2^2]^2} \right]. \quad (16)$$

From Equation (15), after integration the scale factor takes the following form

$$R(t) = e^{\left(\frac{a_1 t + b + 1 + A - S_2}{a_1 t + b + 1 + A + S_2}\right)^{\frac{1}{2S_2}}}. \quad (17)$$

It is observed from Equation (16) that the time derivative of  $H(t)$  is zero when

$$t = t_1 = \frac{-(b+1+A)}{a_1}.$$

If

$$a_1 > 0, \quad b > -(1 + A) \text{ and } t < t_1, \text{ then } \dot{H} > 0;$$

that is, the universe is accelerating or the universe is in a phantom phase and if  $t > t_1$ , one gets  $\dot{H} < 0$  and correspondingly, a decreasing universe i.e. the universe is in a non-phantom phase. There is a transition from phantom epoch to a non-phantom one. At the moment when the universe passes from a phantom to a non-phantom state, again the Hubble's parameter equals

$$H = \frac{-a_1}{s_2^2} \quad (18)$$

### 3.1.3. Case (c): $\Lambda(t) \neq 0$ and $f(\rho) \neq 0$ ,

In this case from Equation (8), the energy density takes the form

$$\rho(t) = \frac{6a_1^2}{\chi^2} \frac{1}{[(a_1 t + \beta)^2 - S_3^2]^2}, \quad (19)$$

where  $\beta = 1 + b + A + \gamma \frac{\chi^2}{6}$  and  $S_3$  is an integrating constant. Hubble's parameter becomes

$$H(t) = \frac{a_1}{(a_1 t + \beta)^2 - S_3^2}. \quad (20)$$

The time derivative of  $H(t)$  gives

$$\dot{H}(t) = - \left[ \frac{2a_1^2(a_1 t + \beta)}{[(a_1 t + \beta)^2 - S_3^2]^2} \right]. \quad (21)$$

Again, from Equation (20) the scale factor takes the following form

$$R(t) = e^{\left(\frac{a_1 t + \beta - S_3}{a_1 t + \beta + S_3}\right)^{\frac{1}{2S_3}}}. \quad (22)$$

From Equation (21) it is again observed that the time derivative of  $H(t)$  is zero when

$$t = t_1 = \frac{-\beta}{a_1}.$$

If

$$a_1 > 0, b > -(1 + A + \gamma \frac{\chi^2}{6}) \text{ and } t < t_1,$$

then  $\dot{H} > 0$ . that is, the universe is accelerating or the universe is in a phantom phase and if  $t > t_1$ , one gets  $\dot{H} < 0$  and, correspondingly a decreasing universe i.e. the universe is in a non-phantom phase. There is a transition from phantom epoch to a non-phantom one.

At the moment when the universe passes from a phantom to a non-phantom state, Hubble's parameter equals

$$H = \frac{-a_1}{s_3^2} \quad (23)$$

For the **3.2. Case (ii)**:  $\omega(t) = \left(\frac{t^n}{\tau^n} - 1\right)$ , again we assume the above three cases: case (a), case (b) and case (c).

**3.2.1. Case (a)**:  $\Lambda(t) = 0$  and  $f(\rho) = 0$ ,

For case (a), and from Equation (8) the energy density takes the form

$$\rho(t) = \left(\frac{3(n+1)^2 \tau^{2n}}{2\chi^2}\right) \left(\frac{1}{(\tau^{n+1} + S_4)^2}\right). \quad (24)$$

where  $S_4$  is an integrating constant. Hubble's parameter becomes

$$H(t) = \left(\frac{(n+1)\tau^n}{2}\right) \left(\frac{1}{\tau^{n+1} + S_4}\right). \quad (25)$$

The time derivative of  $H(t)$  gives

$$\dot{H}(t) = -\left(\frac{\tau^n(n+1)^2 t^n}{(\tau^{n+1} + S_4)^2}\right), \quad (26)$$

and the scale factor from Equation (25) takes the following form

$$R(t) = \exp\left(\frac{(n+1)\tau^n}{2} I\right), \quad (27)$$

where

$$I = \frac{t}{S_4} \text{ hypergeometric } {}_2F_1 \left[ \frac{1}{1+n}, 1, 1 + \frac{1}{1+n}, \frac{-t^{1+n}}{S_4} \right].$$

From Equation (26) it is observed that the time derivative of  $H(t)$  is zero when  $t = t_1 = 0$ . If  $\tau > 0$ ,  $n$  is an odd number and  $t < t_1$ , then  $\dot{H} > 0$ . The universe is in phantom phase and if  $t < t_1$ , one gets  $\dot{H} > 0$ . That is, the universe is in a non-phantom phase.

At the moment when the universe passes from a phantom to a non-phantom phase, Hubble parameter equals

$$H = \left[ \frac{(n+1)\tau^n}{2S_4} \right]. \tag{28}$$

**3.2.2. Case (b):  $\Lambda(t) = 0$  and  $f(\rho) \neq 0$**

In this case the energy density takes the form

$$\rho(t) = \left[ \frac{3(n+1)^2\tau^{2n}}{2\chi^2} \right] \left[ \frac{1}{(\tau^{n+1}+Bt+S_5)^2} \right], \tag{29}$$

where  $S_5$  is an integrating constant and  $B = A(n + 1)\tau^n$ . Hubble's parameter becomes

$$H(t) = \left( \frac{(n+1)\tau^n}{2} \right) \left( \frac{1}{\tau^{n+1}+Bt+S_5} \right). \tag{30}$$

The time derivative of  $H(t)$  gives

$$\dot{H}(t) = - \left( \frac{\tau^n(n+1)((n+1)t^n+B)}{2(t^{n+1}+Bt+S_5)^2} \right), \tag{31}$$

and by integrating Equation (30), the scale factor  $R(t)$  takes the following form

$$R(t) = \exp \left( \frac{[n+1]\tau^n}{2} I \right), \tag{32}$$

where

$$I = \frac{t}{Bt+S_5} \text{ hypergeometric } {}_2F_1 \left[ 1, \frac{1}{n}, 1 + \frac{1}{n}, \frac{-t^n}{Bt+S_5} \right].$$

The time derivative of  $H(t)$  is zero when  $t = t_1 = \left( \frac{-B}{n+1} \right)^{\frac{1}{n}}$ . If  $\tau > 0$ ,  $B > 0$  and  $t < t_1$ , then  $\dot{H} > 0$ ; that is, the universe is accelerating and if  $t > t_1$  one gets  $\dot{H} < 0$  and a corresponding decreasing universe. There is a transition from phantom epoch to a non-phantom one.

At the moment when the universe passes from a phantom to a non-phantom state, Hubbles parameter equals

$$H(t) = \left[ \frac{(n+1)\tau^n}{2} \right] \left[ \frac{1}{\left( \frac{-B}{n+1} \right)^{\frac{n+1}{n}} + B \left( \frac{-B}{n+1} \right)^{\frac{1}{n}} + S_5} \right]. \tag{33}$$

**3.1.3. Case (c):  $\Lambda(t) \neq 0$  and  $f(\rho) \neq 0$ ,**

In this case the energy density takes the form

$$\rho(t) = \left[ \frac{3(n+1)^2 \tau^{2n}}{2\chi^2} \right] \left[ \frac{1}{(\tau^{n+1} + Dt + S_6)^2} \right], \tag{34}$$

where  $S_6$  is an integrating constant and  $D = (A + \gamma \frac{\chi^2}{6})(n + 1)\tau^n$ . Hubble's parameter becomes

$$H(t) = \left( \frac{(n+1)\tau^n}{2} \right) \left( \frac{1}{\tau^{n+1} + Dt + S_6} \right), \tag{35}$$

The time derivative of  $H(t)$  becomes

$$\dot{H}(t) = - \left[ \frac{\tau^n(n+1)((n+1)t^n + D)}{2(t^{n+1} + Dt + S_6)^2} \right], \tag{36}$$

and the scale factor takes the following form

$$R(t) = \exp \left( \frac{[n+1]\tau^n}{2} I \right), \tag{37}$$

where

$$I = \frac{t}{Dt + S_6} \text{ hypergeometric } {}_2F_1 \left[ 1, \frac{1}{n}, 1 + \frac{1}{n}, \frac{-t^n}{Dt + S_6} \right].$$

The time derivative of  $H(t)$  is zero when  $t = t_1 = \left( \frac{-D}{n+1} \right)^{\frac{1}{n}}$ . If  $\tau > 0$ ,  $D > 0$  and  $t < t_1$ , then

$$\dot{H} > 0 \text{ and if } t > t_1, \text{ one gets } \dot{H} < 0;$$

that is, a transition from phantom epoch to a non-phantom one.

At the moment when the universe passes from a phantom to a non-phantom state, Hubbles parameter equals

$$H = \left[ \frac{(n+1)\tau^n}{2 \left( \frac{-D}{n+1} \right)^{\frac{n+1}{n}} + D \left( \frac{-D}{n+1} \right)^{\frac{1}{n}} + S_6} \right]. \tag{38}$$

**3.3. Case (iii):  $\omega = \omega(t)$  and  $\Lambda = \Lambda(t)$**

From the field equations, Equation (3), Equation (4) and with the help of inhomogeneous equation of state Equation (7), we get

$$R\ddot{R} + [1 + 2A + 2\omega(t)]\dot{R}^2 + \frac{\chi^2}{3} \Lambda(t)R^2 = 0. \tag{39}$$

Now to solve Equation (39), we introduce the new variable  $S(t)$  as



$$R = R_0 e^{\int \frac{1}{2(1+A+\omega(t))S} dt} \quad (40)$$

Inserting Equation (40) into Equation (39) we get

$$\ddot{S} - \left[ \frac{\dot{\omega}}{(A+1+\omega)} \right] \dot{S} + \left[ \frac{2(A+1+\omega)\chi^2}{3} \right] \Lambda(t)S = 0. \quad (41)$$

This equation can be identified with

$$\ddot{S} + S^n \dot{S} + \left[ \frac{1}{(n+2)^2} \right] S^{2n+1} = 0 \quad (\text{for } n \neq -2), \quad (42)$$

which is reduced to a linear differential equation by making the substitution [Chimento (1997)]

$$S^n = \left( \frac{n+2}{n} \right) \frac{v^n(t)}{\int v^n(t) dt}, \quad (43)$$

obtaining

$$\ddot{v}(t) = 0 \Rightarrow v(t) = c_1 + c_2 t, \quad (44)$$

where  $c_1$  and  $c_2$  are arbitrary integration constants.

Without loss of generality we choose  $c_1 = -t_0$ , where  $t_0$  is some initial time and  $c_2 = 1$ . With the help of Equation (44) in Equation (43), the general solution of the nonlinear equation (Equation (42)) is found to be

$$S(t) = \left[ \frac{(n+1)(n+2)(t-t_0)^n}{n[C+(t-t_0)^{n+1}]} \right]^{\frac{1}{n}}, \quad (45)$$

where  $C$  is an arbitrary integration constant. Equation (41) and Equation (42) are the same if we define

$$\left[ \frac{-\dot{\omega}}{(A+1+\omega)} \right] = S^n \quad (46)$$

and

$$\left[ \frac{2(A+1+\omega)\chi^2}{3} \right] \Lambda(t) = \frac{1}{(n+2)^2} S^{2n}. \quad (47)$$

By using the value of  $S(t)$  from Equation (45) in Equation (46), after integration we get

$$\omega(t) = \gamma_0 \left[ 1 + \frac{(t-t_0)^{n+1}}{c} \right]^{\frac{-(n+2)}{n}} - (1+A), \quad (48)$$

where  $\gamma_0$  is an arbitrary integration constant.

Again, by using the value of  $S(t)$  from Equation (45) and with the help Equation (48) from Equation (47) we get

$$\Lambda(t) = \frac{3(n+1)^2}{2\chi^2 n^2 c^2 \gamma_0} (t - t_0)^{2n} \left[ 1 + \frac{(t-t_0)^{n+1}}{c} \right]^{\frac{2-n}{n}}. \quad (49)$$

By using the above value of  $S(t)$  and  $\omega(t)$ , the scale factor  $R(t)$  from Equation (40) is given by

$$R(t) = R_0 \exp \left\{ \frac{1}{2\gamma_0} \int \frac{dt}{(t-t_0) \left[ 1 + \frac{(t-t_0)^{n+1}}{c} \right]^{\frac{-(2+n)}{n}}} - E \int \frac{(t-t_0)^n dt}{\left[ 1 + \frac{(t-t_0)^{n+1}}{c} \right]^{\frac{-2}{n}}} \right\}, \quad (50)$$

where  $E = \frac{n+1}{2nc\gamma_0}$ .

Now, taking into account that at late time,  $t \gg t_0$ , we must have  $\omega(t) \rightarrow \gamma_0 - (1 + A)$  for the restriction  $n < -1$  readily follows as can be seen from Equation (48). In addition,  $\Lambda(t)$  vanishes in the same limit. Now, using this approximation we evaluate  $R(t)$  in Equation (40), finding

$$R(t) \approx R_0 \left[ \frac{(n+1)(n+2)}{n} (t - t_0) \right]^{\frac{1}{2\gamma_0}}, \quad (51)$$

This solution has a singularity at  $t = t_0$ . By using Equation (51), Hubble's parameter becomes

$$H(t) \approx \frac{1}{2\gamma_0(t-t_0)}. \quad (52)$$

At  $t = t_1 \rightarrow \infty$ , one has  $H(t) = 0$ .

## 4. Conclusions

In this paper we have studied a Kaluza-Klein type flat model of the universe with inhomogeneous equation of state for different values of parameter  $\omega(t)$ . We have also studied the transition of the universe between a phantom and non-phantom phases. We focus our attention to the variable cosmological constant which has been introduced in the EOS, which becomes inhomogeneous. Then, we solve the equation of motion which lead to the explicit expression of the energy density and consequently to that of the Hubble parameter. The first derivative of the Hubble parameter played a crucial role in this analysis since it allowed us to know which time interval corresponds to a phantom or non-phantom universe. The inhomogeneous form in equation of state helps to realize such a transition in a more natural way.

For the value of  $\omega(t) = a_1 t + b$ , in all three cases i.e. Case (a), Case (b) and Case (c) there occur a passage from a non-phantom era of the universe to a phantom era, resulting in an expansion and a possible appearance of singularities. When the universe passes from a phantom

to a non-phantom era, Hubbles parameter obtained in the Equation (13), Equation (18) and Equation (23) respectively.

For the Case (ii):  $\omega(t) = \frac{t^n}{\tau^n} - 1$ , when the universe passes from a phantom to a non-phantom era, again obtained Hubbles parameter in the equations Equation (28), Equation (33) and Equation (38) respectively.

In Case (i) and Case (ii) it is also observed that, in a phantom phase  $\dot{\rho} > 0$ , the energy density grows and the universe is expanding in a non-phantom phase,  $\dot{\rho} < 0$ , the energy density decreases. However, if the derivative of the scale factor  $\dot{R} > 0$ , the universe expands. Note that in a phantom phase the entropy may become negative. If  $t \rightarrow +\infty$ , then  $H(t) \rightarrow 0$  and  $\rho(t) \rightarrow 0$  so that a phantom energy decreases in both the cases. It is also shown that the universe is located in phantom (non-phantom) phase corresponding to  $\dot{H} > 0$  ( $\dot{H} < 0$ ). We note that  $\dot{H} > 0$  that is universe accelerating and ( $\dot{H} < 0$ ), corresponds to decelerating universe.

For Case (iii)  $\omega = \omega(t)$  and  $\Lambda = \Lambda(t)$ , we observed from the scale factor Equation (51) that it has singularity at  $t = t_0$  i.e. big bang singularity. At the late time  $t \gg t_0$ , we must have  $\omega(t) \rightarrow \gamma_0 - (1 + A)$  for the restriction  $n < -1$  radially follows the Equation (48). From the Equation (49) it is observed that at  $t \rightarrow t_0$ ,  $\Lambda \rightarrow 0$ .

Thus, the universe filled with an inhomogeneous time dependent equation of state ideal fluid may currently in the acceleration epoch of quintessence or phantom type.

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